

# BUILDING STUDENTS' UNDERSTANDING OF QUADRATIC EQUATION CONCEPT USING NAÏVE GEOMETRY

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## *Abstract*

The purpose of this research is to know how Naïve Geometry method can support students' understanding about the concept of solving quadratic equations. In this article we will discuss one activities of the four activities we developed. This activity focused on how students linking the Naïve Geometry method with the solving of the quadratic equation especially on how student bring geometric solution into algebraic form. This research was conducted in SMP Negeri 1 Palembang. Design research was chosen as method used in this research that have three main phases. The results of this research showed that manipulating and reshaping the rectangle into square could stimulate students to acquire the idea of solving quadratic equations using completing perfect square method. In the end of the meeting, students are also guided to reinvent the general formula to solve quadratic equations.

**Keywords:** Quadratic Equations, Design Research, Naïve Geometry, PMRI

## *Abstrak*

Tujuan dari penelitian ini adalah untuk mengetahui bagaimana metode Naïve Geometri dapat membantu pemahaman siswa tentang konsep penyelesaian persamaan kuadrat. Pada artikel ini akan dibahas salah satu aktivitas dari empat kegiatan yang kami kembangkan. Kegiatan ini berfokus pada bagaimana siswa mengaitkan metode Naïve Geometri dengan penyelesaian persamaan kuadrat. Penelitian dilaksanakan di SMP Negeri 1 Palembang. Metode penelitian yang digunakan dalam penelitian ini adalah desain riset yang dilakukan melalui 3 tahap utama. Hasil dari penelitian ini menunjukkan siswa dapat memahami konsep penyelesaian persamaan kuadrat dengan cara melengkapkan kuadrat sempurna melalui metode naïve geometry, yang diinterpretasikan sebagai manipulasi bentuk persegi panjang menjadi bentuk persegi. Pada akhir pertemuan, siswa juga diarahkan untuk menemukan rumus bentuk umum penyelesaian persamaan kuadrat.

**Kata kunci:** Persamaan Kuadrat, *Design Research*, Naïve Geometry, PMRI

In emphasizing the importance of learning algebra, Tall and Thomas (French, 2002) states: "there is a stage in the curriculum when the introduction of algebra may make-Simple Things hard, but not teaching algebra will soon render it impossible to make-the hard things simple ". However, it is not supported by the learning conditions in Indonesia, which is only emphasizes the use of an algorithm or formula, especially on the topic of the solving quadratic equation (Zakaria & Maat, 2010).

Zakaria & Maat (2010) showed that some errors in solving quadratic often occurs due to the weakness in understanding the concept. French (2002) found that the common mistakes made by students is assuming that  $(a + b)^2$  is equivalent to  $a^2 + b^2$ . Zakaria & Maat (2010) research also showed that most of students still make mistakes in terms of transformation and multiplication of

algebraic form. This implies that a learning that supporting the student understanding in concept of solving quadratic equation is needed.

Pendidikan Matematika Realistik Indonesia (PMRI) is a learning approach that adapted from Realistic Mathematics Education (RME). Sembiring et al (2010) stated that PMRI is Indonesian version of RME due to the use of Indonesian culture and RME as a based theory. In PMRI problem situation or context play a key role in developing mathematical concepts (Gravemeijer, & Doorman, 1999; Sembiring, 2010; Van Den Heuvel-Panhuizen, 2003; Zulkardi, 2002). Applying PMRI approach will provide a positive character on teaching and learning in the classroom as follows (Sembiring, 2010):

1. Students will more active in thinking
2. Context and instructional materials directly related to schools and students
3. Teacher will more active in designing the teaching activities.

Problems from history mathematics can be used as a context to make a meaningful learning. This is also supported by a statement of some researchers who suggest the integration of learning mathematics with the history of mathematics (Bakker, 2004; Fauvel & Van Maanen, 2000; Katz, 2000; Panasuk & Horton, 2012; Radford, 2000; Radford & Guerette, 2000). Radford (1996) state the historical construction of mathematical concepts can supply us with a better understanding of the ways in which our students construct their knowledge of mathematics. According to Grugnetti (2000), there are three points of history of mathematics can influence the pedagogical problems.

