DESIGNING A MATHEMATICS CURRICULUM

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Abstract
A decade of PMRI saw the changes in the classroom in some of the primary schools in Indonesia. Based on observation, we can say that though the mathematics syllabus in Indonesia did not change, its curriculum has changed under the movement of PMRI. In this article, we put in writing some of the experience gained through the involvement in designing curricula since 1971. Hopefully, some of the observations made may be of use to the colleagues in Indonesia. The discussion below will cover some deciding factors in designing a curriculum, some practices, and the latest trends. For convenience, we keep the discussion general, and do not refer to a specific syllabus. Also, in many cases, we refer mainly to secondary schools, that is, Grade 7 to Grade 10.

Key words: PMRI, syllabus, mathematics curriculum, assessment, modelling

INTRODUCTION
A decade of PMRI saw the changes in the classroom in some of the primary schools in Indonesia. PMRI stands for Pendidikan Matematika Realistik Indonesia or Indonesian Realistic Mathematics Education. For a report on PMRI, see Sembiring et al (2010). We often refer to syllabus as an official document which describes what to teach in schools. We may refer to curriculum as the package of a syllabus together with the implementation tools such as textbooks and resource materials for teacher training. Then we can say that though the mathematics syllabus in Indonesia did not change, its curriculum has changed under the movement of PMRI. Naturally, the next topic of interest is the design of a mathematics curriculum encompassing PMRI.

Historically, say before the World War II or before 1945, syllabus was nothing but a collection of exercises, and teaching is nothing but going through the exercises. Afterwards, there was a textbook and teachers followed the textbook. The so-called syllabus at the time was exam syllabus, which consisted of a single sheet of items in mathematics designed for students taking a public examination. Now, a syllabus, or
some people call it standards, has the size of a book, spelling out in detail what to teach, and sometimes also how to teach.

In this article, the author put in writing some of the experience gained through the involvement in designing curricula since 1971. Hopefully, some of the observations made may be of use to the colleagues in Indonesia. The discussion below will cover some deciding factors in designing a curriculum, some practices, and the latest trends. For convenience, we keep the discussion general, and do not refer to a specific syllabus. Also, in many cases, we refer mainly to secondary schools, that is, Grade 7 to Grade 10.

DESCRIPTIVE VERSUS PRESCRIPTIVE

Typically, there are two types of syllabuses. One is descriptive and another is prescriptive. Of course, this is over-simplified. We do so for the convenience of discussion. By descriptive, we mean the syllabus is brief. It leaves a lot of room for teachers to interpret the syllabus. It could be due to the culture of the country that teaching is left to teachers to manage, and not to be dictated by those who designed the syllabus. Another type of syllabus is prescriptive sometimes to the extent that items to be excluded would be clearly stated in the syllabus. Teaching out of syllabus is not welcome by students and parents. We shall mention this issue again in Section 4.

It is interesting to observe that some of these descriptive and prescriptive syllabuses are moving closer together. More precisely, those that were descriptive tried to go for subject-specific content, that is, to be more prescriptive. On the other hand, those that were prescriptive went in the opposite direction, that is, to be less prescriptive. Those that had less national examinations now go for more tests. Those that had many national examinations now introduced various measures to downplay the impact on students of the national examinations. In other words, they are moving toward the same goal though from the two opposite ends. One common latest trend is to introduce a new element in the syllabus so that the teaching of processes can be made explicit and in some way be evaluated. We shall elaborate this in Section 7.

There are strands in a syllabus. A common list is: numbers, algebra, geometry, and statistics. Some syllabuses may have more than four strands. For example, one syllabus of an Asian country includes patterns in the primary school syllabus as a strand. The syllabus of a country in continental Europe is likely to include probability,
in place of statistics, as a strand in the secondary school syllabus. Listing an item under a given strand presupposes that a problem will be solved using a method within the strand. Some syllabus requires solving a problem with no solution method given. In other words, it does not belong to any specific strand.

