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Development of Metacognitive and Discursive Activities in Indonesian Maths Teaching

A theory based design and test of a learning environment.

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Abstract

We report on a German-Indonesian design research project, which aims to significantly increase the mathematical skills of secondary school students. Since results of international comparative studies have shown that there exists a relationship between metacognition and learning success, a learning environment for the beginning with secondary school mathematics in class seven has been developed, in order to significantly enhance metacognitive and discursive activities of students and teachers. The effectiveness of the approach has been tested in a secondary school several times.

In this paper the theoretical background for the design of the learning environment is described, some sample exercises are presented and student productions from the project lessons analysed.

Keywords: Metacognition, Microworlds, Mental models, Metaphors, Integers

Abstrak

Dalam artikel ini kami melaporkan proyek penelitian desain Jerman-Indonesia yang bertujuan untuk meningkatkan kemampuan matematika siswa di sekolah menengah secara signifikan. Karena hasil dari studi banding internasional menunjukkan bahwa ada hubungan antara metakognisi dan keberhasilan proses belajar, maka dirancang sebuah lingkungan belajar untuk siswa kelas 7, yang bertujuan untuk meningkatkan kegiatan metakognitif dan diskurs bagi siswa dan guru. Efektivitas pendekatan ini diuji beberapa kali di sekolah menengah.

Dalam artikel ini disajikan latar belakang teoritis yang digunakan untuk pengembangan lingkungan pembelajaran, beberapa contoh tugas yang diberikan kepada siswa dan analisis hasil kerja siswa.

Keywords: Metacognition, Microworlds, Mental models, Metaphors, Integers

Introduction

Researchers from the Universitas Sanata Dharma (Yogyakarta) and the University of Osnabrück, have been closely cooperated since 1982. From the first of October 2009 until the last of December 2010 we jointly conducted the feasibility study

"Development of metakognitive and discursive activities in the Indonesian Mathematics" (MeDIM).

The study was conducted in order to determine to what extent a future pilot study, aiming to increase the mathematical skills of students in year seven of secondary school, could be successful. The pilot study's objective is to test measures which enable a larger percentage of the students to really understand the mathematical concepts and procedures elaborated in class.

Since this is a complex innovation, we considere it appropriate to develop in advance suitable tools for a feasibility study. These comprise learning materials for the students and corresponding teacher trainings, but also methods to monitor the students' learning success and the change in the way of teaching. In addition, the human and financial resources needed for the pilot study should be estimated.

An important role in the innovation plays the way of teaching, featuring two central aspects:

- Giving primacy to the development of sustainable mental models (how to deal with numbers and algebraic transformations) over the mediation of factual knowledge and the practise of calculation techniques.
- Increasing metacognitive and discursive activities in class.

T result of a design research project in Germanyshowed that when these ideas were implemented in a curriculum project, they have led to a significant improvement of students' mathematical skills (Cohors-Fresenborg & Kaune, 2001; Cohors-Fresenborg et al., 2002).

Research has shown that metacognitive and discursive activities play an important role as subject independent indicators for the teaching quality. An overview can be found in Schneider and Artelt (2010). Wang, Haertel and Walberg (1993, pp. 272f.) emphasise the relevance of metacognition for learning achievements in general. In their meta-analysis of empirical studies on the success of school learning, they observe that metacognition should be listed on a high rank regarding the influence on learning achievements.

The prospective pilot study is a design based research project, in which the development of subject-specific didactic theories and the creation of content related teaching-learning-environments go hand in hand (Wittmann 1995) and support each other.

In accordance with Gravemeijer and Cobb (2006, p. 19) the MeDIM project was implemented in three phases:

- Phase 1: *preparation and design phase*:

  As a basis for the intended teaching culture a workbook has been developed, which on the one hand offers mental models appropriate to understand whole numbers, on the other hand a task format had to be developed, encouraging the learner to engage in metacognitive and discursive activities. This workbook should put the teacher in the position to organise the teaching-learning-process in class in a different way than before. So into the design of the workbook, it was to implement the potential for a change in the mathematics education:

  - Written students' dialogues with the focus of different aspects of new mathematical concepts were to be created and incorporated in the workbook.
  - Ideas and misconceptions regarding the new mathematical concepts were to be described. They should be discussed by the students and engage them in cognitive and metacognitive activities.
  - Task solutions had to be presented, serving the students as prototypes for their own work on the task.

  The workbook “contractual arithmetics” (Kaune & Cohors-Fresenborg, 2011) – in conjunction with the intended teaching culture – plays the role of what in Wittman (1995) and Gravemeijer & Cobb (2006) is known as a learning environment.

- Phase 2*: teaching experiment phase:*

  The learning environment was implemented and tested in a secondary school in Yogyakarta with a project teacher. The Project lessons were videotaped and documents from the lessons were kept for further analysis. To evaluate the learning outcome comparative exams were conducted.

- Phase 3*: retrospective analysis phase:*

  On the base of the videotapes and student work the learning processes and effects resulting from the new learning environment have been investigated. Findings from the investigation could be incorporated into the learning environment and were beneficial to the further project development.

Recommendations for the teacher training can be formulated and a concept for an in-service training developed.

This article deals with the first phase. It will also make use of student solutions that were developed in phase 2 of the teaching experiment.

Theoretical Background

Basic mental models and metaphors

An important question for teaching and learning mathematics is how to establish appropriate models of mathematical concepts and methods for students and how they become effective. A common approach is the development of basic ideas (Fischbein, 1989) and mental models (vom Hofe, 1998). This is to say constituting the meaning of a new mathematical concept through the connection to a known context (vom Hofe, 1998, pp. 97f.). Further differentiation occurs in the created new system of mathematical concepts. The term "Grundvorstellung" (basic idea) characterises fundamental mathematical concepts or methods and their interpretation in the context of real situations (vom Hofe, 1998, p. 98).