1. By using old problems, students can compare their strategies with the original ones. This is an interesting way for understanding the effectiveness of our present algebraic process. In observing the historical evolution of a concept, pupils will find that mathematics is not fixed and definitive.
2. History for constructing mathematical skills and concepts. By knowing the history of how a concept was invented or developed, will support improve the skills about that concept.
3. An historical and epistemological analysis allows teachers to understand why a certain concept is difficult for the student. It can support teachers to solve the problem or develop a didactic approach.

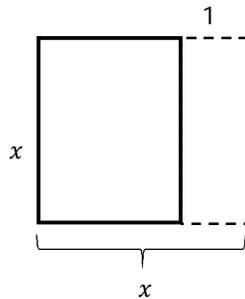
Based on the historical perspective, the concept of solving quadratic equations was built by the geometric foundation (French, 2002; Krantz, 2006; Merzbach & Boyer, 2010). Al-Khwarizmi also describes the geometric proof of solving quadratic equations geometrically in his book *Hisob Al-Jabr wa'l Muqabalah* (Krantz, 2006; Merzbach & Boyer, 2010). The Babylonian geometric method is a geometric method that can be used to solving quadratic equation. This method was identified by J. Hóyrup and he called it *Näive Geometry* (Hóyrup, 1990). In order to show the method, let we discuss one of the Babylonian problems, problem II of a tablet preserved at the british museum and known as BM 13901.

The statement of the problem appear in the tablet is the following:

My confrontation inside of the surface I have torn out:  $14^{\circ}30'$ . 1 the wasitum;  
 You pose. The moiety (half) of 1 you break,  $30'$  and  $30'$  you make span;  $15'$  to  $14^{\circ}30'$   
 you append:  $14^{\circ}30'15'$  makes  $29^{\circ}30'$  equilateral.  $30'$  which you have made span to  
 $29^{\circ}30'$  you append; 30 the confrontation. (Hóyrup, 1990b)  
 note:  $14^{\circ}30' = 870$ ,  $30' = \frac{1}{2}$ ,  $15' = \frac{1}{4}$ ,  $14^{\circ}30'15' = 870\frac{1}{4}$ .

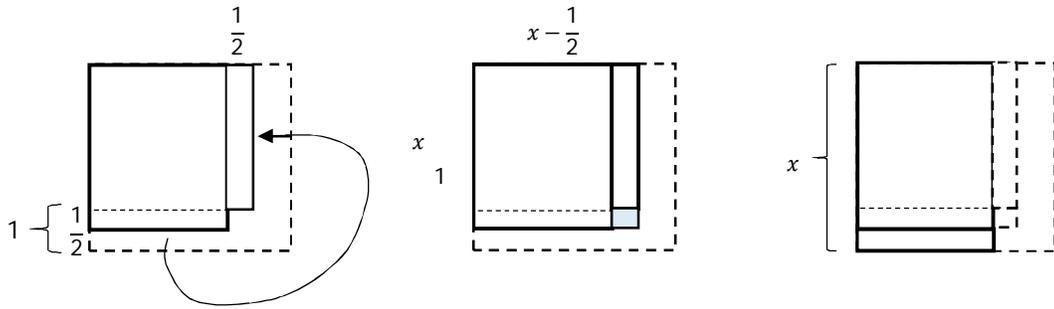
The statement of the problem is, "My confrontation inside of the surface I have torn out:  $14^{\circ}30'$ . 1 the wasitum? Hóyrup explained that the "Confrontation" is a side of the square and the "surface" is a square. The side is not a simple side, but as the side along with it as a side provided with a canonical projection that forms, along with the side (rectangle form) (Radford, & Guerette, 2000). The wasitum is means something going out, including something projecting from a building (Hóyrup, 1990b).

The problem simply is to find the value of the square's side if it is known that the area of the square minus its side is 870 (see Figure 1). Methods of solution of this problems is not fully explained in the text on the tablet. The text shows only a list of instruction concerning a sequence of calculation that allows one to get the answer (Figure 2 is geometric interpretation of the solution explained by Hóyrup (1990b).



**Figure 1. Geometric Interpretation of BM 1391 Problem II**

It should be underlined, that the "Confrontation" or side of square here is not as we know, but a rectangular projection. To get better understanding of the statement in the tablet, Hóyrup (1990b) interpret the geometric shapes as shown below.



