Evaluation of Students’ Mathematical Problem Solving Skills in Relation to Their Reading Levels

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Received: 10 February 2015 / Revised: 12 July 2015 / Accepted: 5 August 2015

Abstract
The purpose of the current study is to investigate the correlation between students’ reading levels and mathematical problem solving skills. The present study was conducted in line with a qualitative research method, i.e., the phenomenological method. The study group of the current research is composed of six third grade students with different reading levels. The data of the study were collected through the reading of texts, the Ekwall/Shanker reading inventory and the problem solving think-aloud protocol. The collected data were evaluated using a descriptive analysis method. Once the study had been completed, it was concluded that problem solving skills varied according to reading level.

Keywords: Reading, Reading level, Mathematical problem solving

Introduction
One of the primary objectives of education is to create individuals who can read instructions in their daily lives, make decisions about issues requiring social participation, read media and are able to overcome potential problems to be encountered in future (Karataş & Güven, 2003; Özsoy & Kuruyer, 2012). In line with this objective, problem solving and reading comprehension instructions given during the education process should attach priority to imparting these skills to students and further developing them. Problem solving refers to the elimination of a problem through the use of required information and operations in cognitive processes (reasoning) (Altun, 1995). Reading
comprehension, on the other hand, refers to the construction of meaning by the reader from context through the use of textual cues (Akyol, 2010; Duffy, 2009). Therefore, like problem solving, reading comprehension relies on the reader’s recognition and perception of symbols in written language, grammar, cognitive skills and real life experiences. The most important requirement for problem solving and reading comprehension is the transfer of solutions to different situations. Just as problem solving requires more than performing operations with numbers, using four main operations and symbols, reading comprehension requires more than word recognition and the accurate vocalization of words. Problem solving and reading comprehension essentially work together in order to reach a goal and do so by utilizing different resources for this purpose. In this regard, reading comprehension skills and problem solving skills are closely interrelated (Fuentes, 1998; Jordan, Hanich & Kaplan, 2003; Vilenius-Tuohimaa, Aunola & Nurmi, 2008).

Solving a problem requires establishing a link between inputs and anticipated outcomes. Reading assigns meaning to a text by determining a suitable goal and method. No problem or text is self-expressive (Akyol, 2005). Therefore, an individual’s eagerness, their interest, setting a goal and use of strategy are of great importance for problem solving and reading comprehension skills.

Evaluation of students’ problem solving skills and reading comprehension skills is as important as the teaching of these skills (Karataş, 2002; Pearson & Hamm, 2005). An individual needs to activate his/her information about the use of problem solving and reading comprehension skills and effect a transfer of information between these skills. For the evaluation of whether such a transfer takes place, these two skills must be observed together, while for the development of problem solving and reading comprehension skills, they need to be observed and evaluated in the long-run.

Problems, including the comprehension of a text, bring about many difficulties for elementary school students due to the complexity of problem solving processes. Problems stated within the context of a story seem to be more complex and difficult for students than problems not embedded in a text (Mayer, Lewis & Hegarty, 1992; Nathan, Long & Alibali, 2002). It is reported that when compared to similar problems stated with numbers, students are 30% less successful in solving problems that include a story (Carpenter, Corbitt, Kepner, Lindquist & Reys, 1980). Even when they know how to solve problems that does not include a story, when they are presented with these problems embedded within a story, solving them can be more challenging, because solving problems that include a story requires the use of various cognitive processes in an integrated manner. For children that do not have an adequate knowledge base or have limited memory capacity, these tasks can be much more challenging (Jitendra, Griffin, Deatline-Buchman & Sczesniak, 2007).

When solving problems that include a story, students are required to understand the language of the problem and the concrete information presented in the problem, to properly conceptualize the problem in his/her mind based on the information given within the problem, to design and follow a plan and to make the calculations required by the solution process of the problem (Desoete, Roeyers & De Clercq, 2003). In short, solving narration problems is closely associated with understanding the relationships involved in the text and the goal stated in the problem. The results of a set of studies conducted on narration problems that included addition and subtraction operations revealed that rather than the syntax of the text, meaning or mathematic structure was viewed to be more important (Carpenter, Hiebert & Moser, 1983). Similar research findings have shown that the problems experienced by students regarding narration problems are related to an accurate comprehension of the text, rather than numbers or operations (Gökkurt & Soylu, 2013).
An examination of the literature reveals that two primary courses have been followed by research dealing with difficulties in narration problems. In one of these, research focuses on the difficulty level of solutions, based on the characteristics of problems. The problematic characteristics examined in these studies are the number of words in the text of the problem, the existence of statements pointing out the operations to be followed in the solution of the problem, as well as the magnitude of the numbers used in the problem (Briars & Larkin, 1984). The second course of research primarily focuses on the cognitive processes required for the solution of the problem (Dellarosa, 1986; Kintsch & Greeno, 1985). In this regard, it is argued that the difficulty of a problem should be explored based on the interaction between the characteristics of the problem and the cognitive capacity of the student. Within the context of this interactive approach, four focal points are proposed for receiving primary focus (Dellarosa, Weimer & Kintsch, 1985):

1. Comprehension of the problem text
   a) Comprehension of words
   b) Comprehension of the meaning of each sentence
2. Comprehension of the problem statement
   a) Activation of the word knowledge related to the problem statement
   b) Activation of the knowledge related to mathematical terms and relationships
   c) Use of the information compiled from item (1) above
3. Selection of a solution strategy based on item (2) above
4. Accurate implementation of the selected strategy