Our approach differs from these positions in the following sense: We start elaborating pieces of knowledge that the students are familiar with (from everyday life or from their previous theoretical considerations) and expand them in the context of their experiences into a system, a microworld. In doing so we follow an analysis of Schwank (1995): "Microworlds are the external places where the external actions – caused by the cognitive activities – take place; the effects of these actions give feedback to the mental organizations and the development of mental models." (p. 104). A microworld has to function and to contain the ideas, theories and procedures that are later on utilised in the mathematical theory (Cohors-Fresenborg & Kaune, 2005). To function means, that in such a microworld an intellectual approach can be tested and experience is gained before the mathematisation is carried out. Subsequently, the behaviour which became evident in the microworld, is clarified and formulated, which is to say what brought into shape, and then written down, i.e. formalized. Whether what has been said or noted resembles the intended meaning is reflected again and again during this process. Thereby one reflects upon how the formulation is going to be used and understood by the reader.

Since this microworld has been developed based on experience, a metaphor system (Lakoff & Johnson, 1980) can be created, which can be utilised anytime by the student as an evidence-base. This intended fall-back mechanism limits the scope and effectiveness of the metaphor system in the sense that the learner truly has to internalise the system.

This is the reason why, for the design of a teaching-learning environment, we consider the development of sustainable mental models to deal with numbers and algebraic transformations more important than the mediation of factual knowledge and the practice of calculation techniques.

The prospective pilot study aims to develop the students' experience of debt and credit into the metaphor system “calculating according to contract”, within which the students have the possibility to organize the reconstruction of their intuitive knowledge about debt and credit. The transition to a contract system (axiomatic system) implies that the experience determines the norm how to behave in future situations. Therefore the application of the mathematical tool is rather generalized than abstracted.

This approach becomes effective only if the students check again and again, to what extent the new contractual reconstruction relates to their previous intuitive knowledge, and to what extend the formal representation resembles this knowledge.

Employing metacognitive and discursive activities, the students discuss again and again the meaning of mathematical notions, the eligibility of mathematical operations and correctness of calculations by drawing on the metaphor system “contractual calculating”.

Differentiation from the concept of realistic mathematics

Although our conception does follow to some extent the subject-specific teaching learning theory “Realistic Mathematics Education” (RME), going back to Freudenthal (1973), (Gravemeijer, 1994), in a critical point we go even further than Freudenthal's followers.

We begin with an analysis of balancing account transactions. This corresponds to a “scenario” as described by Gravemeijer et. al (2003,): "Of course, by saying the starting points are experientially real, we are not suggesting that the students had to experience these starting points first hand. Instead, we are saying simply that **the**

students should be able to imagine acting in the scenario, as king, for example" (p. 53).

In this way mathematics is reinvented. Gravemeijer and Cobb (2006,.) describe the process in their theoretical framework as follows: "This requires the instructional starting points to be experientially real for the students, which means that one has to present the students problem situations in which they can reason and act in a personally meaningful manner.” (p. 63).

The objective of the guided reinvention principle is that the mathematics that the students develop will also be experientially real for them. Learning mathematics should ideally be experienced as expanding one’s mathematical reality. We may further elaborate this point by clarifying the way in which Freudenthal conceives reality: ‘I prefer to apply the term reality to what common sense experiences as real at a certain stage’ (Freudenthal, 1991, p. 17). He goes on to say that reality is to be understood as a mixture of interpretation and sensual experience, which implies that mathematics, too, can become part of a person’s reality. Reality and what a person perceives as common sense is not static but grows, and is affected by the individual’s learning process."

The RME approach does offer a basis to reconstruct properties of mathematical objects, in this case negative numbers and the related operations. It does not, however, reconstruct the mathematical behaviour which is connected to the axiomatic foundation of a number system expansion, as it would be characteristic for 20th century mathematics. In line with RME approach, it is not possible for the students to develop an understanding of how those parts of the integer theory function which do not appear in the respective real scenario. They appear unexplained out of nowhere. In our opinion such an approach is one of the main reasons why at this point in the curriculum learners worldwide come to the conclusion that mathematics cannot be understood.

In order to prevent this, we further develop the practice of RME by expanding the real scenario of balancing an account into a microworld: We invent a contract, to make with the bank, such that all future transactions have to be justified on the basis of this contract. Sense and validity of the contract are obvious. This is followed by the abstraction from the idealised bank to a “Math Bank”. The “Math Bank“'s contract is in this case an agreement on how to operate with integers.

Thus “Reality”, in the sense of Freudenthal (1992, p. 17), in our concept also refers to the experience to make contracts and to keep to them.

Metacognition and Discursivity

Since Pólya (1945) a learner's activities in solving a mathematical problem have been analysed. From this, the construct of metacognition evolved in the field of cognitive psychology. A first systematic documentation of the utilisation of metacognition while learning mathematics can be found in Schoenfeld (1992). Our decomposition of the notion metacognition, which is precisely described in a category system to classify stepwise controllable reasoning (Cohors-Fresenborg & Kaune, 2007), is based on these ideas.

An important component of metacognition is seen in planning problem-solving steps, including the choice of suitable mathematical tools. In addition, in the process of problem-solving the application of these tools has to be controlled, subject relevance and target reference have to be monitored and what is already achieved has to be compared to the target in mind. This activity of control and surveillance is named “monitoring”. Understanding the problem and reflecting on intermediate results is an activity we discriminate from monitoring. It is called “reflection”.

