USING SET MODEL FOR LEARNING ADDITION OF INTEGERS

Umi Puji Lestari[1], Ratu Ilma Indra Putri[2], Yusuf Hartono[2]

[1]Yogyakarta State University, Jl. Colombo No.1, Yogyakarta, Indonesia
[2] Sriwijaya University, Jalan Padang Selasa No. 524, Palembang-30139, Indonesia
e-mail: umipujilestari86@gmail.com

Abstract

This study aims to investigate how set model can help students' understanding of addition of integers in fourth grade. The study has been carried out to 23 students and a teacher of IVC SD Iba Palembang in January 2015. This study is a design research that also promotes PMRI as the underlying design context and activity. Results showed that the use of set models that is packaged in activity of recording of financial transactions in two color chips and card game can help students to understand the concept of zero pair, addition with the same colored chips, and cancellation strategy.

Keywords: addition of integers, set models, PMRI, design research

Abstrak

Penelitian ini bertujuan untuk mengetahui bagaimana *set model* dapat membantu pemahaman siswa tentang penjumlahan bilangan bulat di kelas IV SD. Penelitian dilaksanakan kepada siswa dan guru kelas IVC SD Iba Palembang pada bulan Januari 2015. Metode yang digunakan adalah *design research*. PMRI mendasari desain konteks dan aktivitas. Hasil penelitian menunjukkan penggunaan set model yang dikemas dalam aktivitas mencatat transaksi keuangan dalam keping dua warna dan permainan *remi bilbul* dapat membantu siswa memahami konsep *zero pair*, penambahan dengan keping sewarna, dan *cancellation strategy*.

Kata Kunci: penjumlahan bilangan bulat, *set model*, PMRI, *design research*

Integer is a very important topic for students to learn, due to its usefulness in solving various problems in daily life (Musser, Burger, & Peterson, 2005: 321) as well as a prerequisite material for studying other concepts, like cartesian coordinates (Van de Walle, Karp, & Bay-Williams, 2008: 426) and algebra (Sheffield & Cruikshank, 1996 in Musser, Burger, & Peterson, 2005: 321). However, studies showed that many students still consider this topic difficult. They are often confused to determine the greater numbers and to determine the direction of movement when performing arithmetic operations (Van de Walle, Karp, and Bay-Williams, 2008). This often confused teachers especially to make the learning of integers joyful and meaningful (Putri, 2011).

In order to design learning of addition of integers,two aspects are considered fundamental, such as, historical aspects and learning aspects. Historically, integers has been used by Chinese since 200 BC (Musser, Burger, & Peterson, 2005: 319) in financial transactions using different wood colors (Merzbach & Boyer, 2011: 180). Negative number as part of integers in this case is defined as a number that should be reduced from the quantity or amount of unpaid (Purnomo, 2014).

Having a look from learning aspects, knowledge will be meaningful for students if the learning process is carried out in a context (CORD, 1999). According to Putri (Putri, 2015), in learning using context the students would not learn formulas directly. A situation that can be used as a context to learn the integers is quantity, for example, in golf game and debt credit (Van de Walle, Karp, and Bay-Williams, 2008). A study of Mukhopadhyay et. al (in Menon & Gyan, 2012) showed positive responds of students when being presented the topic of integers in the context of assets and debts.

Contexts for primary school children can be packaged in educative games, because basically primary school students love to play (Somakim, 2008; Fosnot & Dolk, 2001). During the lesson, games proves to be very useful for the development of a second world of mental models (Kaune, et al, 2012). A variety of games has been used in some studies, such as Kaune, et al. (2013) who used domino games for enhancing sustainability of year 7 maths classes in Indonesia, Muslimin, Putri, & Somakim (2012) who used "congklak" game for learning substraction of integers in grade fourth, and Prahmana, Zulkardi, & Hartono (2012) who used Indonesian traditional games for learning multiplication.

The use of model is also needed in mathematics learning, especially to support the process of progressive mathematisation (Wijaya, 2012). In this occasion, a model that is appropriate for quantity context is two colours chips models (Van de Walle, Karp, and Bay-Williams, 2008: 479) or a set model (Musser, Burger, & Peterson, 2005).