Finding the correct solution to a problem does not necessarily mean that the student has the necessary problem solving skills. Though some students may have found the correct answers, they may have followed the wrong approach to a solution; other students might develop the correct solution strategies but nonetheless reach the wrong solution due to simple calculation errors (Soylu & Soylu, 2006). This also holds true for reading comprehension. Correct pronunciation of the text and providing the correct answers to reading comprehension questions does not necessarily mean the possession of the reading comprehension skill by the student. Some students, though pronouncing text accurately and giving the correct answers to reading comprehension questions, may do this by chance and by making guesses without necessarily comprehending the text (Dufy, 2009). Therefore, while evaluating problem solving and reading comprehension skills, individual differences should be taken into consideration (Kuzgun & Deryakulu, 2006).

While evaluating problem solving skills, the steps followed by the student to reach a solution, as well as the critical behaviors expected to be exhibited while following these steps should be considered altogether, as there is no certain way of solving a problem and students may develop their own problem solving strategies (Baykul, 2009). In general, comprehension of the problem, establishment of the mathematical connections between what is given and what is required, determination of the operations to be conducted for the solution, conducting the operations and checking of the accuracy of the solution can be defined as general steps in the solution of a mathematical problem (Erden, 1994; Polya, 1998; Tertemiz & Çakmak, 2003). While following these general steps, students are expected to demonstrate critical behaviors such as writing what is given and what is required in the problem, explaining the problem in their own words, summarizing the problem, drawing a scheme of the problem, determining the operations to be conducted in the problem, predicting the results of the problem, achieving a solution by using the mathematical operations and checking the accuracy of the solution (Baykul, 2009; Polya, 1998). For reading comprehension, the critical behaviors expected from the student are the transferring of prior information into the reading environment, setting a goal for
reading, making predictions before reading, checking the correctness of the predictions after the completion of reading, selecting suitable strategies, using enhancement strategies when difficulties are experienced, making use of context to guess the meaning of unknown words, summarizing the main idea and monitoring comprehension.

When the behaviors demonstrated during problem solving and reading comprehension processes are evaluated together, it is observed that reading comprehension and problem solving skills require the execution of similar processes. For pursuing the stages of problem solving, reading level is important (İlgın & Arslan, 2012). Reading level can be described as loud reading and reading comprehension performance. Reading levels can be evaluated under three headings: free level, teaching level and apprehension level (Shanker & Ekwall, 2000). For good readers at the free reading level, word recognition automatically occurs; they are aware of reading strategies and can use these strategies effectively during the text comprehension process (Pang, 2008). Readers who are at teaching level can read and comprehend as required with the support of a teacher or an adult (Akyol, 2010). Weak readers who are at the apprehension level experience difficulties in word recognition and in the discrimination between words; their attention is distracted throughout reading and they feel anxious, lose track when reading, start reading without making predictions and guesses about the text, do not know what to do when they do not understand the text and cannot create connections between prior and new information.

Students’ success in problem solving is affected by cognitive, affective and experience factors (Haylock & Cockburn, 2014; Van de Walle, Karp & Bay-Williams, 2014). Difficulties experienced in problem solving are usually related to reading difficulties (Reikerås, 2006). According to Grauberg (1998), the problems experienced by students that have reading difficulties while learning mathematics are defined as not recognizing symbols, experiencing difficulties in organization, not being able to speak about the problem and memory. In addition, as reading difficulties directly affect learning, they may lead to problems during the course of teaching the language of mathematics and problem solving (Dowker, 2005). Moreover, it can be argued that students’ different reading levels can affect their problem solving performances. The purpose of the current study is to investigate students’ reading levels and mathematical problem solving skills. This study is of importance in terms of providing information about the interaction between students’ reading levels and problem solving skills. The current study seeks answers to questions such as, “Do basic reading and comprehension skills affect problem solving skills as well as other academic skills?” and “Is there a significant relationship between them?” Investigating these questions will offer different opinions about the teaching of these skills.

**Method**

**Research design**

The present study was conducted in line with a qualitative research method, i.e., the phenomenological method. “The phenomenological method focuses on phenomena which we are aware of but that we do not have a deep and detailed understanding about” (Yıldırım & Şimşek, 2008, s. 72).

**Study group**

The study was conducted with six third grade students. Three of the students were girls and three were boys. The participants were determined by using the purposeful sampling method. For determining the study group, the criterion adopted was students having different reading levels. Third grade students were selected for the current study, as
educational attainments set in relation to mathematical problem solving skills and reading comprehension skills at this grade are different from those set for first and second grades, and are more sophisticated; thus, they were more suitable for the purpose of the current study. In order to determine the participants of the study, meetings were organized to inform the teachers and families of 60 third grade students attending an elementary school, who were randomly selected from elementary schools in the city of Aksaray. Following these meetings, 24 students whose parents approved their participation in the study and who were willing to participate in the study, were administered activities for determining their reading levels. Six students representing different reading levels were determined and included in the study. The real names of the participants were excluded for ethical reasons.

Data collection method

The current study intended to determine the relationship between the reading levels and mathematical problem solving skills of participants. The process of determination was conducted in two stages. In the first stage, the Ekwall/Shanker reading inventory, a word recognition list and reading comprehension inventory were employed to determine the reading levels of participants. In the second stage, the problem solving think-aloud protocol was used to determine the students’ problem solving skills.