Most of the syllabuses are spiral in the sense that a topic may be introduced at a lower grade, but it will be revisited again and again at a higher grade. The opposite is to put algebra together for one grade level, and geometry in another grade level. One country adopted the non-spiral approach not for the reason that it is a better sequence to arrange the topics, but for making it easier for teacher training.

Though the core remains roughly the same, how much to include and when to teach it vary widely. In view of the Maths Reforms in the 60s, or for the Southeast Asian region in the 70s, the amount of geometry to be included in a syllabus varies greatly from one country to another. One special topic under discussion recently is randomness. It is included in some syllabuses and not in the others. It has been suggested that if a topic is important enough, then we should introduce it as early as possible. For example, we should introduce randomness in the primary schools. So far, we do not know how to do it yet. If we go according to the recent research in mathematics, randomness is already an important concept.

A syllabus spells out what to teach, and sometimes how to teach, but never why we teach what we teach. The issue was discussed in Lee (2008a). This could be a factor in designing a syllabus. For example, we may want a topic to be rich in content. If public examination is a factor, then we may also want the topic to be rich in exam questions,

A major event in the history of mathematics education in the past 50 years was the Maths Reforms in the 60s and 70s. Then it was followed by another 20 years of recovery and the era of problem solving. During these 20 years the emphasis was more on the teaching approach rather than the change of content as in the Maths Reforms. Again, this is an over-simplified version of the history. In the last decade, the change was mainly in the classroom. The Maths Reforms has been generally regarded as a failure. However one cannot deny that it helped bring about the changes for some developing countries in teacher training and locally-produced textbooks. The changes would not have happened if there were no Maths Reforms.
One other outcome of the Maths Reforms is that mathematics education now becomes an academic discipline. A consequence of this is that the discussion on a syllabus is no longer the content alone but also the approaches. To some mathematicians, some of the practices introduced into the classroom are really not mathematics. This created a deep divide between mathematicians and mathematics educators.

Having a syllabus is like having the recipe of a dish. We do not know how it tastes until we have cooked it. In what follows, we shall refer to curriculum and not just syllabus.

**TEACHER FACTOR**

A reform can move only as fast as teachers can move. This is a known fact. Hence training of teachers becomes an important component in the implementation of a curriculum. During the Maths Reforms, it was a common practice to conduct workshops for in-service teachers. That seems to be the only way to train teachers to acquire new content knowledge within a short period of time. Somehow this became a standard practice till now without ever questioning whether this is a good way to train teachers. There has been research showing that a better way could be through peer learning within the school environment. However workshop remains a common practice.

For every reform, we need new textbooks. The production of textbooks became another important component in the successful implementation of a syllabus. In some countries, textbooks are produced officially or semi-officially. Some leave it to the private enterprises. The official syllabus may be the intended syllabus. Textbook should be the interpreted syllabus. Often this is not the case if not done officially with or without official sanction. A classical example is the one-thousand-page calculus books. During the 80s, after ten years of the calculus reform in the United States and with many millions of dollars spent, the consensus is to teach calculus graphically, numerically, and analytically. Also, a thin calculus book is preferred. It did not happen. It is hard to persuade publishers to forgo the huge profit that can be made from a thick calculus book.

Any reform takes time to complete a full cycle. Teachers are normally more receptive to changes that reinforce their way of teaching than to replace it. Constant
changes tend to retard the progress. Teachers would simply keep to their original way of teaching. Some schools resort to outsourcing the job to private educational agencies.

One popular means to promote a new approach in teaching is intervention. Hopefully, intervention may help change the habit of teachers and hence the way of teaching in the classroom. Sometimes it works. When it does not, it simply ends up as a fashion. A fashion is something that comes and goes within a short period of time and leaves no lasting effect.

There is a blind spot. We all know about it, but never consider it seriously, at least not as a research topic or presentation at a conference. We refer to private tuition. Ideally, there should not be a need for private tuition. In reality, there is. Though an important component, we do not seem to include this aspect in our discussion about curriculum. We shall not elaborate further here.

The lesson we learn from the past is that we must prepare the teachers before any changes in the curriculum. We cannot afford to rush.