> *"In a set model, chips can be used to represent integers. However, two colors of chips must be used, one color to represent positive integers (black) and a second to represent negative integers (red). One black chip represents a credit of 1 and one red chip represents a debit of 1. Thus one black chip and one red chip cancel each other, or "make a zero" so they are called a zero pair (Musser, Burger, & Peterson, 2005).*

The use of set models can enhance at least two strategies when adding integers, such as, cancellation strategy (Linchevski & Williams, 1999) and addition with same coloured chips (Van de Walle, Karp, and Bay-Williams, 2008). The cancellation strategy is observable when students cancel or take different colors of chips that are equal and forms a zero pair.

There are two main concepts embedded in this study, that is, quantity and opposite (Van de Walle, Karp, and Bay-Williams, 2008). The quantity concept is modeled by a number of chips, while the opposite concept is showed in the use of different colours (Van de Walle, Karp, and Bay-Williams, 2008).

Learning should use scientific approaches and is problem based. It should be students centered instead of teacher centered. Students are demanded to observe from their environments, and are encouraged to ask and answer questions, do experiments, give reasonings, and communicate their mathematical ideas (Permendikbud No. 81A in 2013).

An approach in mathematics that is suitable with the historical and learning aspects is Indonesian Realistic Mathematics Education (PMRI). PMRI emphasizes the significance of

mathematical concepts (Wijaya, 2012: 20). This approach is based on Hans Freudenthal's view that "mathematics is a human activity" where students are not passive recipients. The students need to be have opportunities to reinvent mathematical concepts through activities that they have experienced themselves (Zulkardi, 2002: 29). This approach has 5 main characteristics, such as, the use of context, the use of models, students' creations and contributions, interactivity, and intertwining.

Based on explanation above, this study aims to investigate how set model can help students' understanding of addition of integers in fourth grade.

METHOD

Subjects of this study consisted of 23 students and a teacher of IVC SD Iba Palembang. This study implemented in January 2015. The method of this study is design research that consists of three main steps, such as: preparing for the experiment, design experiment, and retrospective analysis (Gravemeijer and Cobb, 2006: 19). In the preparing for the experiment, researchers conducted reviewed some literatures and designed a Hypothetical Learning trajectory (HLT), which consists of learning objectives, learning activities, and conjecture of students' thinking. In the design experiment, the researchers tested HLT in 2 phases, pilot experiment and teaching experiment. During the design experiment, data were collected through interviews, observation, testing, documentation, and field notes. Afterwards, the collected data were analyzed in a retrospective analysis by the HLT as a reference.

Researchers developed 5 instruction activities that were implemented in design experiment. At the present study, we just focus on two activities (Activity 3 and 4) that conducted in teaching experiment. In activity 3, students record financial transactions in a two colored chips. In activity 4, students played card game.

Hypothetical Learning Trajectory (HLT)

1. Activity of Recording Financial Transactions in a Two Colored Chips

There are 2 problems in this activity. First, students should record financial transactions of 5 members of Entong's family (mother, uncle, aunt, sister, and brother) in a two colored chips. Second, students should represent the end of financial condition of member of Entong's family after they pay debts in a two colored chips then explain how they determine it.

a. Learning Objectives

Learning objective of this activity are :

- Students can represent the amount of money into positive chips (blue) and the amount of debt into negative chips (red)
- Students can understand the concept of addition of integers by financial transaction

b. Conjecture of Students' Thinking

Problem 1

- Students stick 4 blue chips and 3 blue chips for representing mother's financial transaction
- Students stick 3 red chips and 5 blue chips for representing uncle's financial transactions
- Students stick 4 red chips and 3 red chips for representing aunt's financial transactions
- Students stick 3 red chips and 3 blue chips for representing sister's financial transactions
- Students stick 1 blue chip and 4 red chips for representing brother's financial transaction

