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Eliciting Mathematical Thinking of Students through Realistic Mathematics Education

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Abstract

This paper focuses on an implementation a sequence of instructional activities about addition of fractions that has been developed and implemented in grade four of primary school in Surabaya, Indonesia. The theory of Realistic Mathematics Education (RME) has been applied in the sequence, which aims to assist low attaining learners in supporting students' thinking in the addition of fractions. Based on the premise that eliciting and addressing learners' alternative conceptions in mathematics is beneficial in assisting them to improve their understanding, the paper seeks to explore the role that RME plays pertaining to this particular supposition. The paper presents and discusses examples of learners' responses to contextual problems given to them during the course of the instructional activities.

Keywords: Realistic Mathematics Education, mathematical thinking, a sequence of instructional activities

Abstrak

Makalah ini membahas tentang implementasi urutan kegiatan instruksional tentang penambahan fraksi yang telah dikembangkan dan diimplementasikan dalam empat kelas sekolah dasar di Surabaya, Indonesia. Teori Pendidikan Matematika Realistik (RME) telah diterapkan dalam urutan, yang bertujuan untuk membantu peserta didik mencapai rendah dalam mendukung pemikiran siswa pada penambahan fraksi. Berdasarkan pada premis bahwa eliciting dan mengatasi konsepsi alternatif peserta didik dalam matematika bermanfaat dalam membantu mereka untuk meningkatkan pemahaman mereka, makalah ini berusaha untuk mengeksplorasi peran bahwa RME memainkan berkaitan dengan ini anggapan tertentu. Makalah ini menyajikan dan membahas contoh tanggapan peserta didik untuk masalah kontekstual yang diberikan kepada mereka selama kegiatan instruksional.

Keywords: Pendidikan Matematika Realistik, berpikir matematika, urutan kegiatan pembelajaran

In Indonesia, one of the major goals of organizing school mathematics teaching activities is to help the learners to acquire basic mathematical knowledge as well as being good at mathematical thinking. However, the current practice of mathematics teaching activities in Indonesia does not seem to suffice to help develop the students' mathematical thinking. This is due to the fact that in the traditional classroom culture most of the mathematics teachers either are incapable of encouraging their students to express their mathematical thinking freely or the teachers themselves lack a clear understanding on mathematical thinking. To organize mathematics teaching activities for the development of mathematical thinking the teachers who play the most significant role in organizing the activities will need first to alter their own mathematical thinking. They will also need to change their instructional culture from emphasizing rote learning of mathematics content, laws, formula or theories for the development of mathematical thinking to the type of activities that will allow the students to develop mathematical thinking for themselves. This is not an easy task.

To be able to efficiently organize teaching activities for the development of mathematical thinking the teachers will need to rely on some kind of innovation that will enable them to comprehend the significance of change in instructional culture.

This is the reason why; there is a need to emphasize a shift in thinking from procedure to understanding. A progressive innovation program, i.e. PMRI (Indonesian Realistic Mathematics Education), that has been running for more than nine years, has a primary aim to reform mathematics education in Indonesia. This innovation program is adapted from RME (Realistic Mathematics Education) in the Netherlands that views mathematics as a human activity (Freudenthal, 1991) in which students build their own understanding in doing mathematics under the guidance of the teacher. In contrast to traditional mathematics education that used a ready-made mathematics procedure as a starting point for instruction, RME emphasizes mathematics education as a process of doing mathematics in reality that leads to a result, mathematics as a product.

According to this situation, we conduct design research that has purpose to develop theories about both the process of learning and means designed to support that learning (Cobb & Gravemeijer, 2006). The aim of the research is that students will gain more insight into the mathematical subject. In this case, we chosen addition of fractions as a mathematical subject/topic. The research presented in this research is design research which particularly focuses on the relation between fractions as a theme and uses an Realistic Mathematics Educations (RME) approach with measurement length as the context of the activities.