Data collection instruments

For the evaluation of reading level, formal and informal measurement tools can be used. Formal evaluation can be conducted using standard measurement tools and informal evaluation can be conducted using non-standard measurement tools such as word recognition lists and a reading comprehension inventory (Shanker & Ekwall, 2000; Karasu, Gürgin & Uzuner, 2012). As standard measurement tools provide normal values related to reading level, they not only allow for the comparison of students with their peers, but also provide insights about the performance of students. However, they have some disadvantages in terms of collecting detailed information about reading level, because by using them, it is difficult to determine whether students have an awareness of sounds, how they vocalize words, which strategies they use to discover meaning and what mistakes they make while reading; they may also present obstacles for conducting an evaluation of students’ written and oral performances by means of product files and self-assessment tools (Uzuner, 2008). In this regard, within the context of the current study, an informal word recognition list, a reading comprehension inventory and the Ekwall/Shanker reading inventory were employed for determining students’ reading levels. Information about the data collection instruments employed in the current study for determining students’ reading levels is provided below.

Word recognition list. A list of 60 words with different numbers of syllables (ranging from two to six syllables in length) was developed on the basis of a word list created by Karadağ (2005) for the third grade. The scope of the word recognition list covered activities requiring the vocalization of the words on the list, the construction of sentences using the words, thinking about the meanings of the words and the evaluation of the words within a specific context.

Reading comprehension inventory. The reading comprehension inventory comprised five reading comprehension questions, two of which required simple comprehension and three of which required deep comprehension. This inventory was developed by the researchers taking into account the objectives of the third grade. The reason for the use of this reading comprehension inventory was to collect detailed information about the students' reading levels.
Ekwall/Shanker reading inventory. In order to determine loud reading errors and loud reading levels, a guidance table, adapted by Akyol (2005) from Ekwall and Shanker (1988), was used. This allowed for the rapid evaluation of students’ reading performance and monitoring of the loud reading process. During administration of the Ekwall/Shanker reading inventory, students were asked to read the entire text out loud to determine their loud reading performance. The researchers recorded this loud reading. While the student continued reading, areas where he/she committed mistakes and where they made corrections were marked and notes were taken about the student’s reading. Students’ responses to the simple and deep comprehension questions asked within the framework of the inventory were also recorded.

Reading texts. The texts used during the application process of the study were selected from third grade reading texts recommended by the Board of Education and Discipline. The text used within the framework of the Ekwall/Shanker reading inventory for determining the loud reading levels of students was composed of 170 words. The text used within the framework of the reading comprehension inventory for determining the reading comprehension levels of the students consisted of 148 words. As the application of these texts was conducted among third grade students, the length of the texts selected needed to be between 100 and 200 words (Akyol, 2005). Both of the texts were narrative texts. The texts were written using 1.5 line-spacing and a size 14 font.

Problem solving think-aloud protocol. The problem solving think-aloud protocol consisted of five problems and was developed by the researchers considering the objectives of third grade and Polya’s problem solving stages. The purpose of using a think-aloud protocol was to provide detailed information about the approaches students applied during the process of problem solving.

Instructions are listed below, while details regarding the implementation of the problem solving think-aloud protocols follow immediately after.

1. What is given and required in the problem?
2. Can you briefly explain the problem?
3. Can he/she use visuals to explain the problem?
4. What operations will you perform while solving the problem and why?
5. Can you predict the result of the problem?
6. Can you tell the result of the problem?
7. Can you solve the problem in a different way?
8. If he/she solved the problem incorrectly, were they able to explain the reason for doing so?
9. Can you construct a problem similar to this one?

Data analysis.

Analysis of the data can be evaluated under two headings: a) analysis of the data related to reading levels; b) analysis of the data related to mathematical problem solving skills.

Analysis of the Data Related to Reading Levels

When evaluating data related to the word recognition list, the following aspects were taken into consideration: the duration of vocalizing words on the word recognition list, whether students knew the meanings of words, their performance in terms of constructing meaningful sentences and any mistakes on their part. When evaluating the data collected via the reading comprehension inventory, students’ responses to simple and more complex comprehension questions were analyzed.
Data collected using the Ekwall/Shanker reading inventory were evaluated by considering the mistakes committed by students during their loud reading performances; this was assessed on the basis of word recognition levels and a percentage determination guideline adapted by Akyol (2005) from Ekwall and Shanker (1988). In order to detect the percentage of mistakes committed where words were concerned, the following procedure was adopted according to the guideline. Mehmet read a 170-word text within the inventory. While reading the text, he committed eight mistakes. According to the guideline, eight mistakes committed within a 166-170-word text represent word recognition of 95%. Responses to the simple and more complex comprehension questions asked within the inventory were scored as 3, 2, 1 and 0, while responses to simple comprehension questions were scored as 2, 1 and 0. The highest score to be taken from the comprehension questions was 13. In order to calculate the comprehension percentage, the sum of the scores taken was divided by the sum of the scores that were required to be taken. For example, Mehmet received eight points from the five questions in order to measure deep comprehension (8/13=0.615); that is, his comprehension percentage was 61%.