Problem 2

- Students represent the end of mother's financial condition in 7 blue chips. They explain contextually (4 moneys plus 3 moneys is equal to 7 moneys), using a model (4 blue chips + 3 blue chips = 7 blue chips), or formally $(4 + 3 = 7)$.
- Students represent the end of uncle's financial condition in 2 blue chips. They explain contextually (5 moneys minus 3 moneys to pay the debt), using model (5 blue chips + 3 red chips = 2 blue chips or cancel every pair of blue and red), or formally ($5 + (-3) = 2$ or $5 - 3 = 2$).
- Students represent the end of aunt's financial condition in 7 red chips. They explain contextually (4 debts plus 3 debts), using a model (4 red chips + 3 red chips = 7 red chips), or formally $(-4 + (-3) = -7)$
- Students represent the end of sister's financial condition in the blanks for stating a value of zero. They explain contextually (3 moneys is used to pay 3 debts), using a model (3 blue chips + 3 red chips = 0 or cancel every pair of blue and red), or formally ($3 + (-3) = 0$ or $3 - 3 = 0$)
- Students represent the end of brother's financial condition in 3 blue chips. They explain contextually, using a model, or formally $(1 + (-2) = -1)$

2. Activity of Playing Card Game

In this activity, students play card game. There are 2 kinds of card, blue chips card and red chips card (Figure 1). Blue chips card represents the amount of money, while red chips card represents the amount of debt. When playing card game, students take 2 cards then determine the result of the addition of first card and second card.

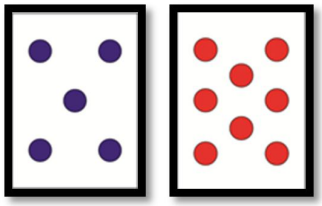


Figure 1. Blue and Red Chips Card

a. Learning Objectives

Learning objective of this activity is students can determine the result of addition of integers using set model.

b. Conjecture of Students' Thinking

- When getting blue chips on the first card and blue chip on the second card, students will add them by adding a lot of blue chips on the first card and a lot of blue chip on second card. Then the students determine the result as a positive number.
- When getting red chip on first card and red chip on second card, students will add them by adding a lot of red chips on first card and a lot of red chip on second card. Then the students determine the result as a negative number.
- When getting blue chips on first card and red chip on second card, students will cancel each different colors pair. Then the students determine the result as a positive number if the remaining chips is blue and negative if the remaining chips is red.
- Students can determine that the player who has a greater than others as the winner of the game

RESULT AND DISCUSSION

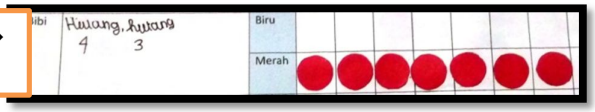
Recording Financial Transactions in a Two Colored Chips

In this activity, the students were working in group to record financial transactions of member of Entong's family (mother, uncle, aunt, sister, and brother) in a two colored chips. In order to solve the problem in this activity, the students sticked the provided chips properly. This is suitable with what is predicted in the HLT. In this occasion, the students' answer to this problem can be observed in Figure 2 as follows:

Mother : Have 4 pieces of five-thousands and 3 pieces of five-

(a)

Aunt: Debts ➔ 4. debts ➔4

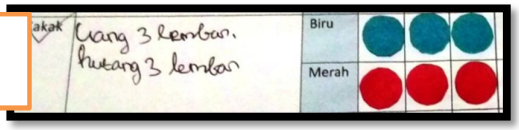


(b)

Uncle: 5 pieces of money, 3 pieces of debts

(c)

Sister: 3 pieces of money, 3 pieces of debts



(d)

Sister: get 1 pieces of five-thousands, 2 pieces of ten-thousands

(e)

Figure 2. Student's Representation for Mother's Financial Transaction (a), Student's Representation for Aunt's Financial Transaction (b), Student's Representation for Uncle's Financial Transaction (c), Student's Representation for Sister's Financial Transaction (d), and Student's Representation for Brother's Financial Transaction (e)

As observable in Figure 2, the students first sticked 4 blue chips and then 3 blue chips to represent mother's financial transaction (Figure 2a), 4 red chips and then 3 red chips to represent aunt's financial transaction (Figure 2b), 5 blue chips and then 3 red chips to represent uncle's financial transaction (Figure 2c), 3 blue chips and then 3 red chips to represent sister's financial transaction (Figure 2d), and 1 blue chip and then 4 red chips to represent brother's financial transaction (Figure 2e).