As we have extended the focus from problem solving to concept formation and understanding as well as from an individual perspective to interactions in class, additional mental activities have to be considered, which we subsume under the category of reflection: Reflection on the adequacy of concepts and metaphors, on the choice of the mathematical approach, on conceptions and misconceptions, and on the interplay between what was said, meant, and intended (the presentation and the conception). To the category, monitoring the control of arguments has been added. In the category “planning”, planning metacognitive activities plays a role, as it might occur for example by choosing a suitable task or presenting a student's solution at the beginning of a new instruction section.

A deeper understanding of concepts, procedures chosen and applied tools is only possible if the monitoring and reflection precisely refers to what is discussed in class at the moment. To this end a contribution’s reference point has to be made obvious to those involved in the lesson's discourse and understanding of what is said has to be supported by an adequate choice of words. We subsumed the activities essential for this under the notion of discursive activities. A discursive teaching culture is central when it comes to encouraging metacognitive activities of learners.

In order to read and write mathematical knowledge accurately and to reason in the mathematics lesson the ability to realise and articulate the difference between what has been presented and what had been the intension. For this purpose it is necessary to follow the lines of an argument, estimate its applicability and to strategically place doubt and counterarguments. This shows that metacognitive and discursive activities have to be interwoven.

An insight into the usage of this category system for metacognitive and discursive activities can be found in Cohors-Fresenborg & Kaune (2007). An in-depth documentation including excerpts of mathematics lessons, which have been classified, from the first to the thirteenth school year can be found in Kaune & Fresenborg (2010).

Therory Based Development Of A Learning Environment

Initial situation

An analysis of the school books in use (Adinawan, 2006 and Marsigit, 2008) has shown that their conception do not support the development of adequate conceptions to handle whole numbers. This is due to the fact that neither explanatory texts nor the work on the exercises establish a relationship between the contents, contexts and individual mental structures. Both books introduce addition and subtraction of integers by means of an illustration with arrows. The arrows are initially shown on two different number lines, later on the same one. Here the length of the arrow represents the magnitude of the number and the direction the number’s sign.

This representation is only of limited use, as the summand zero cannot be depicted. Furthermore a formal notation of the calculation rules to operate with integers, which is developed according to this illustration, (cp. Adinawan, 2006, p. 7) favors the formation of a widespread misconception: “-a is a negative number” (Sjuts, 2002).

Outline of the learning environment's design

The design of the learning environment allows the students to develop two completely different mental models.

On the one hand we created a banking environment which is mainly based on balancing debt and credits. This model has been widely used in European textbooks for decades: “Attempts were made to reconcile ideas of assets and debts, or above and below sea level and so on, with the formal-operational structure of the negative numbers, by founding the sign-rules on concrete models” (Streefland, 1996). On the other hand we have modified the game “Hin und Her” (“back and forth”), a jumping game introduced into the German didactics of mathematics (Kliemann et al., 2009), which enables the learner to build a microworld based on the movement experience, which as well can be used later for the metaphor system to operate on integers. The game’s board and rules also contribute to develop a better orientation in the number system and mental models for the numbers by actively experiencing actions on the number line. While the banking environment pronounces more the cardinal aspect, the game stresses more the ordinal aspect**.**

In the following the design of the learning environment first is introduced emphasising the content. Here we focus on the two microworlds. Then the design is introduced in terms of a special exercise format, which aims to bring forth student’s metacognitive and discursive activities. Both times Students’ solutions to the exercises will be presented in order to show how the design is intended and how it is adopted by the students.

Construction of the microworld „debiting and crediting“

The starting point of the lesson is students’ intuitive knowledge how to deal with being in debit. First, students are made aware of their prior knowledge. To begin with, the students record deposits and withdrawals on so-called account cards of a “Math Bank”.

There are some entries missing in this account card:

Nr.	Date	Old balance	Transactions		New balance
			Withdrawal	Deposit	
1.	12.10.09	250.000		25.000	275.000
2.	17.10.09	275.000		17.500	257.500
3.	28.10.09	257.500	130.250		127.250
4.	31.10.09	127.250		18.375	145.625
5.	02.11.09	145.625		20.025	165.650

a) Fill in the card appropriately.
b) Make up a short story matching the account card.

Fig. 1: Account card which has been filled in correctly by a student

Thereby the known pieces of knowledge start to develop into a microworld. This steps' format of choice is the “account card”. The students should be able to switch between this formalisation and the content of deposits and withdrawals. This is the purpose of part b of the exercise. At the same time this part also serves to ensure that the students learn to write about mathematical content in their everyday language. We cite the following transaction story to the account card given in Figure 1 as an example. Student’s solution in the original Indonesian is provided in the appendix.

> *Ardi has got some savings in a bank. His balance is Rp 260.000,00. On 12.10.09 Ardi paid in Rp 25.000. His balance became Rp 275.000,00. On 17.10.09 Ardi withdrew Rp 17.500,00 to buy sandals. His balance became Rp 257.500. On 28.10.09 he withdrew another Rp 130.250, so that his balance became Rp 127.250. On 31.10.09 Ardi paid in Rp 18.375,00. His balance became Rp 145.625,00. On 2.11.09 Ardi paid in Rp 20.025,00. His new balance changed to Rp 165.650*

Fig. 2: Bayu’s transaction story

We did not advance in mathematical concept formation by the means of abstraction, but the intellectual approaches will be tried out and experience will be gathered before they become mathematised.

