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BIRTHDAY CAKE ACTIVITY STRUCTURED ARRANGEMENT FOR HELPING CHILDREN DETERMINING QUANTITIES

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Abstract

Few researches have been concerned about relation between children's spatial thinking and number sense. Narrowing for this small research, we focused on one component of spatial thinking, that is structuring objects, and one component of number senses, that is cardinality by determining quantities. This study focused on a design research that was conducted in Indonesia in which we investigated pre-school children's (between 2 and 3.5 years old) ability in making structured arrangement and their ability to determine the quantities by looking at the arrangements. The result shows us that some of the children were able to make such arrangement. However, the children found difficulties either to determine quantities from those arrangements or to compare some structures to easily recognize number of objects.

Keywords: structures, structured arrangement, cardinality

INTRODUCTION

Recent years, theories about relation between spatial thinking and number sense have been developed in the educational research community. The developers in this domain believe in early mathematical thinking young children mentally apply spatial configuration to determine an amount (Nicol et al., 2004; Mulligan et al., 2004). Some studies have been concerned on primary group children that are more than and equal to 4 years old (van Nes et al., 2006; Nicol et al., 2004; Mulligan et al., 2004), spreading out in some countries in Europe, America, and Australia. These studies showed the role of structured arrangement for the development of children's number sense.

Considering the important of that domain and realizing that lack of research about this domain in Indonesia, we designed a study to examine young children's ability in determining quantities by looking at the arrangements they made. I worked with pre school children, between 2 and 3.5 years old in order to rich the results of the domain. Besides, most children develop fundamental number sense before they receive formal education in primary school (Jordan et al., 2006), and pre school

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children mostly learn basic counting principles (Fuson and Wynn in Jordan et al., 2006). This report discusses an experimental study in which we aimed to better understanding emergence of the relationship between spatial thinking and number sense during children's activities.

THEORETICAL FRAMEWORK

We used some theoretical backgrounds to underpin this project, those were: Structuring objects and determining quantities – as concepts behind our research goals; Realistic Mathematics Education (RME) – as an approach of mathematical lesson; Social Constructivism Paradigm – as we found that this paradigm has a similar essence with RME; and psychological theory for young children (between 2 – 4 years old) – as a basic of our analysis and evaluation of the results.

1. Structuring Objects and Determining Quantities

Structuring objects is a part of spatial ability. We defined the word structuring objects practically referring to arranging objects in such away so that others can easily recognize the quantities and find easy ways to count the objects.

Furthermore, this research would avoid the word 'structure' practically. The teacher is suggested using another word to describe the word 'structure'. She can use 'arrangement' or 'nice arrangement' to represent that word. As long as the teacher understand what we mean with the word 'structure'. Hiele (1986) defined structures into two categories. They are rigid structures and feeble structures. Mathematical structures are very rigid if the rule of the structure is given. You can extend them without making mistakes. Meanwhile, feeble structures do not have the structure of mature work so that it is difficult to recognize the early work.

What we mean by the word 'structured arrangement' in this study refers to word 'structures'. Even van Hiele (1986) did not give exact definition of structure. He stated some characteristics to recognize what structure is. Structure is objective: Other people see a structure just as we do. Specifically, van Nes (2006) in her power point presentation gave some familiar examples for structures, i.e. dice and egg box.

This study is expected leading children to make rigid structures when they arrange the candles. Even though in special cases they probably create their own arrangement unstructurally, we can define it as 'the structure' for them. More or less, definition of feeble structures can argue as a basic concept of the meaning of structures for this kind of children.

Two key concepts of number sense are ordinality and cardinality. Ordinality is something that related with order of numbers, and cardinality is aspect to understand the meaning of a number. One part of cardinality aspect is determining quantities. For preschool children, ability to determine quantities as a result of counting is difficult to reach. Mostly, children at this age (between 2 until 3.5 years old) do not understand the meaning of 'how many' question (Griffin, 2005). As main goal of this research, by making their own arrangement, they will be able to apply it to determine the quantities. What we expected is they will be helped by structured arrangement and gave them challenge to determine the quantities at a glance by the help of the arrangement.