From students' work, we know that they have been able to represent the amount of money in positive chips (blue) and the amount of debts in negative chips (red). It indicates that they have understood the meaning of positive numbers that can be used to represent the amount of someone's money and the meaning of negative numbers that can be used to represent the amount of debts to be paid.

An interview with a student, given in the following transcript, shows the students' strategies and thinking.

Dialogue 1

1.	Researcher	:"How many red chips that aunts have?"
2.	Student	:"7."
3.	Researcher	:"Why 7?"
4.	Alisa	**:"Because aunt has debt 4 pieces of five-thousands and 3 pieces of five-thousands. And if we add them, we get 7. "**
5.	Researcher	:"How if this? (brother's financial transaction)."
6.	Alisa	:"Red one?"
7.	Researcher	:"Red and blue. Why the number of blue is 1 and red is 4?"
8.	Alisa	:"Because brother gets 1 piece of five-thousands. For red one, because in ten-thousands there are 4 pieces of five-thousands."
9.	Researcher	:"How?"
10.	Alisa	:"Eh, **in twenty-thousands there are 4 pieces of five-thousands**."
11.	Researcher	:"Where is twenty-thousands from?"
12.	Alisa	:"**Twenty-thousands is from adding 10 and 10**."

Based on line 4 and 12 from Dialogue 1, we know that students have understood that addition is adding of a similar type set. A debt plus a debt will result in more debts. Beside that, based on line 10, students have intertwined addition, multiplication, and divition. They know that there are 4 pieces of five-thousands in twenty-thousands. It is suitable with one of the PMRI's characteristic, intertwinement.

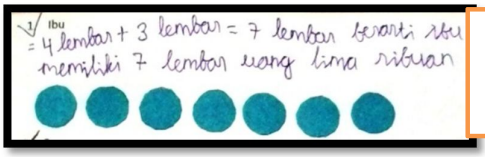
In the next problem, students should represent the end of financial condition of member of Entong's family after they pay debts in a two colored chips then explain how they determine it. First, students met difficulties to solve that problems. This is the interview which shows how the students able to solve the problem after making dialogue among their friends and teacher.

Dialogue 2

1. Teacher :"Do you understand?"
   Students are smiling and thinking.
2. Teacher :"In mother's transaction. Fixed or changed?"
3. Students :"Fixed."
4. Teacher :"So, you have to stick it as many as it because (mother's transaction is) fixed.
   How about uncle? **He has 5 pieces of money and 3 debts.**
5. Alisa **:"The remain is 2."**
6. Teacher :"So, how many chips will be sticked?"
7. Students :"2."
8. Teacher :"Good… The next is same."
9. Students :"O…."
10. Kayla :"How about this?"
11. Teacher :"Where is it?"
    Students showed brother's transaction.
12. Teacher :"Brother, **he has 1 pieces of money and 4 debts**. It means?"
13. Students :"**He still has debts**."
14. Teacher :"How many debts?"
15. Alisa :"20…15…. Eh…."
16. Teacher :"Please, look at the picture."
17. Alisa :"4…. E, 5."
18. Kayla :"What it means?"
19. Teacher :"Please, help them."
20. Alisa :"This is a pieace of five-thousands. He has ten-thousands of debts. Min 4. Eh…
21. Teacher :" Please, help Alisa…. Brother has 1 piece of money and 4 debts. If he want to
    pay the debts, how many debts that he able to pay?"
22. Hafid :"1."
23. Alisa :"Ohya, 1."
24. Teacher :"So, is he still has money or debts?"
25. Students :"**Debts**."
26. Teacher :"How many debts?"
27. Hafid :"**3**."
28. Teacher :"What color of 3 chips?"
29. Students :"**Red**."

Dialogue 2 shows how the teacher guided the students to understand the problems. This situation shows the good interaction among students and teacher. Beside that, it shows that the students has understood that if someone pays the debts, the money and the debts will be decrease. It means students understood zero pair concept contextually.