Theoretical Framework

1. Eliciting Mathematical Thinking

From the researcher point of view, mathematical thinking can be seen through many theoretical frameworks. Based on Inprasitha and other (2003) conducted a research to investigate elementary and secondary students' mathematical processes which emphasizing their mathematical thinking during solving open-ended problems. It revealed that the major obstacle to the students' successful participation in the mathematical problem solving activities was that almost all of the mathematics problems used in Indonesian schools are the exercises designed to drill the students in what they have been taught only.

In addition, the exercises usually provide only one correct answer. This has essentially inhibited students from entering into varying ways of thinking and to use different methods for working together to solve problems. Quite contrarily, the teachers should have used the open-ended problems instead of the exercises because through such approach, problems can yield various answers and offer various processes for solving the problems. The problems also can develop into other problems for solving. Such characteristics of the open-ended problems make them look like situational problems from which students can create problems for themselves.

This is a crucial condition in which the students can work together to solve the problems, and to participate in the problem-solving activities for a longer period than doing the old-pattern exercises. Furthermore, as the students engage in the solving of problems they have created, the teachers can observe their students' processes of learning and student's mathematical thinking.

The research findings also pointed to the fact that the Indonesian social and cultural context has greatly influenced the students' mathematical thinking, especially the role of mathematics teacher which seems to inhibit free expression of mathematical thinking by the students. Therefore, a change in the way the teachers administer their classroom activities from the one emphasizing presenting new subject content, giving examples and making summaries at the end of each period, to a new approach of learning activities through open-ended activities; and to change their role as providers of answers or transmitters of knowledge to those of encouraging the students to appreciate the significance of thinking. They can do this by switching from the type of questions aiming at making certain that the students make correct answers to that of questioning for the purpose of stimulating the students to reflect on or to review their own thinking.

2. Addition of Fractions

a. Interpretation of Fractions

There are some interpretations of fractions such as ratio, operator, quotient, and measure. The operator and measure interpretations are considered necessary for developing proficiency in additive operations on fractions (Fosnot & Dolk, 2002; Charlambos, et al., 2005).

In the measure aspect, a fraction can represent a measure of a quantity relative to one unit of that quantity. Lamon (1999) explained that the measure interpretation is different from the other constructs in that the number of equal parts in a unit can vary depending on how many times you partition. This successive partitioning allows to "measure" with precision. We speak of these measurements as "points" and the number line provides a model to demonstrate this. More specifically, a unit fraction is defined (i.e., 1/a) and used repeatedly to determine a distance from a preset starting point (Lamon, 2001). For example, 3/4 corresponds to the distance of 3 (1/4 units) from a given point. No wonder why this latter personality of fractions has systematically been associated with using number lines or other measuring devices (e.g., rulers, hand spam) to determine the distance from one point to another in terms of 1/a units.

In the operator aspect, Charlambos & Demetra (2006) explained that a fraction can be used as an operator to shrink and stretch a number such as 3/4 x 12 = 9 and 5/4 x 8 = 10. It could also be suggested that student lack of experience with using fractions as operators may also contribute to the common misconception that multiplication always makes bigger and division always makes smaller.

b. Addition Fractions through Measurement of Length

There are five cluster that precede operations with fractions, namely producing fractions and their operational relations, Generating equivalencies, Operating through a mediating quantity, Doing one's own productions, and On the way to rules for the operations with fractions (Streefland, 1991).