At this point it is crucial which external form of representation, for the to-be-formed concept of a “positive or negative whole (and later: rational) number”, has been chosen. This representation is developed in two steps. In Figure 3 one can see the first level of abstraction: the irrelevant information on the account card is omitted. Relevant information is depicted in a first formal shorthand notation. It still includes two designators: D for debt and K for credit (in Indonesian: debet and kredit).

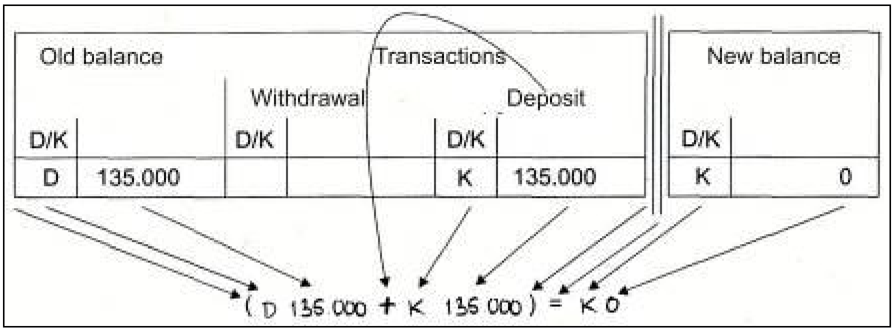
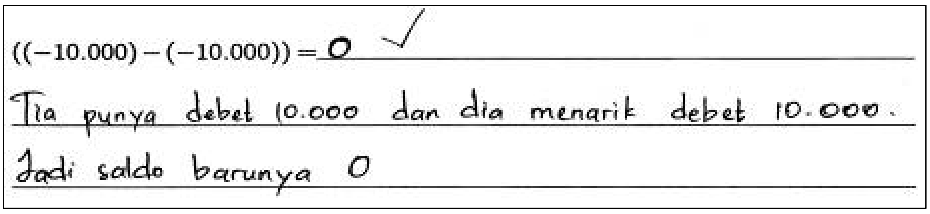


Fig. 3: A formalisation is developed on the base of the account card

Figure 4 shows how we reach the usual notation through an intermediate formalisation. In this paper we do not discuss in detail why and for how long we use the outer parentheses in the terms, and how we later on leave them out in a for the student understandable way. This is explained in the textbook.

Postings in shorthand can be written down faster than those on account cards. We can make the paperwork even shorter: Until now we have used two signs for balances: ***C*** *for* ***C****redit and* ***D*** *for* ***D****ebt (In Indonesian:* ***K*** *for* ***K****redit and* ***D*** *for* ***D****ebet). But it is enough to use only one sign. We decide to use the sign for debts. Credits will not be marked. Therefore, numbers without any sign are credits. Debts are "the opposite" of credits. The Mathbank writes (-55.000) instead of D 55.000. We call (-55.000) the inverse of 55.000.*

Exercises, in which the students have to translate formal representations into colloquial stories of transactions and vice versa, ensure that the student can recall a corresponding transaction story while reading the formal notation.


((−10.000) − (−10.000)) = 0 ✓

Tia punya debet 10.000 dan dia menarik debet 10.000.

Jadi saldo barunya 0

Fig. 4: A student's transaction story corresponding to the given formal representation:
Tia's debts come up to 10.000 and she withdraws another debt 10.000. The new balance is 0.

Tia was a student who ticked the correct calculation. To encourage the students to monitor (their own calculations) is intended by the project and is also required in the exercise. Monitoring one also has to reflect whether the writing fits to the intended meaning, how it could be used and understood by the reader.

Figure 6 shows a correct solution. Here it should be noted that the numbers in the task have been chosen to be equal on purpose. While working on the various sub-tasks the focus must be on the distinguished formal notations of deposit and withdrawal transactions.

My debts amount to Rp. 350.000 and I withdraw Rp. 400.000.	((-350.000) – 400.000) = (-750.000)
My debts amount to Rp. 350.000 and I pay in Rp. 400.000.	((-350.000)) + 400.000) = 50.000

Fig. 5: Formal notation to a given transaction story.

Transition from the microworld "Crediting and debiting" to a microworld "contractual calculating"

Next, an agreement for the accounting practice of the Math Bank is made which precisely codifies the current intuitive knowledge. We chose the same notation as for an axiomatic system in the mathematics and also use it as such for the Math Bank transactions. The designator for the paragraph's name prepares for the later utilization in the calculation contract:

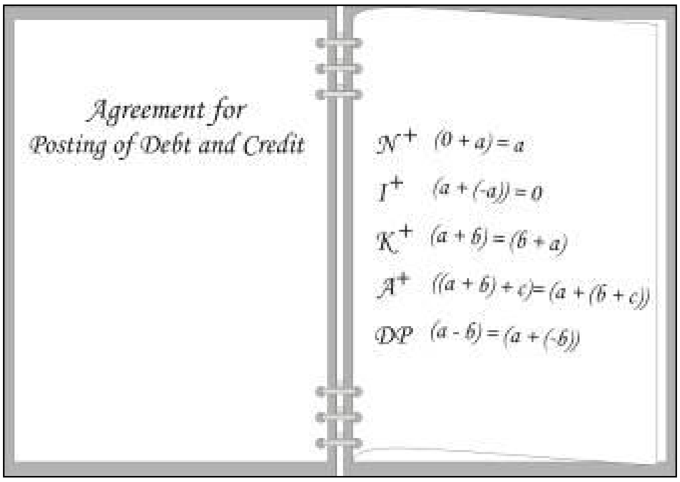


Fig. 6: Contract with the Math Bank for debiting and crediting

N$^+$: additive identity
(in Indonesian: elemen **N**etral penjumlahan)