2. Realistic Mathematics Education

Realistic Mathematics Education (RME) is a teaching and learning theory in mathematics education that was first introduced and developed by the Freudenthal Institute in the Netherlands. The present form of RME is mostly determined by Freudenthal's view on mathematics (Freudenthal, 1991). Two of his important points of views are mathematics must be connected to reality and mathematics as a human activity. In teaching mathematics realistic, a teacher should use a kind of context to start a class activity. This context should be able to promote mathematizing – the activity of interpreting, organizing, and constructing meaning of situations with mathematical modeling. This mathematizing involves the setting up of quantifiable and spatial relationships (Gravemeijer, 1994).

This study brings the idea of relation between spatial thinking and number sense. Some researchers in this domain believe that children's ability in number is related with their spatial skills when they mentally arrange objects to easy themselves counting those objects. Briefly, one component of spatial thinking – structuring objects – is used to examine their number sense – determining quantities, whether that ability help them in easy counting or not.

Starting with a contextual situation about birthday party, this activity was designed following the path of Hypothetical Learning Trajectory (HLT). We will go further about our HLT in the next section. There are three components in context that allow students to mathematize the given situation (Fosnot and Dolk, 2001):

1. The potential to model the situation must be built in.
2. The situation needs to allow students to realize what they are doing.
3. The situation prompts learners to ask question, notice patterns, wonder, and ask "why and what if".

3. Social Constructivism Approach

We will try to serve this project in socio-constructivist approach. As Woolfolk (2007) said, social constructivism – such as Vygotsky believe – is a social interaction, cultural tools, and activity that shape individual development and learning. By participating in a broad range of activities with others, learners appropriate the outcomes produced by working together; they acquire new strategies and knowledge in their world. Rather than seeing learning process as individual construction as in constructivism, social constructivism see the learning process as a social constructed knowledge. Hence it is built on what participants contribute and construct it together. There is collaboration between the students during the process of learning and it occurs through socially construction opportunity. Teacher's role is not only as a facilitator but also as a co-participant and a co-construct different interpretation of knowledge.

This study brought the idea of social constructivism. Instead of working individually, the children were asked to work in pair. They were expected to share each other and make the arrangements together. They had to realize that the arrangement is the result of their collaboration. The whole design provides such activities in where they need to share with their partner. Even though it is difficult for them to share their idea, sharing in this study is more about sharing in social society as an individual of a social community.

There are two main similarities between RME and socio-constructivist in mathematics education (de Lange, 1996). First, both the socio-constructivist and realistic mathematics education are developed independently of constructivism. Second, in both approaches students are offered opportunities to share their experiences with others. In addition, de Lange (1996) stated that the compatibilities of socio-constructivist and RME are based on a large part or similar characterizations of mathematics and mathematics learning. Those are: (1) both struggles with the idea that mathematics is a creative human activity; (2) that mathematical learning occurs as students develop effective ways to solve problems; and (3) both aim at mathematical actions that are transformed into mathematical objects (Freudenthal, 1991).

4. Psychological Theory of Young Children.

Pre school children are able to develop early number sense. Van de Heuvel-Panhuizen (2001) generally classified development of an elementary number sense at pre school level along three broad levels: (1) the level of context-bound counting-and calculating; (2) the level of object-bound counting-and calculating; and (3) the level of pure counting-and calculating. This classification allows the teacher recognizing children's level thinking in a way that they mostly work at such level.

Van Hiele (1986) also had classification of level thinking. However, practically we will not use Van Hiele's levels of thinking to define what in children's minds for our analysis. Since we argue that van Hiele's worked in Geometry domain, it is not appropriate to be used to examine our children in this project. Therefore, we prefer drawing on van de Heuvel-Panhuizen's levels of thinking to determine children's mathematical thinking, since we worked with the role of structures to build up children's number sense.