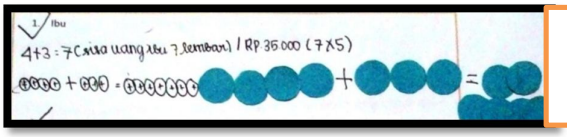
Students represented the end of mother's financial condition in 7 blue chips. In explaining the answer, there are students who explain contextually (Figure 3a), using a chip (Figure 3b), and formally (Figure 3c).



4 pieces + 3 pieces = 7 pieces. It means mother has 7 pieces of five-thousands

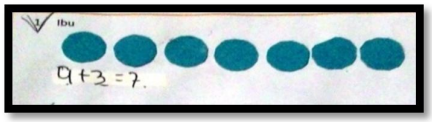
Because mother doesn't has debt

(a)



4+3=7 (the remain of mother's money is 7 pieces) / Rp.35.000 (7 x 5)

(b)



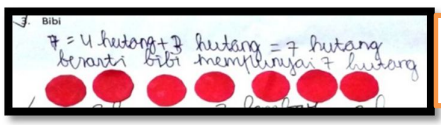
(c)

Figure 3. Context Based Explanation for Representing the End of Mother's Financial Transaction (a), Explanation Using Chips for Representing the End of Mother's Financial Transaction (b), and Formal Explanation for Representing the End of Mother's Financial Transaction (c)

In context based explanation, students explain the answer by connecting in the context of money and debt (Figure 3a). Students determine the end of mothers's financial condition in 7 blue chips that is obtained from 4 moneys plus 3 moneys. In addition, because the mother does not have debts, so the mother does not have to pay any debts. Therefore, mother's money in the end is 7.

In explanation using chips, students explain the answer by drawing positive chips (Figure 3b). Students drew 4 positive chips, and then added 3 positive chips again. Thus, they ended up with 7 positive chips. The other, students explained the answer formally, using mathematical symbols (Figure 3c). They added 4 and 3, and found 7. Commonly, students have understood that addition is adding of a similar type set. In this case, the amount of money owned plus the amount of money owned then the amount will be more and more.

In representing the end of aunt's financial condition, the students sticked 7 red chips. Some students explained their answers contextually (Figure 4a), and some others used chips (Figure 4b).



7 = 4 debts + 3 debts = 7 debts. It means that mother has 7 debts.

(a)

-7 (remain of aunt's debts is 7 pieces of five-thousands).

(b)

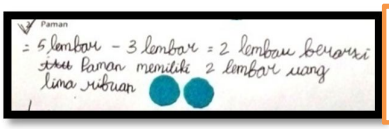
Figure 4. Contextual Explanation for Representing the End of Aunt's Financial Transaction (a), Explanation Using Chips for Representing the End of Aunt's Financial Transaction (b)

As can be observed in Figure 4, in contextual explanation, the students explained their answer by connecting in the context of money and debts (Figure 4a). In this case, the students added 4 debts and 3 debts and found 7 debts altogther. Since the aunt did not have money to pay the debts, the total debt became 7.

In explanation using chips, students explained their answer by drawing negative chips (Figure 4b). In this manner, they drew 4 negative chips, and then add 3 more negative chips. Thus, they had 7 negative chips in the end.

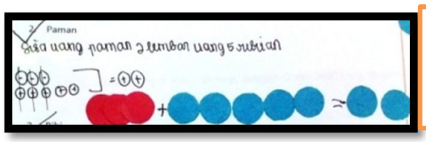
When students represented the aunt's financial transactions, none of them provided a formal explanation. It seemed that the students still require other activities that should lead them to mathematical symbols, especially in addition of two negative numbers.

Afterward, students were asked to determine the uncle's financial condition after he paid his debt with 2 blue chips. In order to determine the amount and the color of chips, some students referred back to context (Figure 5a), some others use chips representation (Figure 5b), while the rest presented a formal explanation (Figure 5c).



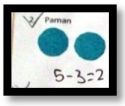
Uncle
5 pieces – 3 pieces = 2 pieces
It means that uncle has 2 pieces of five-thousands

(a)



Uncle
The remain of uncle's money is 2 pieces of

(b)



(c)

Figure 5. Context Based Explanation for Representing The End of Uncle's Financial Transaction (a), Explanation Using Chips for Representing The End of Uncle's Financial Transaction (b), and Formal Explanation for Representing The End of Uncle's Financial Transaction (c)

As can be seen in Figure 5, in contextual based explanation, students explained their answer by connecting the problem with the context of money (Figure 5a). They subtracted 5 moneys that the uncle owned with 3 moneys that he used to pay the debts. So, they found 2 moneys left.