I$^+$: Additive inverse element
(in Indonesian: elemen **I**nvers penjumlahan)

K$^+$: commutative law
(in Indonesian: "Hukum **K**omutatif)

A$^+$: associative law
(in Indonesian: "Hukum **A**sosiatif)

DP : definition of subtraction
(in Indonesian: **D**efinisi **P**engurangan

So the axiomatic system for the additive group of the integers (and later rational numbers) does not evolve through abstraction of many examples, but rather as kind of construction task to specify and formalise existing knowledge. In this context, the opening of a bank account is regulated by paragraph N$^+$, the closure by paragraph I$^{+.}$ Paragraph K$^+$ and A$^+$ describe the balance' independence on the order of the transactions, and paragraph DP defines that withdrawals can be recorded as special deposits (of the additive inverse).

In this way students are instructed to control the calculation rules and their use in the microworld “crediting and debiting”.

In a final step, we again present students with the agreement entitled “Contract to calculate with numbers“(Figure 7),.

The task to design this contract is not being understood as an application-oriented approach to teaching. Rather, in the sense of Freudentahl's “conceptual mathematisation”, the intuitive knowledge becomes precise in a real context. It is systematised and formalised and only then it develops into the mathematical concept of the integers. “The problem is not in the first meant to be solved for problem solving purposes, but the real meaning lies in the underlying exploration of new mathematical concepts.“ (De Lange 1996, p. 90)

Establishing the microworld „jumping game“

Figure 8 shows the board of the game “Jumping on the number line” and its rules. Unlike in those published versions known to us (Kliemann et al., 2009), the addition of the instruction “if you moved your man forwards or backwards, you turn it back to front” ensures that after having rolled the pair of dice (in the current move) the instruction of the “sign dice” has to be executed as an action itself. This is to guarantee that the meaning of the unary and binary functions, both of which are represented formally by the minus sign, are connected in the mind of the learner with different actions. The moves are noted in the same way as previously the transactions.

You need: one pawn with face for every player, two dice, and a playing field.

Here we go:
Every player puts his pawn beside the start field. You throw the dice and move the pawn by turns. The player whose birthday is next may start. You must get “0” for putting the pawn in the playing field. You have to put the pawn in such a way that its face looks to the front side. First you throw the plus/minus die.

The meaning of these signs is as follows:

+ : turn your pawn, so that its face looks in the positive direction;
− : turn your pawn, so that its face looks in the negative direction.

The meaning of the number die:

3 : move the pawn three steps forward in viewing direction;
(−2) : move the pawn two steps backward in viewing direction.

After you have moved your pawn, turn it in such a way that its face looks to the front direction. The winner is the person who reaches the finish first. It is not important to reach the finish by exact number. Is it clear? Have fun!

Note your move as shown in the following table:

Initial position	Plus/minus die	Number die	End position
(0	−	(−2))	2
(2	+	(−1))	1
(1	−	3)	(−2)
((−2)	−	0)	(−2)

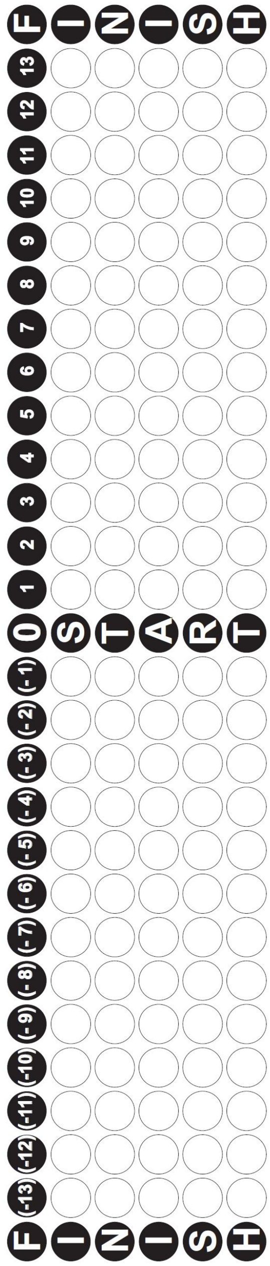


Fig. 7: The game's board and rules

Since the game is meant to be played in pairs, its implementation is a contribution to cooperative learning.

Fig. 8: Working in pairs: rolling the dice, moving the men, then representing the moves formally

The following transcription excerpt shows the discussion of:

> Maria moves her pawn two steps into the positive direction. What dice did she roll?__

Fig. 9: Exercise to clarify x + 2 = x – (-2) in the mental world of the game

The transcription starts immediately after the exercise has been read aloud.

Students	Positive. Positive.
Teacher	Positive ...?
Students	(*noisy*) Two. Positive. Null. Eh, dummy. Positive two. Plus two.
Teacher	What is the first die? What?
A student	Positive
Teacher	Positive...
Students	two
	[*Teacher writes + 2 on the whiteboard*]
	+ 2
Andre	How about minus minus two?
Teacher	Andre said…
A student	minus two
Teacher	minus...
Andre	minus minus two
Teacher	minus two [*writes – (–2) on the whiteboard*]
	+ 2 – (-2)
Students	They are the same, the same...
Teacher	Is it also possible, minus minus two?
Students	Possible...
Teacher	Agree?
Students	Agree...
Teacher	Why?
Andre	The same.

Fig. 10: Discussion: Is x + 2 = x – (-2)?