Streefland (1991) formulates the sequence of addition of fractions are described as follows:

a. Producing fractions

The activities here are concentrated on providing rich contexts at the concrete level. In solving the contextual problem, fractions are produced by means of partitioning and measuring context (Keijzer, 2003; Streefland, 1991). Attaching a length to a given unit also measures. The fraction that at first described the part-whole relationship now becomes a fraction in a measure. Through this activity, students will realize about the interpretation of fractions such as measure and operator.

b. Generating equivalencies

Partitioning as activity for producing fractions has its sequel in the treatment of situations in which division is better concealed. This also holds for increasing precision in the comparing and equivalent of fractions (Streefland, 1991). This means that the mathematical ideas under consideration will be applied more broadly. This also takes place in problem involving distance (length) relate to addition of fractions problem.

c. Operating through a mediating quantity

The point of this is to determine the length of all sort of combinations in which fractions appear. This is an indirect method of determining the addition of fractions (Streefland, 1991; Fosnot & Dolk, 2002). The idea of common whole or common denominator can be of service in mediating quantity.

d. Doing one's own productions

In this stage, attention is paid to take fractions apart and put them together in order to acquire skills in producing equivalent fractions and to sharpen students' own concept of the operations. It means that students are able to solve problems in a more and more refined manner at the symbolizing level. This takes place through using a variety of 'model of situations' and through applying production methods which become more formal. The visual models here can be of service in illustrating length. A number line and bar can also be applied for this purpose.

e. On the way to rules for the operations with fractions

Free productions at a symbolizing level focuses the attention on taking fractions apart and putting them together, keeping in mind production of equivalent of fractions and developing ideas for the operations (Streefland, 1991).

Phrasing of formal rules as an activity is not considered up to this stage. On the other hand, as many activities as possible are directed towards stimulating the students to contribute their own informal ways of working.

Realistic Mathematics Education

Realistic Mathematics Education has its roots in Hans Freudenthal's interpretation of mathematics as a human activity (Freudenthal, 1983; Gravemeijer, 1994). To this end, Freudenthal accentuated the actual activity of doing mathematics; an activity, which he envisaged should predominantly consist of organizing or mathematizing subject matter, taken from reality. Learners should therefore learn mathematics by mathematizing subject matter from real contexts and their own mathematical activity rather than from the traditional view of presenting mathematics to them as a ready-made system with general applicability (Gravemeijer, 1994). These real situations can include contextual problems or mathematically authentic contexts for learners where they experience the problem presented as relevant and real. The verb "mathematizing" or the noun "mathematization" implies activities in which one engages for the purposes of generality, certainty, exactness and brevity (Treffers, 1987; Gravemeijer, Cobb, Bowers & Whiteneack, as cited in Rasmussen & King, 2000).

Through a process of progressive mathematization, learners are given the opportunity to reinvent mathematical insights, knowledge and procedures. In doing so learners go through stages referred to in RME as horizontal and then vertical mathematization (see Figure 1). Horizontal mathematization is when learners use their informal strategies to describe and solve a contextual problem and vertical mathematization occurs when the learners' informal strategies lead them to solve the problem using mathematical language or to find a suitable algorithm (Treffers, 1987; Gravemeijer, 1994). For example, in what we would typically refer to as a "word sum", the process of extracting the important information required and using an informal strategy such as trial and error to solve the problem, would be the horizontal mathematization. Translating the problem into mathematical language through using symbols and later progressing to selecting an algorithm such as an equation could be considered vertical mathematization, as it involves working with the problem on different levels.

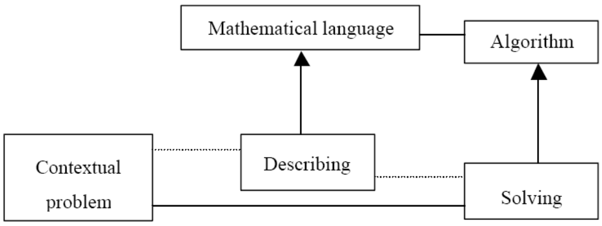


Figure 1. Horizontal mathematization (⋯⋯); vertical mathematization (⟶)(adapted from Gravemeijer, 2004)

Treffers (1987) defined five tenets for Realistic Mathematics Education, namely:

a. Phenomenological exploration

The mathematical activity is not started from formal level but it is started from a situation that is experientially real for student.

b. Using models and symbols for progressive mathematization

The second tenet of RME is bridging from concrete level to more formal level by using models and symbols.

c. Using students' own construction

Students are free to use and find their own strategies when solving problems and their strategies and products can be used to develop the next learning process.

d. Interactivity

The learning process of students is not only an individual process, but it is also a social process.

e. Intertwinement

The connection of various domains can be taken as an advantage when designing mathematical activity.