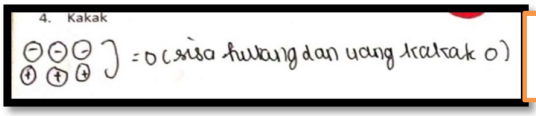
In explanation using chips, the students explained the answer by drawing positive and negative chips. They drew 3 negative chips and then 5 positive chips then canceled every pair of positive and negative chips, since they knew that the value of each pair of positive and negative chips is zero.

In formal explanation, the students explained the answer formally, directly using mathematical symbols. Students subtracted 3 from 5 to obtain 2. In this case, there is no students who represent the financial transactions in the form of addition with negative numbers.

The student then showed the final condition of sister's financial in an empty display. In explaining the answer, some studentsexplained their answers by refrerring to the context(Figure 6a), chips (Figure 6b), and formal (Figure 6c).

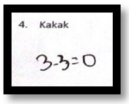
4. Sister ➔ because sister has 3 pieces of debts and she pays 3 pieces. So, the remain is zero

(a)



0 (the remain of debts and sister's money 0)

(b)



(c)

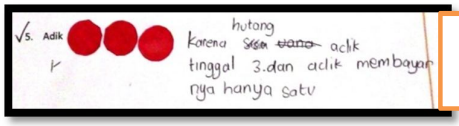
Figure 6. Contextual Explanation for Representing of the End of Sister's Financial Transaction (a), Explanation Using Chips for Representing of the End of Sister's Financial Transaction (b), Formal Explanation for Representing of the End of Sister's Financial Transaction (c)

In contextual explanation, the students explained their answers by bringing back the problem into the context of the debt (Figure 6a). Students determined the final conditions of financial end in zero because three pieces owned by the sister were used to pay debts.

In explanation using chips, the students explained the answer by drawing positive and negative chips. Students draw 3 negative chips, and then 3 positive chips. They have had an idea that the value of positive and negative chips is zero.

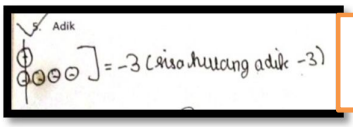
In formal explanation, the students used mathematical symbols to represent theirs. The students substracted 3 from 3, and found zero. In this case, students used substraction operation. In this case, there was no students who write the problem in an addition operation with negative numbers.

Then the students determine that the end of brother's financial condition in 3 red chips. In explaining the answer, there are students who explain contextually (Figure 7a), using chips (Figure 7b), and formally (Figure 7c).



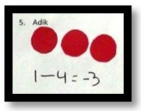
5. Brother
Because brother's debt is 3 and he pays it just 1.

(a)



5. Brother
-3 (the remain of brother's debts is -3)

(b)



(c)

Figure 7. Contextually Explanation for Representing of The End of Brother's Financial Transaction (a), Explanation Using Chips for Representing of The End of Brother's Financial Transaction (b), and Formally Explanation for Representing of The End of Brother's Financial Transaction (c)

In contextual explanation, the student explained it by connecting in the context of the debt. Students determined that the end of brother's financial condition is 3 debts because he could pay 1 debt only.

In explanation using chips, the students explained the answer by drawing positive and negative chips. Students represent it in a positive chip and 3 negative chips then cancel a pair of negative and positive chips. They know that it's value is zero.

In formally explanation, students used mathematical symbols. Students subtracted 4 from 1 to obtain -3. In this case the students chose to use substraction operation. There was no students who wrote the problem in addition operation with negative numbers.

At the end of Activity 1, students presented the results of group discussion. It supports to do classical discussion that is guided by the teacher. It helps the students to deepen their understandings of the material being learned and share strategies with other students.

Playing Card Game

In this activity, the students played card game and then write the results in a table that has been provided. When doing card game, students got blue chips on first card and blue chips on second card (Figure 8).