As a possible outcome is named: 

Fig. 11: Proposal 1 of several students

The teacher recorded this suggestion on the black board in shorthand notation +2. The second possibility was discussed in line20: 

Fig. 12: Andre's Proposal 2

The teacher also wrote this suggestion on the board. Without further request of the teacher some students noted that both the representations were equal. We assume they mean that both outcomes effect the man in the same way and lead to the same final score.

This short scene ended with a question from the teacher in line 31, which served to reassure and with a request to give a reason in line 33.

On the whole, one can say after this short excerpt: the exercises exhibit a cognitively activating effect on the learner, the learners adopt the microworld. They reason about the equality of terms accordingly in this microworlds?. The teacher's role in this scene is the one of a mediator, who first records the student's positions on the board and relates them to each other, but does not solve the question him/herself.

Moreover, it should be mentioned, that a further difficulty is brought about by the formulation of the exercise as the initial position is not given. For any given position on the game's board (number x) one has to find an applicable action (binary function) and another number y, such that starting from the initial position (number x) a position two boxes to the right (number x+2). On the formal level one has to find an interpretation for the function symbol $\circ$ and the term y, which transfer the equation $x \circ y = x + 2$ into a true proposition for all x. Even a student addresses this hidden difficulty in the class discourse:

54	Panta	So this exercise gives a wrong problem? (*Other students are laughing*) Lho, because it says nothing about the starting…

Fig. 13: Panta's Reflection on a difficulty in the exercise shown in Fig. 9

Whether the students have understood the basic mathematical concepts becomes evident if they are able to solve problematic situations from different contexts using these tools. This is the reason why the workbook includes a follow-up chapter, comprising examples for the use of negative numbers in the mathematisation of realistic situations.

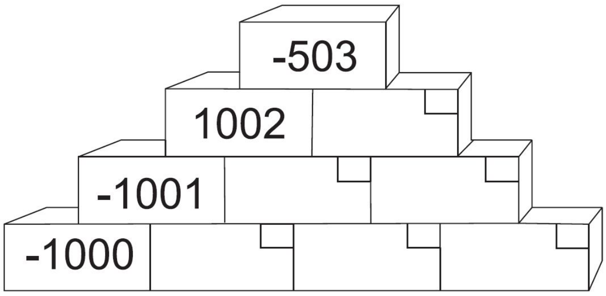
Exercises as A Means to Encourage Metacognitive Activities

Exercises are important means for the teacher to encourage the students' individual metacognitive activities. Moreover, the class discussion can help to change the way of conversation. This is the reason why we have invested a great effort into the development of xxxx. On the basis of some example tasks from (Kaune & Cohors-Fresenborg, 2011) it will be demonstrated how the theories described above interact with each other and have been used to design the tasks:

Tasks which evoke planning activities

The essence of the task is a so-called number wall. A number wall is constructed in such a way that the numbers on two adjacent stones are added up, and their sum is written on the stone centered above them.

There are six numbers missing in this addition number wall:



a) There are some possibilities to start solving this number wall. Write a “1” in the right upper corner of those bricks, which you could start with in the first step. Calculate these numbers and fill them in the number wall.
b) Mark the last brick to solve with a “3”. Calculate the rest of the numbers and fill them in in the number wall.

Fig. 14: Exercise stimulating planning activities

The task is cognitively activating as it permits different ways to complete the number walls. What makes this task special is, that it does not require students to follow one solution strategy but initiates planning activities. The learner should plan different approaches before choosing one solution.

International studies have shown how difficult it is to train a learner's planning activities such that he later uses them in a self-directed and target-oriented way (Depaepe et al., 2010). This exercise indicates our efforts to achieve that the learners plan, develop strategies on their own, and internalise them as their own ones.

Tasks which evoke monitoring activities

It is known from research that there exists a relation between monitoring activities encountered while working on a mathematical exercise and the performance: “The lack of use of metacognitive monitoring activities in practice can be regarded as an important factor to explain failure of German students in the PISA-2000E study” (Cohors-Fresenborg et al., 2010). Hence, this raises the problem to design exercises which teach the students to monitor themselves. It became evident that exercises, in which as a start only other peoples' calculations have to be monitored, are a suitable precursor before monitoring one's own thought processes and are suitable to change the way of teaching.

Most of the mathematics textbooks present perfect solutions to exercises introducing a new topic. This is important, as these exercises and their corresponding solutions help to acquire an understanding for the topic, if for example a student has been missing due to illness. This is also the case in the Indonesian textbooks we analysed. In the designed learning environment however, we confront the student with so-called find-the-mistake exercises, which require the learner to control precisely each and every transaction (calculation). This requires the students' monitoring activities. At the beginning of the teaching unit the students are asked to solve exercises of the following type (Fig. 15, 16):

There are some mistakes in the following account card. Mark the mistakes.