According to the Ekwall/Shanker reading inventory, students’ reading levels are at free level when their word recognition percentage is 99+ and comprehension percentage is 90+; reading levels are at teaching level when word recognition percentage is 95+ and comprehension percentage is 75+, and at apprehension level when their word recognition percentage is 90-, with comprehension percentage at 50- (Shanker & Ekwall, 2000). On the basis of these evaluation criteria, Mehmet’s reading level was evaluated to be at teaching level, as his word recognition percentage was 95 and his comprehension percentage was 61.

**Analysis of the data obtained in relation to mathematical problem solving skills**

The collected data were evaluated using a descriptive analysis method. The purpose for using this method was to present the findings in a summarized and interpreted manner to the reader. The steps followed during the analysis process are presented below.

- A framework was constructed for the data analysis within the conceptual context of the study.
- Based on the constructed framework, the data were read and organized.
- Excerpts were included to define the organized data.
- The defined data were explained and associated or compared with the obtained phenomena.

Within the theoretical framework of the study, the collected data were defined under headings pertaining to the findings related to reading level and findings related to mathematical problem solving skills. The findings related to reading comprehension and reading level was subsumed under the heading of reading level. The skills demonstrated by students while solving problems and their responses to questions were evaluated under the heading of problem solving skills. The findings are presented and supported with direct observations and quotations. In order to demonstrate how the analysis process was conducted, the think-aloud protocol, conducted with a student named Esra, is presented as an example.

**Researcher:** Esra, you have read the problem. What is given and asked for in the problem?

**Esra:** I need to find [out] how much a [kilogram] of banana[s] is.

... 

**Researcher:** Which operation will you use [to solve] the problem?

**Esra:** Addition.
Researcher: Can you explain why?

Esra: [The question] asks how much [a kilogram] is.

Researcher: Can you guess the result of the problem?

Esra: No.

Researcher: Go on, please.

Esra: Now I add 10 to 10 [and] the result is 20; now I add 20 to 5 and the result is 25; I add 25 to 2 and the result is 27 and I add 27 to 3 and the result is 30.

Researcher: Are you sure that the result is correct?

Esra: Yes.

Researcher: Now, let's read the problem and solve it together (with the help of the researcher the problem is solved again). Is the result that you found different from the result we found together?

Esra: Yes.

Researcher: Can you explain why the results are different?

Esra: Hmmm… (she thinks for a while.) No.

By examining the skills demonstrated by the student named Esra while solving the problem and the responses given by her within the context of the think-aloud protocol, how the student solved the problem and skills demonstrated by her are presented below.

A seller buys 10 kg apples, 10 kg oranges and 5 kg bananas for the green grocery every day. One kilo of apples costs 3 liras and one kilo of oranges costs 2 liras. The seller paid 100 liras in total; thus, how much is one kilo of bananas?

<table>
<thead>
<tr>
<th>Student</th>
<th>Student’s solution to the problem</th>
<th>The way followed by the student while solving the problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Esra</td>
<td>10+10=20</td>
<td>She defined what was required as the cost of one kilo of bananas. She summarized what was given as kilos and liras. She said that the operation to be used for the solution of the problem would be addition. She was sure of the correctness of the solution. After seeing the correct solution to the problem, she could not identify where she had made a mistake.</td>
</tr>
<tr>
<td></td>
<td>20+5=25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>25+2=27</td>
<td></td>
</tr>
<tr>
<td></td>
<td>27+3=30</td>
<td></td>
</tr>
</tbody>
</table>

In order to establish reliability and validity in qualitative research, it is necessary to precisely present all the decision-making stages and strategies utilized during the study (Yıldırım, 2010). Within the current study, the data transferred into the computer environment and transcribed were evaluated by the researchers and an independent expert, in light of the literature findings and within the framework of themes in order to establish the reliability and validity of the study. While reading the data collected from each interview, all the codes and themes were systematically and repeatedly compared to the conceptual framework.

Process and setting

All the applications were conducted with students at predetermined times outside of school time and in a quite environment. The application setting was organized in such a way as to facilitate working with each student individually. The students were made to feel relaxed during the application. The word recognition test, the reading comprehension inventory and Ekwall/Shanker reading inventory were administered in three sessions. Each session lasted nearly 25 minutes. The problem solving think-aloud protocol was implemented in a single session. Each session lasted nearly 40 minutes and was video recorded.
Findings

This section presents: (1) findings related to the reading levels of students; (2) findings related to the problems involved in the think-aloud protocol; (3) findings related to mathematical problem solving skills. The findings related to the reading levels of the students are presented in Table 1.

Findings related to reading levels

Table 1. Findings related to the reading levels of the students in the study group