**Figure 2. The Step of Naïve Geometry Method**

It appears that 870 refers to result of reduction area of Fig.1. By doing the geometric manipulation (Fig. 2) we got a new square shape (by adding small square). Next, to find the value of  $(x - \frac{1}{2})$  we just look for the square root of the area or the new square. Further, we can easily get the value of  $x$ . That geometric interpretation also makes the problem easier to understand. In this paper we present one of four instructional activities developed based on that conducted in second cycle of teaching experiment in building the concept of solving quadratic equation.

The purpose of this paper is to explore how the naïve geometry method supports the development of students' understanding the concept of solving quadratic equation. We are also interested to know how students interpret their geometric manipulation into algebraic form. Therefore, we formulate the general research question as: How naïve geometry method support students' understanding the concepts of quadratic equations in eighth grade junior high school students?

**METHOD**

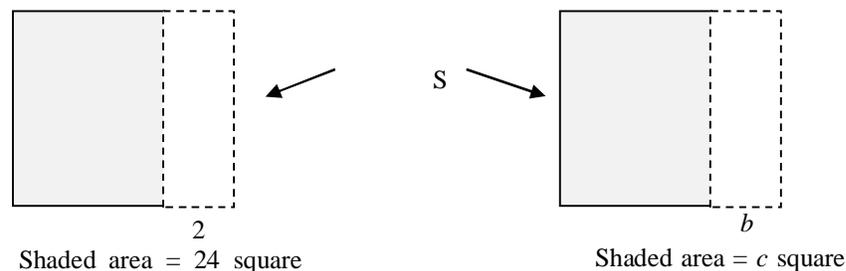
This study involves 32 students of 8<sup>th</sup> grade students of State Junior High School 1 Palembang. This study also involves an 8<sup>th</sup> grade classroom teacher of State Junior High School 1 Palembang, Indonesia. Design research was used as method in this research. The core of this type of research is formed by classroom teaching experiments that center on the development of instructional sequences and the local instructional theories that underpin them (Gravemeijer, 2004). There are three main steps undertaken in this research, preliminary design, teaching experiment, and retrospective analysis (Gravemeijer & Cobb, 2006; Gravemeijer, 2004).

In the first phase, researcher formulate a Hypothetical Learning Trajectory (HLT) that is made up of three components: (a) learning goals for students, (b) planned instructional activities and the tools that will be used, (c) a conjectured learning process in which one anticipates how students' thinking and understanding could evolve when the instructional activities are used in the classroom (Gravemeijer, 2004; Simon & Tzur, 2004). In teaching experiment, HLT was tested through two phases, pilot experiment (first cycle) and teaching experiment (second cycle). During this phase, data

was collected to improve the HLT. The last, HLT and students' actual learning process was compared in retrospective analysis.

We developed four instruction activities to be implemented in the classroom. The present study we just focus on one (last activity) of four instructional activities that conducted in a second cycle of teaching experiment that focused on linking between geometric methods and solving quadratic equation in algebraic form. In activity IV, we facilitate students to make a connection between the geometric interpretations (naïve geometry) that they used to solve the problem in worksheet and developed their algebraic skills. First, students are asked to solve the problem using geometric method that they have learned in previous activity and then transform it into algebraic interpretation.

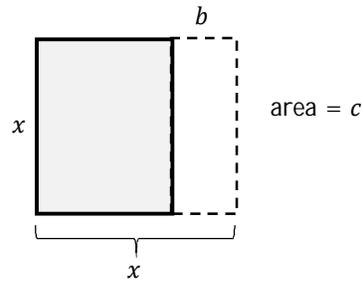
In the first meeting, learning started by introducing the context and steps must be followed so that students can solve the problem using geometric method (ignoring algebraic procedure). The second problem, students must solve the second problem (determining the length and width of a rectangle) using the same method they used in the first meeting. However, the steps needed are the opposite way. In first meeting (first activity), to solve the problem they must reshaping square into rectangle but in second meeting (second activity) is the opposite. Then, in third and fourth meeting the problem used is similar (determining side of a square), they also need to reshaping rectangle into a square but the difference is that concrete number numbers are given neither for the area nor base of the rectangle. In the last activity, we focus on how student bring the geometric procedure to the algebraic form and understand the concept of solving quadratic equation through Naïve Geometry.



**Figure 3. Problem in Worksheet 4 (Activity III) and 4 (Activity IV)**

## RESULTS AND DISCUSSION

This section will discuss the analysis of student work on activity IV, where students focus on how interpret geometric manipulation they had done into algebraic symbols. In this activity students are asked to determine the side of a square (value of  $x$ ) if the shaded area is  $c$  (see Figure 4).