**STUDENT PROFILE**

Society has changed. So has the student profile. For example, the older people listen to radio, whereas the younger people watch television. The older people write by hand, whereas the younger people work on the computer. Students are at the receiving end of the teaching process. The environment definitely affects their learning. Queena Lee-Chua et al (2007) in their research identified ten best practices in student learning. Top of the list is learning habit or more precisely discipline. Hence it is important that we know our students and design a curriculum with students in mind.

It is a trend to talk about the competencies for the twenty-first century. See, for example, OECD (1997). In the past, we sent children to schools to be educated. Now, we want them to learn a trade so that they can get a job after schooling. The new jobs require them to be a thinking person and an independent learner. Hence teaching mathematics is not just teaching mathematics alone. It is also part of training for the work place. In order to be creative, one has to teach out of syllabus. In other words, teaching out of syllabus will soon become a norm and not an exception. Syllabus is a home, and not a prison. Furthermore, students must be taught to follow rules first
before they learn to break them. This adds a heavy responsibility to the designers of a curriculum.

Negative number is a difficult concept. In the 70s, textbook writers and teachers made great effort to distinguish the negative symbol from the minus sign. Finally, they gave up. A common practice now is to explain addition of negative numbers using the number line. For multiplication of negative numbers, they appeal to the patterns as using the number line will be too demanding. This is done in view of the change in student profile.

Another example is teaching of fractions. In the past, there were more fractions and less decimals. Now we still teach simple fractions, and move on quickly to decimals, then come back to fractions again. Computation of fractions is gradually replaced by decimals. This is due to the fact that students use calculators. The habit of students has changed. Hence the design of a curriculum should also change.

Often student profile is not a top priority in designing a curriculum. Knowing the learning habit of our students is as important as selecting the topics to be included in a syllabus. In one sentence, we may wish to pay greater attention to the student profile while designing a curriculum.

DIFFERENTIATED SYLLABUSES

The issue of Mathematics for All was discussed extensively at the international conferences in the 90s. The essence was that all students should study mathematics and up to a certain level. This created immense difficulty for some students. The questions were and still are how to help them and whether they really need it. This is a general problem in education and not mathematics alone. Most countries practise some kind of streaming and some do it more rigorously than others. One consequence of such severe streaming is that after 25 years we may produce a group of students who would never make it anywhere within the existing educational system. Consequently, special schools had to be set up to cater to this group of students.

The practice of differentiated syllabuses or differentiated curricula is not new. School Mathematics Project (SMP) of Southampton, England, produced a series of secondary school textbooks during the Maths Reforms. The first series was numbered 1, 2, 3 etc. Then a second series was produced labelled as a, b, c etc. This was
obviously differentiated curricula. What is interesting is that the second series was meant to complement the main series, which was the first series. As it turned out, the second series overtook the first as the main series. What is even more interesting is that 30 years later the textbooks gradually shifted back to something more traditional. It is the kind of practices that can win over more hearts stayed on.

Every country has its own way in dealing with this problem. It is said that Japanese syllabus is a minimum syllabus in the sense that every teacher goes beyond it. Similarly, the U. S. standards is a maximum syllabus in the sense that nobody reaches it. Singapore adopted differentiated syllabuses (Lee, 2008), like many other countries did it in one way or the other. Two characteristics stood out when Asian curricula were reviewed in the literature. One is the practice of explicit systematic instruction (U. S. Department of Mathematics, 2008, p.48). Another is the attention of the affective domain. It seems that confidence, belief, and commitment play an important role in learning, especially in the Asian culture.

Assume that every student needs to learn some mathematics. In connection with the design of a curriculum, a solution could be differentiated syllabuses or differentiated curricula.

**ASSESSMENT**

Initially, assessment was meant to be assessment of learning. Then people started advocating assessment for learning. In the past two years, people invented another new term assessment as learning. Roughly speaking, assessment of learning means “we assess what is learned”. Assessment for learning means “we learn from what is assessed”. Assessment as learning means “assessing is a way of learning”. It is too early to say whether the third version of assessment will work as well as the first two. Teaching must always come before learning. Few people made it to the top by self-learning. In Chinese, teaching-and-learning is one word, and not two words. That says a lot about the meaning of teaching and learning.