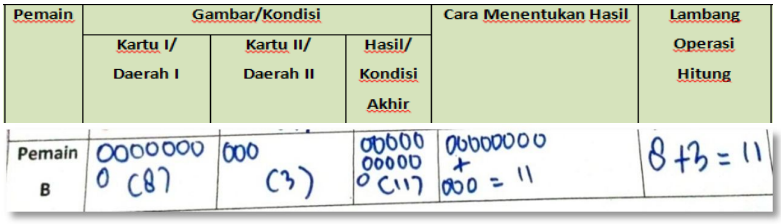


Figure 8. Addition of Blue and Blue Chips

From Figure 8 above is known that students have been able to determine the result of addition of positive chips (blue) and positive chips (blue). Students used the strategy of adding with the same colored chips to solve this problem. In determining the result, students drew blue chips to represent positive integers. In addition, students can also represent the problems in the game in a formal mathematical symbols ($8 + 3 = 11$).

In addition, students also got red chips on first card and red chips on second card (Figure 9).

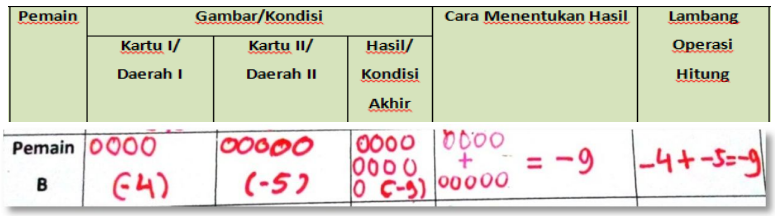


Figure 9. Addition of Red Chips and Red Chips

From Figure 9 above is known that students have been able to determine the result of addition of negative chips (red) and negative chips (red). Students used the strategy of adding with the same colored chips to solve this problem. In determining the result, the students drew red chips to represent negative integers. In addition, students could also represent the problems in a formal mathematical symbols and involved an addition of negative numbers $(-4 + (-5) = -9)$.

The students also got blue chips on first card and red chips on second card (Figure 10).

Figure 10. Addition of Blue Chips and Red Chips

From Figure 10 above is known that students have been able to determine the result of addition of positive chips (blue) and negative chips (red). Students used the cancellation strategy to solve this problem. In determining the result, students drew it in 9 blue chips and 1 red chips then canceled 1 pair of blue and red chip so they got 8 red chips. In addition, students could represent the problems in formal mathematical symbols and included addition of negative numbers $(9 + (-1) = 8)$.

Students also got red chips on the first card and blue chips on the second card (Figure 11).

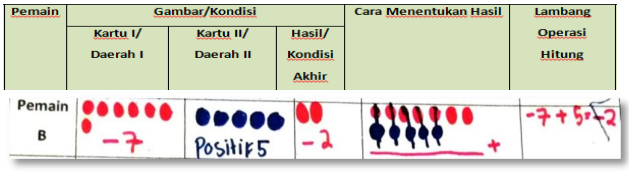


Figure 11. Addition of Red Chips and Blue Chips

From Figure 11 above is known that students have been able to determine the addition of negative chips (red) and positive chips (blue). Students used the cancellation strategy to solve this problem. In determining the result, the student drew 7 red chips and 5 blue chips then canceled 5 pairs of blue and red chips to obtain 2 red chips. In addition, students could also represent the problems in a more formal mathematical symbols $(-7 + 5 = -2)$.

In playing the game, students also determined the winner, that is the player who has the the greatest. For this occasion, the students applied what they have learned previously, which is order of integers.

CONCLUSION AND RECOMMENDATION

Based on the results and discussion that has been described, it can be concluded that the use of set models that are packaged in the activity of recording financial transactions in two color chips can stimulate students to represent the amount of money in positive chips (blue) and the amount of debt to be paid in negative chips (red). Through the activity of paying debt, students began to understand the cancellation strategy and the concept of zero pair. Some students explained their answer by referring back to context, using chips, and employing mathematical notations. Furthermore, the use of set models that are packaged in a card game can also help students in deepening the concept of zero pair, addition with the same colored chips, cancellation strategy, and in representing the problems in a more formal notation.