Gravemeijer (1994) mentions that there are three principles that are important in designing mathematics education based on RME, namely: guided reinvention, didactical phenomenology, and emergent models.

a. Guided reinvention

The students should experience the learning of mathematics as a process similar to the process by which mathematics was invented (Gravemeijer, 1994).

b. Didactical phenomenology

Bakker (2004) said that a phenomenology of a mathematical concept is an analysis of that concept in relation to the phenomena it organizes.

c. Emergent models

There are four levels of emergent modelling from situational to formal reasoning, namely: situational level; referential level; general level and formal level.

The levels of emergent modelling from situational to formal reasoning are shown in the following figure:

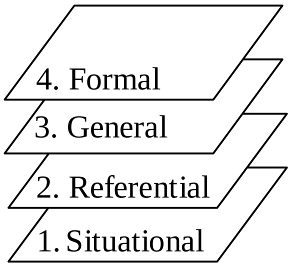


Figure 3. Levels of emergent modelling from situational to formal reasoning (Gravemeijer, 2004)

The implementation of the four levels of emergent modelling in this research is described as follows:

1. Situational level

Situational level is the basic level of emergent modelling where domain-specific, situational knowledge and strategies are used within the context of the situation. In this level, students still use their own production of symbolizing and model of thinking related to the situation.

2. Referential level

The use of models and strategies in this level refers to the situation described in the problem or, in other words, referential level is the level of models-of. A class discussion encourages students to shift from situational level to referential level when students need to make representations (drawings) as the models-of their strategies and measuring tools in the measuring activity. As an addition, the "draw number line" activity also served as referential activity in which students produced their own draw (line) to represent their way in measuring length.

3. General level

In general level, models-for emerge in which the mathematical focus on strategies dominates over the reference to the contextual problem. Student—made line produced in “making our own number line” became model-for measurement when they turned to be "blank number line" as means for measuring. In this level, the blank line were independent from the students' strategies in the measuring activity.

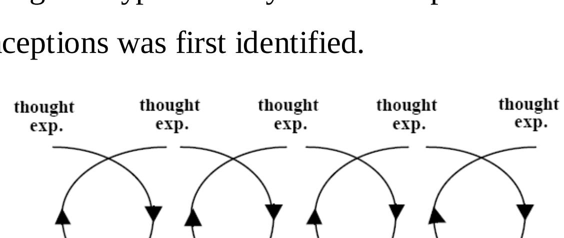
4. Formal level

In formal level, reasoning with conventional symbolizations is no longer dependent on the support of model-for mathematics activity. The focus of the discussion moves to more specific characteristics of models related to the concept of addition of fractions.

Methodology and Subject

1. Design Research Methodology

The RME theory is one that is constantly "under construction", being developed and refined in an ongoing cycle of designing, experimenting, analyzing and reflecting (Cobb & Gravemeijer**,** 2006). Design research plays a central role in this process and, in contrast to traditional instructional design models, focuses on the teaching-learning process, zooming in specifically on the mental processes of learners (Rasmussen & King, 2000). Cyclic processes of thought experiments and instructional experiments form the crux of the method of design research and serve a dual function (see Figure 2 where exp. serves as an abbreviation for experiment). They both clarify researchers' learning about learners' thinking and address the pragmatic affairs of revising instructional sequences (Cobb & Gravemeijer**,** 2006). Instructional sequences are designed by the curriculum developer who starts off with a thought experiment (abbreviated to "thought exp." in Figure 2) that imagines a route that learners could have invented for themselves. The lesson is implemented and the actual process of learning that takes place in relation to the anticipated trajectory is analyzed. This analysis can then provide valuable information in order to revise the instructional activities. It was during this type of analysis that the potential value of using RME to elicit alternative conceptions was first identified.