<table>
<thead>
<tr>
<th>Student</th>
<th>Reading Level</th>
<th>Detailed information about reading level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hatice:</td>
<td>Free Level</td>
<td>She can vocalize the text accurately. She can adjust her reading speed according to the text; she read the text at the correct speed and with the correct intonation. She can accurately vocalize the words involved in the word recognition test.</td>
</tr>
<tr>
<td>Leyla:</td>
<td>Free Level</td>
<td>She can vocalize the text accurately. She can adjust her reading speed according to the text; she read the text at the correct speed and with the correct intonation. She can accurately vocalize the words involved in the word recognition test.</td>
</tr>
<tr>
<td>Mehmet:</td>
<td>Teaching Level</td>
<td>He reads the text very slowly. He vocalizes the words erroneously; he spends a significant amount of time on word recognition and discrimination. He erroneously vocalizes the words involved in the word recognition test. With an increased number of syllables in words, he experiences more difficulties and spends more time vocalizing the words.</td>
</tr>
<tr>
<td>Esra:</td>
<td>Teaching Level</td>
<td>She can vocalize the text accurately. She can adjust her reading speed according to the text. She can accurately vocalize the words involved in the word recognition test.</td>
</tr>
<tr>
<td>Ömer:</td>
<td>Apprehension Level</td>
<td>He reads the text very slowly. While reading, he makes some additions, skips lines, follows text using her finger and cannot discriminate the words. He erroneously vocalizes the words involved in the word recognition test. With an increase in the number of syllables in words, he experiences more difficulties and spends more time vocalizing the words.</td>
</tr>
<tr>
<td>Mustafa:</td>
<td>Apprehension Level</td>
<td>He reads the text very slowly. He skips words and lines, follows text using his finger, spends more time on word recognition and makes some additions. He erroneously vocalizes the words involved in the word recognition test. With an increase in the number of syllables in words, he experiences more difficulties and spends more time vocalizing the words.</td>
</tr>
</tbody>
</table>
Findings related to the problems in the think-aloud protocol

Table 2. Findings related to the first problem in the think-aloud protocol

<table>
<thead>
<tr>
<th>Student</th>
<th>Student’s solution to the problem</th>
<th>The student’s approach to solving the problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leyla</td>
<td>10+10+5+2=30 100-30=70</td>
<td>She expressed what is required as the cost of one kilo of bananas. She listed the data. After thinking for a while about a solution, she said that the result was 70. For an alternative solution, she said that she would subtract 70 from 100. She realized that she was sure of the result. After seeing the correct answer, she realized the mistakes she made while solving the problem.</td>
</tr>
<tr>
<td>Hatice</td>
<td>The problem was solved with help.</td>
<td>What is required was determined (how much is a kilo of bananas?)</td>
</tr>
<tr>
<td>Mehmet</td>
<td>The problem could not be solved.</td>
<td>What is required and given was not determined. No predictions were made about the mathematical operations to be conducted.</td>
</tr>
<tr>
<td>Esra</td>
<td>10+10=20 20+5=25 25+2=27 27+3=30</td>
<td>She defined what was required as the cost of one kilo of bananas. She summarized what was given as kilos and liras. She defined the operation to be used for a solution to the problem as addition. She was sure of the correctness of her solution. After having seen the correct solution to the problem, she could not identify where she had made a mistake.</td>
</tr>
<tr>
<td>Mustafa</td>
<td>10+10=20</td>
<td>He was not able determine what was required and given. He said that he would use addition to solve the problem and explained the reason for selecting this operation as it being an easy approach. Though he stated that he could solve the problem in another way, he did not provide any information about what this approach might be. When he saw the correct solution to the problem, he did not make explain where he had made mistakes.</td>
</tr>
<tr>
<td>Ömer</td>
<td>10+1=10</td>
<td>He read the problem statement incorrectly. He could not summarize what was given and required. He could not define the operation he would use to solve the problem. He could not determine where he committed a mistake after seeing the solution to the problem.</td>
</tr>
</tbody>
</table>

When the responses given to the first problem are examined, it can be argued that the students, with the exception of Esra and Leyla, were unable to determine what was required. Though Esra and Leyla were able to determine what was required, they could not find a way to solve the problem and thus, were unable to do so. When the mathematical operations conducted by the students for the problem are analyzed, it can be seen that the students preferred to add the numbers given in the problem. Only Leyla recognized the mistake she made after seeing the correct solution to the problem. Ömer read the problem statement incorrectly. Therefore, it can be argued that he was unable to develop an approach to solving the problem.
Table 3. Findings related to the second problem in the think-aloud protocol

*Can is nine years old and Ece is seven years old; what is the sum of the ages of Can and Ece six years later?*

<table>
<thead>
<tr>
<th>Student</th>
<th>Student’s solution to the problem</th>
<th>The student’s approach to solving the problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leyla</td>
<td>9X6=54 (1st solution) 9+6=15 (2nd solution) 7+6=13 15+13=28</td>
<td>She defined what was required as the sum of the ages of Can and Ece six years later. She defined what was given as the ages of Can and Ece. First, she stated that she needed to conduct a multiplication operation to solve the problem. After finding a result of 54, she recognized that the result was incorrect; she then changed her approach to the solution. She was sure of the correctness of her second solution. She also stated that another solution may be to add all the numbers together.</td>
</tr>
<tr>
<td>Hatice</td>
<td>9+6=15 7+6=13 15+13=28</td>
<td>She answered that first, she would multiply 9 with 6 and then multiply 7 with 6 to solve the problem. She stated that the result of the problem could be 36 or 40. She also stated that after multiplying 9 with 6, she found a result of 54 and therefore stated that her first prediction was wrong. She then read the question again and when she was asked whether she was sure about her result, she said that she had to conduct an operation of addition instead of multiplication; she then solved the problem correctly. She could not produce any other solution.</td>
</tr>
<tr>
<td>Mehmet</td>
<td>9+7=16 (1st solution) 15+13=28 (2nd solution)</td>
<td>He defined what was required as the age of Can. He stated that he could solve the problem by using an operation of addition.</td>
</tr>
<tr>
<td>Esra</td>
<td>9+7=16 365+15=380</td>
<td>She defined what was required as how old Can and Ece would be six years later. She listed what was given as the ages of Can and Ece. She stated that the operation to be used for solving the problem would be addition. After finding the sum of the ages of Can and Ece, he tried to calculate how many days there were in six years. She did so to add 6 years to 16. As she could not do this operation, she added the sum of the ages of Can and Ece to the days in one year. She was unsure of the correctness of the result she found. She recognized where she had made a mistake after seeing the solution to the problem.</td>
</tr>
<tr>
<td>Mustafa</td>
<td>9+7=60 6+6=12 60+12=72</td>
<td>He could not determine what was given and requested. He correctly expressed how to solve the problem. However, when he added 9 to 7, he said that the result would be 16, but he wrote 60 as his result instead; thus, he could not reach the correct solution. When he checked his solution, he recognized that he had made a mistake but could not determine where he had done so.</td>
</tr>
</tbody>
</table>
Table 3 (Cont.). *Findings related to the second problem in the think-aloud protocol*