**Figure 4. Problem Worksheet 4**

The following transcript show how the students' discuss about how they interpret geometric problem given into algebraic form.

- Teacher : What did you get?(algebraic interpretation of the problem)  
 Irfaan :  $x$  times  $x$  equals  $x^2$ ,  $x$  times  $b$  equals  $bx$  are equal to  $c$  (the student wrote  $x^2 - bx = c$ )

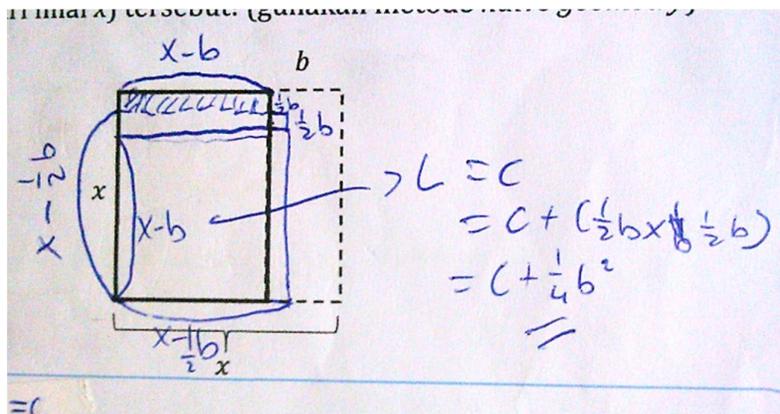
The discussion above shows that they had no difficulties turning the geometric problems into the form of algebraic form.

Further, students worked to determine the value of  $x$  by using the *naïve method* that involves algebraic symbols. They begin to determine the extent of the new square is formed. Here are the transcript of the video.

*after some moments and students did naive method*

- Teacher : What is the area of this small pieces added here?  
 Irfaan :  $\frac{1}{2} b$  times  $\frac{1}{2} b$   
 Teacher : what is the product of  $\frac{1}{2} b$  kali multiplied with  $\frac{1}{2} b$ ?  
 Islam and Irfaan :  $\frac{1}{4} b^2$   
 Teacher : then, what is the area of this new square (the result of manipulated geometry)?  
 Irfaan :  $c + \frac{1}{4} b^2$   
 Teacher : That is right, just replace it.  
 Irfaan : So..  
 Teacher : What is the area of this square?  
 Irfaan : this area... ehm...(humming)  
 Teacher : What is the area formula of this square?  
 Islam : A square, length of its edge times lengths of its edge)) the product of its lengths of edges..  
 Teacher : Yes, a length of its edge times a length of its edge, or the length

- of its edge is?
- Islam : squared
- Teacher : Indeed, or the square of the length of its edge. Then, what is the length of the edge of the new square (refers to the creation of naïve geometry)?
- Irfaan :  $x - \frac{1}{2}b$
- Teacher : Therefore, this square of the length of the edge is equal to what? You can write it directly
- Teacher : As you knew it before this area is equal to what??
- Irfaan :  $c + \frac{1}{4} b^2$



**Figure 5. Students' Work in Determining the Area in Algebraic form**

Furthermore, with the teacher's guidance, students trying to find the value of  $x$ .

- Teacher : What is the area formula?
- Irfaan : the area formula is the square of the length of its edge)  
(writing square of the length of its edge )

(after a few minutes)

- Irfaan : The square of the length of its side is equal to (then humming)

$$c + \frac{1}{4} b^2$$

- Teacher : So, what is the length of its edge?

Irfaan :  $x - \frac{1}{2}b$

- Islam : it means we should take a root of it

- Teacher : What do you do?

- Irfaan : Square it

- Teacher : Please, repeat it again
- Islam : we take a root, don't we?
- Teacher : Yes, later on we can move the root) (gave a wrong guidance)
- Irfaan :  $\left(x - \frac{1}{2}b\right)^2 = c + \frac{1}{4}b^2$
- Teacher : can we omit the power?
- Irfaan : yes, we can
- Teacher : then?
- Irfaan :  $x - \frac{1}{2}b = \sqrt{c + \frac{1}{4}b^2}$  (after the researcher gave some guidance)
- Irfaan and Islam :  $x = \sqrt{c + \frac{1}{4}b^2} - \frac{1}{2}b$

From the results of geometric manipulation, students get a gnomon form and add it to the small square into get a square shape. Because of the area of the gnomon is  $c$  unit area (equal to the initial area before manipulating) then the new square has an area of  $c + \frac{1}{4}b^2$  unit area, because the added  $\frac{1}{4}b^2$  areas required to complete a square shape (Figure 5).