Nr.	Date	Old balance	Transactions		New balance
			Withdrawal	Deposit	
1.	13.09.09	130.000	25.000		105.000
2.	14.09.09	105.000		31.500	136.500
3.	18.09.09	136.500	17.500		120.000
4.	21.09.09	120.000	20.000		100.000
5.	25.09.09	100.000		30.000	70.000

Fig. 15: Find-the-mistake exercise

Fill in this table:

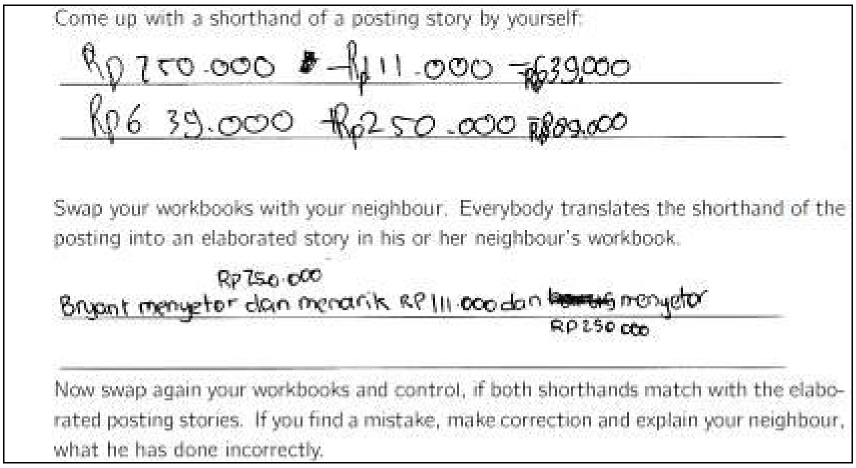
	correct	false
I have done all the postings correctly.		
There is a mistake in my postings. The first mistake is in row 1.		
There is a mistake in my postings. The first mistake is in row 2.		
There is a mistake in my postings. The first mistake is in row 3.		
There is a mistake in my postings. The first mistake is in row 4.		

Fig. 16: A measure to monitor one's own calculation

Exercises in which false arguments have to be unmasked and facts have to be corrected go beyond the control of calculations and the detection of errors. In these kind of exercises the own point of view has to be justified by means of mathematical arguments. This requires the students to critically examine a subject using language and to train their skills in reasoning and communication.

Tasks which evoke reflection activities

Tasks as in Figure 17 demand reflection activities; furthermore they require monitoring activities and encourage collaborative learning.


Come up with a shorthand of a posting story by yourself:

Rp 750.000 − Rp 11.000 = Rp 639.000

Rp 639.000 + Rp 250.000 Rp 889.000

Swap your workbooks with your neighbour. Everybody translates the shorthand of the posting into an elaborated story in his or her neighbour's workbook.

Bryant menyetor Rp 750.000 dan menarik RP 111.000 dan menyetor RP 250.000

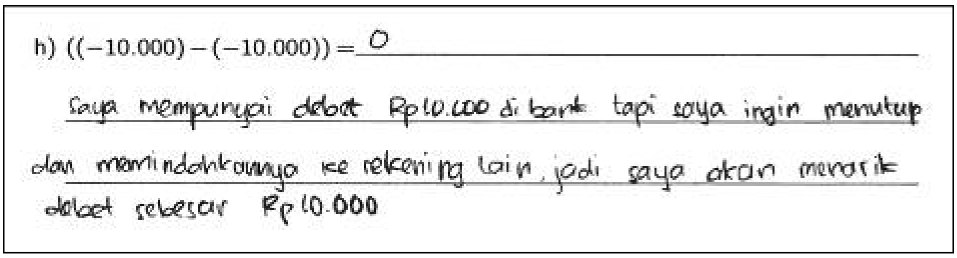
Now swap again your workbooks and control, if both shorthands match with the elaborated posting stories. If you find a mistake, make correction and explain your neighbour, what he has done incorrectly.

Fig. 17: Task designed for cooperative learning. Ruben's solution:

Bryant deposits Rp 750.000 and withdraws Rp 111.000 and deposits Rp 250.000

The exercise requires a student to analyse the structure of a formal representation, which has been written down by a classmate. The representation has to be interpreted in the microworld "crediting and debiting". This demands reflection activities. If a student identifies a mistake in the classmate's solution, this mistake has to be discussed. In the given example the students will have to discuss, whether the units (symbols for the currencies) have to be written down in the formal notation. Finally the student, who previously wrote down the formal notation, has to check whether his mathematical expression agrees with the interpretation written by his classmate. Also this subtask requires reflection and monitoring.

Figure 19 shows a student's solution to another exercise, which initiates reflection activities. In this exercise one has to write down a transaction story to a given shorthand notation of the Math Bank.


h) $((-10.000) - (-10.000)) =$ 0

saya mempunyai debet Rp10.000 di bank tapi saya ingin menutup dan memindahkannya ke rekening lain, jadi saya akan menarik debet sebesar Rp 10.000

Fig. 18: Fiola utilises one of the microworlds for her reflection:

I am Rp 10.000 in debt, but I want to close my account and move it to my other account. So, I withdraw Rp 10.000 debt.

The student solution indicates that the use of microworlds is beneficial for the operation on integers. The written transaction story provides a reason for the new balance, thus for the correctly recorded result of the mathematical exercise.

The availability of the metaphor system “Contract to operate with integers” reveals a possibility to promote the learners' reasoning skills in the stepwise controllable handling of mathematical content and to motivate learners to speak and write about mathematics and to precisely control (in a teacher independent way) their own and the others' learning outcomes.

The following two tasks show, how calculations, term rewritings, and proofs are interrelated (Cohors-Fresenborg & Kaune, 2005).