<table>
<thead>
<tr>
<th>Student</th>
<th>Student’s solution to the problem</th>
<th>The student’s approach to solving the problem.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ömer</td>
<td>9+7=16 16+13=29</td>
<td>He could not list what was given and requested. However, he said that he needed to use the operation of addition to find a solution to the problem. He could not guess the result of the problem. In the second stage of the solution to the problem, as he incorrectly conducted the operation of 6+6, he found an incorrect result. After seeing the correct solution to the problem, he could not explain why he had solved the problem incorrectly. When he was asked whether he could solve the problem in another way, he stated that he could use a subtraction operation.</td>
</tr>
</tbody>
</table>

When the responses to the second problem are examined, it can be seen that Ömer completed the first stage of the problem correctly. He tried to carry out the second stage of the problem, that is, the operation of 6+6, in his mind. As he calculated the result of this operation incorrectly, he was unable to find the correct solution. Mustafa committed a calculation error in the solution of the problem; therefore, though his approach to a solution was correct, he found the wrong result. Mustafa and Ömer were unable to solve the problem correctly. Though they were shown the correct solution, they could not determine where they had made mistakes. Mehmet first added together the ages of Can and Ece and then changed his mind; he then reached the conclusion that he needed to add the ages of Can and Ece six years later. Esra experienced a problem related to the concept of a year in the solution of this problem; he argued that since one year was 365 days, he needed to add 365 days to nine years, which prevented him from finding the correct result. Hatice and Leyla solved the problem correctly.

Table 4. *Findings related to the third problem in the think-aloud protocol*

**Third Problem.** Cem has 210 liras. How much does he need to add to have 500 liras?

<table>
<thead>
<tr>
<th>Student</th>
<th>Student’s solution to the problem</th>
<th>The student’s approach to solving the problem.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leyla</td>
<td>500-210=290</td>
<td>She defined what was requested as what was needed to make Cem’s money 500 liras. She said that she needed to conduct a subtraction operation to solve the problem. She did not propose another solution. She said she was sure of the correctness of her solution.</td>
</tr>
<tr>
<td>Hatice</td>
<td>500-210=290</td>
<td>She defined what was required as, “I need to [establish] how much [money] Cem needs. I can count from 210 to 500.” When she was asked to solve the problem using another approach, she said that she could subtract 210 from 500.</td>
</tr>
<tr>
<td>Mehmet</td>
<td>100+200=300</td>
<td>He read 500 as 50 100 liras. He experienced difficulties reading the problem statement. He could not list what was given and requested. He said that he would not be able to solve the problem.</td>
</tr>
<tr>
<td>Esra</td>
<td>500-210=390 (First solution) 500-210=290(Second solution)</td>
<td>She defined the problem as how much money was needed to make the amount given 500 liras. She said that she would subtract 210 from 500 liras to solve the problem. She recognized her mistake and corrected it. She was not sure of the correctness of her result. She said that she could not solve the problem in any other way.</td>
</tr>
</tbody>
</table>
Table 4 (Cont.) *Findings related to the third problem in the think-aloud protocol*

<table>
<thead>
<tr>
<th>Student</th>
<th>Problem could not be solved</th>
<th>While reading the problem, he read the number 210 as 2101. He said that he needed to conduct an addition operation to solve the problem. He could not make any guesses about the result. He expressed what was requested as the sum of Cem’s money.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ömer</td>
<td>Problem could not be solved</td>
<td>He said that he needed to conduct an operation of addition to solve the problem. He could not make any guesses about the result.</td>
</tr>
</tbody>
</table>

When the responses given to the third problem are examined, it can be seen that it was correctly solved by Esra, Hatice and Leyla. Esra recognized the mistake she committed in finding a solution to the problem and corrected it. As Mustafa, Ömer and Mehmet read the numbers given in the problem statement incorrectly, they were unable to present correct solutions. Leyla was sure that she had solved the problem correctly.