(Gravemeijer & Cobb, 2002)

Figure 2: Developmental research, a cumulative cyclic process

Cobb et al., (in Bakker; 2004) mentions five features of design research. The first feature is to develop theories about learning and means to support that learning. An instructional theory for measurement of lengths is designed in this research and the Indonesian traditional games are used as the starting point for the learning process of measurement of length. The second feature is interventionist nature. Design research is flexible because the designed instructional activity can be changed during research to adjust to the situation. The third feature of design research is that design research has a prospective and reflective component. After implementing the designed activity, the conjectures of each hypothesized learning process is compared to the actual learning process. The fourth feature of design research is the cyclic character of design research; invention and revision form an iterative process. The actual learning process can be used as the base for revising the next activity. The fifth feature of design research is that the theory under development has to deal with the real work.
There are three phases in this design research, namely:

1. Preliminary design
2. Teaching experiment
3. Retrospective analysis

2. Data Collection

The data of this research are written and audio visual data.

3. Subject

The research is being held in the fourth grade of SD Islam At Taqwa, Surabaya, Indonesia.

Results and Discussions

This part provides our findings in actual learning and our analysis (retrospective analysis) of the implementation. In this chapter we focus on one meeting (the last meeting) of six meeting in our implementation teaching. In the first meeting until the fifth meeting student had already learned about the interpretation of fractions (i.e. fractions as measure and operator), Comparing and equivalence of fractions, and common denominator.
The sixth activity was started by working with worksheets that preceding the class discussion. The worksheet contained three problems and had been solved by 24 students that worked in group consisting six students. The problems were *A racer*

followed the race bike. At the time of the race, the rain fell very heavy. After pedalling the bike around 2/3 of the track the racer fell because the track is slippery. And then he continue the race. But, after a quarter of the track, he fell again and he cannot continue the race because the bike was heavily damaged. First question: Could you make draw about the situation? Second question: How long the track such that you can determine every part (2/3 and ¼ of the track)? Third question: How much of the track taken by racer from the start until finally he could not continue the race?. At the end of learning, students were asked to represent their work in front of class. This activity was preceded by representation students' work to investigate students' thinking and reasoning in solving addition of fractions with different denominator.

The following excerpt is an example of a student who gave reason about using a bar as model of situation.

> *Akzal : from this to this is 2/3 of the track,*
> *Teacher : you mean that the racer fell at the first time at that point, 2/3 of the track. And then?*
> *Fahri : the racer continue the race until ¼ of the track. He fell again and could not continue the track because the bike was heavily damaged.*

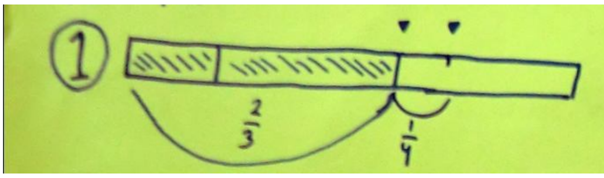


Figure 3. A bar model used by students to visualize the contextual situation.

This drawing showed that two possibilities. First, students drawn the situation by approximation. it means that the length of part is not represent the actual proportion. Second, students did not realize that the second distance is a quarter of the length of the track rather and not a quarter of the remaining path. Moreover, based on their writing on their poster, at the first time they thought that the second distance was a quarter of the rest. But in solving the second question they commenced realize that the second distance was a quarter of the track.