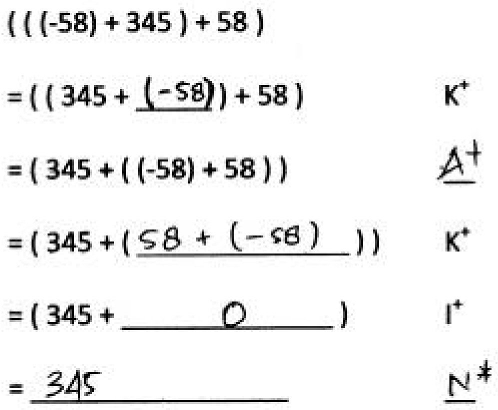


Fig. 19: Contractual Calculating

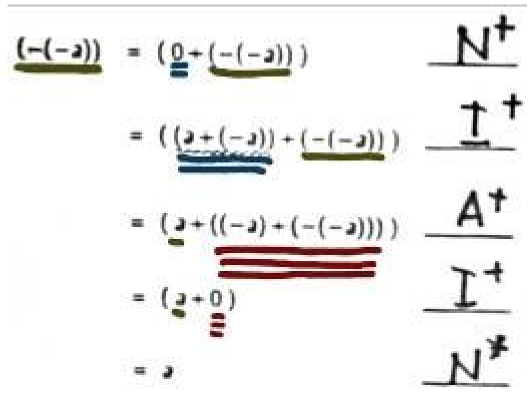


Fig. 20: A contractual proof of (-(-a))=a

In both cases it was the students' task to reflect on each of the single steps in the calculation/proof For each step they had to quote one paragraph, which legitimises this single step or had to perform an applicable transformation based on the quoted paragraph. The abbreviation of the respective name is listed at the end of each line.

Figure 20 showed the coloured underlings of sub-terms in consecutive lines of the proof. This is already the result of a first reflection on term structure and a methodical support to further structure the terms. Given this as a basis the next reflection step, this is to identify the effect of a paragraph in a given transformation, is considerably easier.

A reflection on syntax and semantic clarifies the relation between a (mathematical) object, its name and the symbol for its name. The following sentences are an excerpt from the workbook, which deals with the problem that the string (-0) does not have a meaning yet:

> *We have made an agreement with the Mathbank, saying the balance Rp 0 will be posted as a C 0. We have not yet explained, how to imagine debts of Rp 0. In mathematics the term (-0) exists as the inverse of 0. We have just derived this fact from the agreement. What is the meaning of (-0)? Let’s reflect about it.*

At the end the subsequent considerations are summarized in a three-line proof. The learners' task then consists of reflecting on each of the steps and of providing support for them by means of the respective paragraph in the contract.

Decide for every row, whether a paragraph, a theorem, or an additional agreement is used. Write the name on the line in every case:

$(0+a)$	$= a$	________
$(0+(-0))$	$= (-0)$	________
0	$= (-0)$	________

Fig. 21: Proof: $(-0) = 0$

Conclusion

The focus of the feasibility study was the development of a special learning environment for students in secondary school at the beginning of year seven in Indonesia. The design particularly draws on theories which deal with the benefit of microworlds and metaphor systems for the development of a sustainable understanding in the student's mind. In the above described learning environment it was the metaphor system “Contract to operate with integers” which has been used. Additionally, the approach provides the students with a consistent basis to integrate the concept formations necessary within the usual school mathematics, such as extensions of the number fields, term rewriting, theory of equations, and the methodology of proving. One might want to develop further mathematical content on this basis. This would have the advantage that knowledge to be acquired in the future does not have to be learned in fragments, but instead could be linked to an already existing semantic network. An increase in comprehension and understanding would be expected (Cohors-Fresenborg & Kaune, 2005).

The objective to significantly increase the students' mathematical abilities should also be achieved by changing the way of teaching, such that metacognitive and discursive activities become central aspects of the lessons. With regard to this, exercises, which are designed according to these theoretical considerations, are an important tool in the hand of the teacher. In the presented examples it has been shown by means of student examples, how the utilisation of these different kinds of tasks worked out in the experimental lessons.

A further essay will report on the details of the test and an evaluation of the results achieved. It became evident that the developed tools are beneficial and suitable for a pilot study. The pilot study is going to be conducted in several schools on Java starting in 2011/12.

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Appendix

Ardi menabung di bank dengan saldo awal Rp 250.000,00
Pada tanggal 12-10-09, Ardi melakukan penyetoran Rp 25.000
sehingga saldo baru menjadi Rp 275.000,00. Pada tanggal
17-10-09 Ardi melakukan penarik Rp 17.500,00 untuk membeli
sandal sehingga saldo baru menjadi Rp 257.500. Pada tanggal
Rp 28-10-09, Ardi kembali menarik uang Rp 130.250 sehingga
tabungannya menjadi Rp. 127.250. Pada tanggal 31-10-09
Ardi kembali menyetorkan Rp. 18.375,00 pada bank sehingga uang
di saldonya menjadi Rp. 145.625,00. Pada tanggal 2-11-09 Ardi
ke menyetorkan lagi
uang Rp 20.025,00 ke
bank sehingga
saldo barunya
Rp 165.650.

Fig. 2: A transaction story to the account card given in Figure 1

Transcription in Indonesian:

	Siswa	Positif. Positif.
6	Guru	Positif..?
	Siswa	(*Ramai*) Dua. Positif. Nol. Eh ngawur. Positif dua. Plus dua.
8	Guru	Dadu yang pertama? Apa?
	Siswa	Positif
10	Guru	Positif...
	Siswa	dua
12		[*Guru menulis + 2 di papan tulis*] +2
14	Andre	Kalau min min dua, gimana?
	Guru	Kata Andre…
16	Siswa	min dua
	Guru	min...
18	Andre	min min dua
	Guru	min dua [*menulis – (–2) di papan tulis*] +2 −(−2)
20		
	Siswa	Ya sama aja, sama aja...
22	Guru	Bisa ndak, kalau min min dua?
	Siswa	bisa....
24	Guru	Setuju?
	Siswa	setuju...
26	Guru	Mengapa?
	Andre	Sama