Table 5. *Findings related to the fourth problem in the think-aloud protocol*

<table>
<thead>
<tr>
<th>Fourth problem. The sum of two different numbers is 350. One of these numbers is 234; what is the second number?</th>
<th>Student</th>
<th>Student’s solution to the problem</th>
<th>The student’s approach to solving the problem</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Leyla</td>
<td>350-234=116</td>
<td>She defined the problem as the determination of the second number. She said that she must conduct an operation of subtraction to solve the problem. She also stated that she was sure of the correctness of her solution.</td>
</tr>
</tbody>
</table>
|                                                                                                                | Hatice  | 234+Δ=350
Δ=116                                                         | She did not make use of prediction strategies. She showed the known number with a box and tried to calculate the unknown number in her mind. She was sure of the correctness of her solution. |
|                                                                                                                | Mehmet  | 350+234=596                   | He experienced problems reading the problem. He defined what was required as the sum of two different numbers. Thus, he expressed that he had to conduct an operation of addition. After he had explained the problem and saw the correct solution, he recognized where he had made a mistake. |
|                                                                                                                | Esra    | 350-234=116                   | She defined what was required as the determination of the second number. She expressed the solution as the subtraction of 234 from 350. She was sure of the correctness of her solution and did not produce any other solution. |
|                                                                                                                | Mustafa | The problem could not be solved | He read the word Farklı as ‘faith’. He expressed the number 234 as 400 3 2. He said that all the numbers should be added together. |
|                                                                                                                | Ömer    | The problem could not be solved | While reading the problem statement, he read the number 350 as 175. After rereading the problem statement, he said that an addition or subtraction operation should be conducted to solve the problem. When he was asked which operation should be selected for this problem, he was unable to provide an answer. |
When the answers given to the fourth problem are examined, it can be seen that Mehmet, Mustafa and Ömer experienced difficulties in reading the problem statement. While Esra, Hatice and Leyla solved the problem correctly, Mustafa and Ömer were unable to find the correct answer. For the solution to this problem, Hatice made use of prediction strategies and visuals. Esra, Hatice and Leyla said that they were sure of the correctness of their solutions.

**Table 6. Findings related to the fifth problem in the think-aloud protocol**

<table>
<thead>
<tr>
<th>Student</th>
<th>Student’s solution to the problem</th>
<th>The student’s approach to solving the problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leyla</td>
<td>50-4=6 (First solution)</td>
<td>She defined the problem as how much should be distributed to Doğa and her four friends. She said that an operation of subtraction should be conducted to solve the problem. After thinking about the problem for a while, she recognized that the operation she had conducted was wrong and then stated that she should conduct an operation of division. She was sure of the correctness of her solution. She suggested no other solution.</td>
</tr>
<tr>
<td></td>
<td>50:5=10 (Second solution)</td>
<td></td>
</tr>
<tr>
<td>Hatice</td>
<td>50:5=10</td>
<td>She said that she needed to divide 50 by 4, then recognized that her prediction for a solution was wrong and said that the number of people was 5. She then found a result by conducting this division operation. She was sure of the correctness of her answer. She offered no other solution.</td>
</tr>
<tr>
<td>Mehmet</td>
<td>46+56</td>
<td>He experienced difficulties reading the problem. He could not list what was given and required. He tried to add together the numbers mentioned in the problem statement. He could not explain why he solved the problem in the way he did.</td>
</tr>
<tr>
<td>Esra</td>
<td>50:5=10</td>
<td>She could not summarize what was given. She said that first she needed to add 6 to 50 to solve the problem. After thinking for a while, she explained what was required in the problem and suggested a solution. First, she distributed the money to the people by 5 liras. She then concluded that she needed to divide 50 liras among five people. She conducted the division operation and found the result.</td>
</tr>
<tr>
<td>Mustafa</td>
<td>The problem could not be solved</td>
<td>He could not read the problem statement.</td>
</tr>
<tr>
<td>Ömer</td>
<td>The problem could not be solved</td>
<td>He could not read the problem statement correctly. He tried to add the numbers in the problem statement together in order to solve the problem.</td>
</tr>
</tbody>
</table>

When the students’ responses given to the fifth problem are examined, it can be seen that Mehmet, Mustafa and Ömer experienced problems in reading the problem statement. Esra, Hatice and Leyla solved the problem correctly. Leyla initially made a mistake in her solution before realizing her mistake and correcting it. Only Leyla stated that she was sure of the correctness of her approach for solving the problem and the consequent result.

When the students’ responses to five problems within the context of the think-aloud protocol and problem solving skills are examined, it can be argued that only Leyla and
Hatice capitalized on guessing and self-correction strategies. Esra capitalized on guessing and self-correction strategies for only one problem. Additionally, Esra, Leyla and Hatice stated that they were sure of the solutions and results of the problems. For Esra, this held true for the problems that she could not solve. When the mistakes made by Mehmet, Mustafa and Ömer are examined, it can be seen that their biggest challenge was the difficulties experienced while reading the problem statements, as well as the mistakes made in mathematical operations. When the students were asked to construct a similar problem after seeing the correct solution to the problem, it was observed that all the students, except for Leyla, were unable to construct a similar problem. In this regard, it can be maintained that the students thought that the construction of a similar problem inferred simply changing the numbers of the already given problem. Furthermore, while Leyla, Esra and Hatice were able to explain why they had made mistakes (once they saw the mistakes they had committed in the problems that they were unable to solve), Ömer, Mehmet and Mustafa were unable to recognize the mistakes they had committed.

**Findings related to Mathematical Problem Solving Skills**

The findings related to the students' reading comprehension levels and problem solving skills are summarized in Table 7.