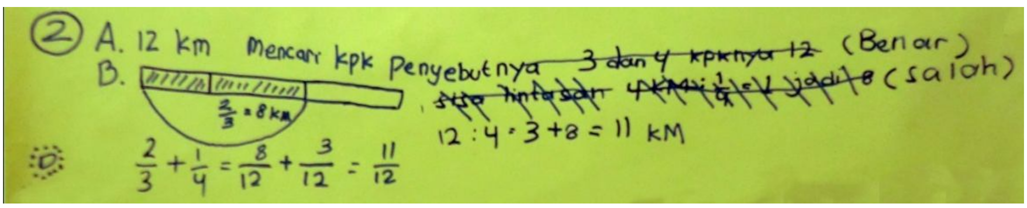


Figure 4. A bar model used by students to reason about their idea and strategy in solving problem

The following excerpt is an example of a student who gave reason about the idea of common denominator.

The problem: How long the track such that you can determine every part (2/3 and ¼ of the track)?

Akzal : 12 kilometres
Teacher : explain your answer!
Akzal : 12 is lcm of the denominators
Teacher : what are the denominators?
Akzal : 3 and 4
Teacher : what is the lcm of 3 and 4
Akzal : 12

The phrase "12 is l cm of the denominators**"** show that Akzal connected her knowledge about the idea of less common multiply of both denominator as a length of the track so that the length could be divided by 3 and 4. This phrase also show that students commenced to acquire the idea of common denominator.

The following excerpt is an example of a student who gave reason about the strategy in solving addition of fractions with different denominator.

The problem: How much of the track taken by racer from the start until finally he could not continue the race?

Akzal : because the length of the track is 12 kilometres. 2/3 of the track is 8 kilometre, because 12 divided by 3 is 4, so 1/3 of 12 is 4 kilometres
Teacher : oh, 1/3 of 12 meters is 4 kilometres?, then?
Akzal : because it is 2/3, so 2 times 4 is 8 kilometres.
Teacher : 8 kilometres, the?
Akzal : then, …
Teacher : how can the denominator is 12?
Fahri : 12 divided by three and multiply with 2.
Teacher : yes, where does the 12 come from?
Fahri : lcm of 3 and 4
Teacher : oh… from the first answer. Then
Fahri : 12 divided by 3 and multiply with 2
Teacher : then…
Akzal : 12 divided by 4 is three, and then add 8 and 3, it equals to 11. So the result is 11/12.
Teacher : 11/12. Ok.

The phrase "*2/3 of the track is 8 kilometres, because 12 divided by 3 is 4, so 1/3 of 12 is 4 kilometres*", "*because it is 2/3, so 2 times 4 is 8 kilometres***"** and their drawing show that students used their interpretation of *fractions as operator* and *measure* to determine the first distance (multiplication fractions with whole number). This phrase also show that students used *measuring length by using unit fractions as unit measurement* as strategy to multiply fractions with whole number, 1/3 of 12.

The phrase *"12 divided by three and multiply with 2"*show that students commenced to *acquire the formal way to determine multiplication* of fractions with whole number. The phrase "*12 divided by 4 is three, and then add 8 and 3, it equals to 11. So the result is 11/12*" show that students used *a bar model* to help their thinking to add fractions with different denominator. They worked with two numbers, fractions and whole number. To find the result, they used the idea of *part of a whole* and *measuring length using unit fractions as unit measurement.*

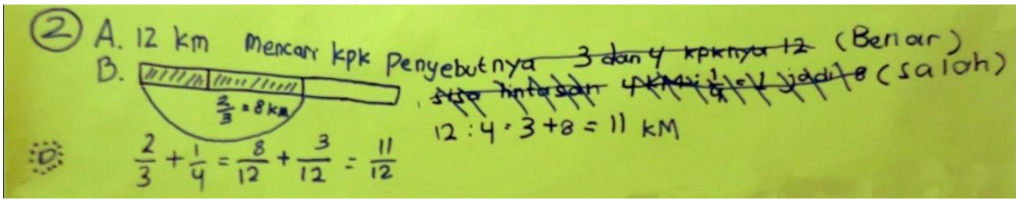


Figure 5. Work of Akzal's group in solving the addition of fractions with different denominator problem, 2/3 + ¼

What they wrote on their poster show that two conjectures of students' strategies. First, they added fractions by determining the equivalent fractions using the idea of *common denominator* and strategy in adding fractions with same denominator. Second, they worked with whole number and moved back to fractions using *a bar model.* In moving back to fractions, they used *measuring length using unit fractions as unit measurement* as strategy.