PMRI approach with 3 principles and 5 characteristics can create a meaningful learning environment for students. Therefore, the researcher recommended the use of this approach both in learning addition of integers and other materials. In addition, because this study only discusses the addition of integers, the researcher suggested to other researchers to design learning of subtraction of integers using set models.

REFERENCES

CORD. (1999). *Teaching Mathematics Contextually*. Waco: CORD.

Fosnot, C.T & Dolk, M. (2001). Young Mathematics at Work. Constructing Number Sense, Addition, and Subtraction. United States of America: NCTM.

Gravemeijer, K & Cobb, J. P. (2006). Design research from a Learning Perspective. In Educational Design research, 7-51. London: Routledge.

Kaune, C., Cohors-Fresenborg, E., Nowinska, E., Marpaung, Y., Handayani, N. (2012). Developing Metacognitive and Discursive Activities in The Indonesian Mathematics Education. Journal on Mathematics Education (IndoMS-JME), 3(1), 1-16. Palembang: IndoMS.

Kaune, C., Nowinska, E., Paetau, A., & Griep, M. (2013). Games for Enhancing Sustainability of Year 7 Maths Classes In Indonesia. Journal on Mathematics Education (IndoMS-JME), 4(2), 129-150. Palembang: IndoMS.

Linchevski, L., & Williams, J. (1999). Using Intuition from Everyday Life in 'Filling' The Gap in Children's Extension of Their Number Concept to Include the Negative Numbers. In Educational Studies in Mathematics, 131-147. Netherlands: Kluwer Academic.

Menon, U., & Gyan, J. (2012). Extending Numbers with Number Sense. Proceeding in The 12th International Congress on Mathematical Education, Topic Study Group 7, 1845-1854. Korea: ICME.

Merzbach, U.C. & Boyer, C.B. (2011). A History of Mathematics. Third Edition. USA: John Wiley & Sons, Inc.

Muslimin, Putri, R.I.I., & Somakim. (2012). Desain Pembelajaran Pengurangan Bilangan Bulat Melalui Permainan Tradisional Congklak Berbasis Pendidikan Matematika Realistik Indonesia di Kelas IV Sekolah Dasar. Jurnal Matematika Kreatif-Inovatif (Kreano), 3(2), 100-112. Semarang: UNNES.

Musser, G.L, Burger, W.F., & Peterson, B.E. (2005). Mathematics for Elementary Teacher. USA: John Wiley & Sons, Inc.

Peraturan Menteri Pendidikan dan Kebudayaan Republik Indonesia Nomor 81A Tahun 2013 Tentang Implementasi Kurikulum Pedoman Umum Pembelajaran. Jakarta: Kemdikbud.

Prahmana, R. C. I., Zulkardi, Hartono, Y. (2012). Learning Multiplication Using Indonesian Traditional Game in Third Grade. Journal on Mathematics Education (IndoMS-JME), 3(2), 115-132. Palembang: IndoMS.

Purnomo, Y.W. (2014). Bilangan Cacah dan Bulat. Bandung:Alfabeta.

Putri, R.I.I. (2011). Implementasi Alat Peraga Operasi Bilangan Bulat Bagi Guru Sekolah Dasar (SD) se-Kecamatan Ilir Barat I Palembang. Jurnal Pendidikan Matematika, 5(1). Palembang:Unsri.

Putri, R.I.I. (2015). Professional Development of PMRI Teachers for Introducing Social Norms. Journal on Mathematics Education (IndoMS-JME), 6(1), 11-19. Palembang: IndoMS.

Somakim. (2008). Teori Belajar Dienes. In Bahan Ajar Cetak: Pengembangan Matematika Sekolah, 1-42. Jakarta: Departemen Pendidikan Nasional.

Van de Walle, J.A., Karp, K. S., & Bay-Williams, J.M. (2008). Elementary and Middle School Mathematics, Teaching Developmentally. USA: Pearson.

Wijaya, A. (2012). Pendidikan Matematika Realistik, Suatu Alternatif Pendekatan Pembelajaran Matematika. Yogyakarta: Graha Ilmu.

Zulkardi. (2002). Developing a Learning Environment on Realistic Mathematics Education for Indonesian Student Teachers. Published Dissertation. Enschede: University of Twente.