To understand the idea to find the value of  $x$  is by using formula of square area, students still need guidance although they understand that the new square's area they get is  $c + \frac{1}{4}b^2$  and the area of the square is squared of its side. That may occurred due to students have not accustomed to solve problems involving geometric and algebraic manipulations together. However, students didn't get serious problem in calculating until they obtain the form  $\left(x - \frac{1}{2}b\right)^2 = c + \frac{1}{4}b^2$ . Base on the fragment above, in the end they get that the value of  $x = \sqrt{c + \frac{1}{4}b^2} + \frac{1}{2}b$ .

In general, the purpose already achieved based on the conjecture of the HLT has been made. Through that problem, students noticed that the side of square they found is the formula to find the value of  $x$  if the quadratic equation given in the form  $x^2 - bx = c$ . From here, they also learned that in naïve geometry method, the idea of "reshaping into square form" is equal to "completing perfect square" in terms of quadratic equation.

The next step is to give the general equation  $ax^2 + bx + c = 0$  and ask students to find the formula to solve this equation. They noticed that the equation must be divided by  $a$  to get the similar form with the previous one.

The image shows two stages of handwritten algebraic manipulation. The first stage shows the equation  $ax^2 + bx + c = 0$  with a horizontal line under the entire left side and a vertical line under the coefficient  $a$ , resulting in  $x^2 + \frac{bx}{a} + \frac{c}{a} = 0$ . An arrow points to the second stage, which shows the equation  $x^2 - \left(\frac{b}{a}\right)x = -\frac{c}{a}$ , where the terms are rearranged and signs are adjusted.

**Figure 6. Student’s Work in Algebraic Manipulation**

By substituting the forms above to the formula that had been discovered earlier ( $x = \sqrt{c + \frac{1}{4} b^2} + \frac{1}{2} b$ ), students got the form  $x = \sqrt{-\frac{c}{a} + \frac{1}{4} \left(-\frac{b}{a}\right)^2} + \frac{1}{2} \left(-\frac{b}{a}\right)$ . Furthermore, to be able to convert the formula into  $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  teacher’s guidance is needed.

The image shows a series of handwritten steps in the derivation of the quadratic formula. It starts with  $x = \sqrt{-\frac{c}{a} + \frac{1}{4} \left(-\frac{b}{a}\right)^2} + \frac{1}{2} \left(-\frac{b}{a}\right)$ , then simplifies to  $x = \sqrt{-\frac{c}{a} + \frac{b^2}{4a^2}} + \frac{1}{2} \left(-\frac{b}{a}\right)$ , then  $x = \sqrt{-\frac{c}{a} + \frac{b^2}{4a^2}} - \frac{b}{2a}$ , then  $x = -\frac{b}{2a} + \sqrt{-\frac{c}{a} + \frac{b^2}{4a^2}}$ , then  $x = -\frac{b}{2a} + \frac{\sqrt{4ac + b^2}}{2a}$ , then  $x = -\frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}$ , and finally the general formula  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ . A separate box on the right shows the final formula  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ .

**Figure 7. the Reinvention of General Formula**

**CONCLUSION AND SUGGESTION**

Based on that learning conducted in a second cycle of teaching experiment, we can conclude that through these activity students can realize the idea of naïve geometry can be used to solve quadratic equations and finding a general form to solve quadratic equations. Through the geometric idea (reshaping into a square), also supports students to perform symbolic operations are meaningful because they are familiar with the context involved. However, we want to stress the fact that this

approach cannot avoid the problems due to understanding of algebraic symbols. For example, the guidance from the teacher becomes very necessary because of the low ability student in algebra also the learning style that students are still not familiar with. Moreover, many students had not enough time to complete the task in worksheet. Actually, this is only intended instructional design provide a useful context to support the students develop a meaning for symbols that are useful for developing their algebraic skills.

Based on the results obtained for better further research quality then the researchers suggest that collaboration between researchers and teacher to be improved so teacher gain knowledge on how to handle PMRI classroom based on the HLT. Thus, the error in the study can be minimized.

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