**Table 7. Findings related to the students’ reading levels and problem solving skills**

<table>
<thead>
<tr>
<th>Student</th>
<th>Mathematical problem solving skills</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hatice</td>
<td>She was able to determine what was required but not what was given. She could not properly explain which operations to use. She could not make predictions about the solutions to a problem and attempted different approaches to problem-solving. Her solutions to the problems and results were correct. She was able to recognize her mistakes when the correct solutions were demonstrated.</td>
</tr>
<tr>
<td>Leyla</td>
<td>She was able to solve the problem accurately. While solving a problem, she was able to proceed in line with the problem solving stages. She was able to think about different ways of solving the problem. She made use of prediction and self-correction strategies. She was mostly sure of the correctness of her solution and its result.</td>
</tr>
<tr>
<td>Esra</td>
<td>She was able to determine what was required but not what was given. She was unable to properly explain which operations to use. Her solutions to the problems were incorrect. She was able to recognize her mistakes when the correct solutions were demonstrated. She was mostly sure of the correctness of her solution and its result.</td>
</tr>
<tr>
<td>Mehmet</td>
<td>He was unable to determine what was given and required in the problem. He was unable to summarize the problem. He solved two problems correctly.</td>
</tr>
<tr>
<td>Ömer</td>
<td>He was unable to determine what was given and required in the problem. He did not know which operations to follow in order to find a solution. The approaches he followed to solve the problems and his results were false.</td>
</tr>
<tr>
<td>Mustafa</td>
<td>He was unable to determine what was given and required by the problem. He was unable to summarize the problem. He did not know which operations to follow to find a solution. The approaches he followed to solve the problems and his results were incorrect.</td>
</tr>
</tbody>
</table>
When the problem solving skills of the Hatice and Leyla were analyzed, it was observed that they were able to determine what was given and required and correctly explained the mathematical operations required for solving the problem. It was observed that in addition to being able to predict the solution to the problem, they were able to recognize their mistakes when the correct solution to the problem was demonstrated. These students were able to think about different ways for solving the problem, checked the accuracy of the problem they solved and asked similar problems based on the solved problem. Esra and Mehmet, who were found to be at the teaching level, were able to determine what was given and required in the problem; however, they were unable to make predictions about the result of the problem or test the accuracy of the problem they had solved.

Mustafa and Ömer, who were at the apprehension level, were unable to determine what was given and required by the problem and did not know which mathematical operations to conduct in order to solve the problem. These students were unable to develop different approaches for solving the problem. When they were shown the correct way of solving the problem, they were unable to answer why they had solved the problem incorrectly. In addition, the students did not mention the difficulties they experienced when reading the problem statement while evaluating their mistakes in those problems they were unable to solve. This may have been due to the fact that they were unaware that their reading levels affected their readings of the problem statement, as well as their problem-solving approaches.

Discussion, Results and Suggestions

Basic reading skills are imparted to students in elementary school during first grade. When students proceed through higher classes, acquired skills are developed further and higher skills are inculcated in students. A similar process is followed in the inculcation of mathematical skills. Here, it should be noted that the main skill to be imparted to students, both in mathematics and in all the other academic disciplines, is that of thinking; this can be realized by means of the language used by the student. People try to understand and make sense of life and academic skills through language and transfer this information together with other skills into different contexts and situations. Both language skills and mathematical problem solving skills thrive in tandem with thinking skills that capitalize on language. In short, mathematical thinking skills have become more important for modern people than mathematical operation skills. Furthermore, thinking and language skills provide the basis for this skill.

The purpose of the present study was to investigate the relationship between reading level and problem solving skills. When the behaviors demonstrated by the students during the process of solving problems were examined, it was found that their problem solving skills varied depending on their reading level. In light of the findings of this study, it can be argued that, particularly in the case of students whose reading level is at the apprehension level, difficulties are experienced in terms of reading problem statements. These students were unable to make use of strategies during their problem solving performances. In addition, it can be argued that students were distracted while reading and solving the problem. They were not aware of the mistakes they had made while reading and problem solving. Moreover, while these students were able to solve problems that had relatively shorter problem statements, they were unable to construct original problems. The students whose reading level was at the teaching level were able to vocalize problems correctly and summarize what was given and required; however, they could not determine the operations required for solving the problem. During the solution process, they made errors and did not apply strategies. The students whose reading level was at the free level were observed to use strategies during reading and problem solving in order...
to recognize their mistakes and correct them, to be sure of their answers and to be able to try different ways of solving the problem. Furthermore, regardless of reading level, students at times made errors in some problems. This was because the use of knowledge about operations and how students perceived them varied according to individual differences.

Problem solving requires reading, reading comprehension and the use of mathematical knowledge, as well as the use of mathematical operations (Bender, 2012). Students who experienced difficulties reading a problem were unable to provide correct answers. In addition, correct vocalization of the problem may not be enough for finding the correct solution to a problem, because the mathematical language involved in the problem must also be understood. There is also a need for students to develop strategies besides understanding the language of the problem and the mathematical language involved in the problem. In addition to the development of strategies, how to administer these strategies to new situations must be mastered. Strengthening teacher-student and student-student relationships is of great importance for the understanding of problems (Mercer & Sams, 2008). Therefore, reading level and the problem solving skills of students should be handled together and instructional activities should focus on the concurrent teaching of these two skills.

References


Evaluation of Students’ Mathematical Problem Solving Skills / Özsoy, Kuruyer & Çakıroğlu