There is nothing wrong to teach for examination. Here by examination, we mean public examinations. What is wrong is to teach for examination only. There is nothing wrong to go for tuition. What is wrong is to rely heavily on tuition. As far as we can see, examination will stay at least in the near future. It is entirely within our reins to downplay the importance of examination. We may even make positive use of
examination. If examination can drive learning, examination can also drive the change in learning. In other words, examination definitely plays a role in the design of a curriculum.

MODELLING

The latest change in the classroom is to teach in context and to emphasize on the processes. This is not new. This was exactly the way mathematics was learned in ancient China over two thousand years ago till the end of the nineteenth century. However this is not reflected explicitly in the current syllabus. In 2010, one Asian country and one European country introduced independently a new element into their revised syllabuses, to be announced in 2011. One calls it learning experience and the other knowledge requirements. The idea is to make explicit the processes to be learned for the topics in the syllabus. More precisely, other than topics listed, an attempt is made to put in words experience that students need to go through and the expected outcome. It may be easy to assess the product of a process. It may not be so easy to assess the process itself. This new inclusion is in some way to formalize the common practices in the classroom.

Geometry with proofs and mechanics were the two essential subjects in school mathematics before the Maths Reforms. Both involve processes more so than algebra. Now algebra dominates the school mathematics. We lost two subjects rich in content and rich in exam questions. In other words, we lost two rich training grounds to teach thinking process and to teach applications. So far, there is no replacement.

To specify experience that students need to go through is one way to remedy the loss. Another way to compensate for the situation is to incorporate modelling into the curriculum. Many countries have done it, for example, Australia and Germany. Singapore in 2010 organized a school out-reach event on modelling trying to introduce modelling into the curriculum. The success or otherwise of the modelling project depends on how the participants play it. The hope is that learning experience and modelling might help bring back some of the things that we have lost in geometry and mechanics. We can only wait and see.
CONCLUSION

A curriculum is a good curriculum only when we have implemented it successfully. To be successful, it has to be considered in connection with teachers, students, and many other determining factors. We have included teacher factor and student profile in Sections 3 and 4 respectively. We highlighted one common practice, namely differentiated syllabuses, in Section 5. Finally, we mentioned certain new trends in Sections 6 and 7. A short summary is given below.

- Mathematics is the core and not how to teach it.
- A reform can move only as fast as teachers can move.
- We train our students for the work place different from our own.
- We need to cater to students of different abilities.
- We should explore the positive aspect of the examination system.
- Make explicit in a syllabus teaching in context and emphasis on processes.

We omitted at least two subjects in this article. We did not dwell upon the process of reviewing a syllabus and the revision of the curriculum that follows. Also, we did not mention the evaluation, if any, of the implementation of newly revised syllabuses. The review was not always transparent. The approaches were often diverse. One does not know the detail unless one is personally involved. After the revision, implementation takes place. The evaluation of the implementation does not always happen. The evaluation is an important aspect of any reform. However this is a neglected area, especially in the developing countries. As this is beyond the scope of designing a curriculum, we shall not discuss here.

It is common to take a generation or 25 years to find out whether we have made a correct decision on a major educational policy. Further, it would take another 25 years to correct it if the decision was found to be wrong. We can give a few examples. Perhaps designing a curriculum is not so serious. The fact that it takes time to find out right or wrong is the same.

When we review our syllabus, we look around among other countries and compare with them. Perhaps we should also look back in time. Find out what we have done right and what we have done wrong. In order to do that, we need good documentation of the past events. In other words, we must have institutional memory. Activity-based teaching of PMRI may have made in-roads into the classroom in
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Indonesia. It will be the design of a locally-produced curriculum that will lead the movement of PMRI to become a main stream in the reform of mathematics teaching, in particular, and education, in general, in Indonesia.

REFERENCES


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