Based on students' answers in the worksheet and students' reasoning, it is conjectured that the students could add fractions with different denominator. The progress of students' reasoning in explaining their answer showed that Realistic Mathematics Education (RME) can contribute to developing learning to a more progressive learning. In our research, RME has supported the classroom activities and we have seen how students learned better in such an environment. The use of measurement contexts have supported students thinking and reasoning in solving addition of fractions. With a good context, students can construct their understanding about mathematical ideas that is meaningful so that it makes sense for them. The emergence of models supports students' transition from concrete situational problems to more formal mathematics. The model can be a bridge between informal to formal mathematics. it is a long-term learning process from a *model of* the students' situated informal strategies towards a *model for* more formal mathematical reasoning. In RME classrooms, the contributions from the students are highly promoted. Students learn to share and listen to each other’s idea through a discussion where strategies are discussed and compared to determine which ones are more sophisticated. In a discussion, students can learn from their peers and the collaborative development of knowledge among students can be made possible.

Furthermore, the implementation of RME in this design research reflects from how the principles of RME underlay the activities in this research. This implementation will be elaborated on in the following chapters: Didactical Phenomenology: measurement activity as supporting activities for thinking and reasoning addition of fractions, Guide Reinvention: teacher’s role and students’ social interaction and Emergent Modelling.

Didactical Phenomenology. The study showed that measurement context could support students’ thinking in adding fractions. In solving addition of fractions with different denominator, students also made *a bar* as visualization/model of situation. Student’s thinking process showed that how measurement context provokes students thinking in addition of fractions from the daily life problem (informal) to more formal mathematical concept of addition of fractions.

Guide Reinvention: teacher’s role and students’ social interaction. The teacher, as the facilitator of the class discussion, should stimulate students to present their ideas as the starting point of the class discussion. Teacher can stimulate students to express their idea by asking “how can you ……” or “explain your answer?”.In supporting students’ reasoning, it is also important for the teacher to help children communicate and develop their ideas by elaborating upon what they already know from their pre-knowledge or their finding in measuring activity. An example of this manner was when the teacher encouraged students to perceive the idea of equivalent fractions using doubling or multiplication as strategy. The teacher connected the comparing two kind of coloring stick to compare fractions activity by posing the following questions: “Do you remember when we compare using comparing stick? What are your findings? what can you conclude?”.

Emergent Modelling. The research also found that there was a students' model that emerged when they solved the contextual problem related to addition of fractions with same denominator and different denominator called a bar model. In general, students have accomplished the situational level of emergent modelling when they explained their interpretation and solution of measuring contextual problem (bike race problem) using drawing a bar which was partitioned as representation of fractions. Afterwards the accomplishment of the referential level was showed by describing strategies for reasoning in the measuring context with jumps on *the bar*. Moreover, the *bar* became the base of the emergence of student-made representation of situation as the *models-of* the situation that relates to the addition of fractions problem. The "*making drawing*" to explain their reasoning when they solved the addition fractions problem, 2/3 + 1/4, promoted the accomplishment of the next levels of emergent modelling. The fractions relations with jump on the bar showed how students commenced to describe their strategy for reasoning. The use of the *bar* as the *models-for* reasoning showed that *general level* of modelling has been attained by students. Students commenced to accomplish the *formal level* when they reasoned within a framework of number relations without the support of the bar. So, the *emergent modelling* (i.e. a bar model) played an important role in the shift of students reasoning from concrete experiences (informal) in the situational level towards more formal mathematical concept of addition of fractions.

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