Within what are, for them, meaningful context situations, children are able to count at least ten, arrange numbers in the correct order, make reasonable estimates, and compare quantities as being more, less or equal (van de Heuvel-Panhuizen, Buys, and Treffers, 2001). At this age level, comparing two quantities is a large leap for them, even though they have to see the quantities of the objects with their naked eyes to do so. Griffin (2005) stated that children who are successful with this sense appear to know (1) that numbers indicate quantity and therefore (2) that numbers themselves have magnitude, and (3) that numbers, which come later in the sequence, indicate larger quantities. In this study, we choose number 5, because the counting of quantities greater than five can not taken in at a glance (van de Heuvel-Panhuizen, Buys, and Treffers, 2001). Therefore, structuring the objects is expected help them to build up their number sense as in our hypothesis.

GOAL OF THE RESEARCH

The Main Goal of This Research is to help young children realize that making a structured arrangement of objects can help them easily determining quantities.

Besides, we have some goals related with our research and the learning experiment of young children:

a. Mathematical Goals:

1. Children will discover as many structures of 5 candles as possible by placing them on a cake.
2. Children will work to find out the easiest structure that they can use to easily recognize 5 objects without counting them.

b. Didactical Goals:

1. Children should be able to find the solution on *their own*.
2. Children should be able to reason their own decision.
3. Children should be able to make an agreement among them.

HYPOTHESIS AND RESEARCH QUESTION

Our own hypothesis, which underlies this study, is making a structured arrangement of objects can help young children easily determining quantities.

Related with the hypothesis, we formulated a research question for this study, that is: "Can making a structured arrangement of objects help young children to easily determine quantities?"

The answer of this research question will be a conclusion of the whole study as well as the brief attainment of the goals above.

HYPOTHETICAL LEARNING TRAJECTORY

Based on the theory of RME, we know that giving a meaningful situation to young children can keep their activeness in the learning process. In this case, the role of the teacher is extremely important. We thought about how the teacher could proactively support children's mathematical development. It is proper with what Simon (in Gravemeijer, 2004) called as "Hypothetical Learning Trajectories (HLT)". The teacher has to be able to see how the thinking and learning in which the students could engage as they participate in certain instructional activities relates to the chosen goals.

However, it is not easy task to do for the teacher. That is why we also thought about what kind of support that we could give to the teacher. Of course, the support materials are not fixed things. The design that we wrote down in the teacher manual should be flexible and giving the teacher chance to do improvisations in her classroom situation. Therefore, we offered some framework of reference in the teacher manual, including of the learning goals, the instructional activities, our expectations and our hypothesis of what in children's mind, and the tools and imagery, as a source of inspiration.

In this one-day study, we used some tool that can support children's mathematical thinking. The tools and the imagery are explained in the following table:

Table 1
Overview of the Proposed Role of Tools in the Instructional Sequence

Tool	Imagery	Activity	Expectation
Pictures of birthday cakes	-	Observing candles' position on the cake	Realizing the candles on the cake
5 candles and a cake	Record of the candles' position from the pictures	Placing the candles and making nice arrangements	Making unstructured and structured arrangements
Children's arrangement	Record of their own arrangement	Comparing the arrangement to easily determine the quantities	Finding the easiest structures for them to recognize the quantities in a glance

The activities would be conducted in a frame of these following three sections:

1. **Bringing the Contextual Situation**
2. **Creating the Structures of Candles**
3. **Finding the Easiest Structures**

Furthermore, I explain each section and its relation with the goal and the activities. I also put some essential features that showed our expectation of each path in our HLT, in where each previous feature influenced the following futures.

Table 2
Overview of the HLT in the Instructional Sequence

Sections	What the teacher should do	The Goal of the activities	Awareness	Essential Features
1. Bringing the Contextual Situation	• Tell the story to the children, e.g a story about a birthday party of five year boy • Show the slide of birthday cake pictures • Pose the problem to the children (see design activities) • Show a birthday cake and five candles. • Making rules for the activities: floor the social norm, e.g. attend to the other comments, raise thumb when they have question or need help, etc.	To involve children in the problem by bringing contextual situation about birthday cake and candles in a classroom.	• You can create your own story which is **related with candles and birthday cakes**. • **Be careful with the context** you bring to appear the problem!! • Strengthen the reason that is in order to make invited people easily determine the age of kid who celebrate her/his birthday.	Understand the problem and give the feedback for the story, (e.g. they raise their hands and start telling their experiences about birthday party). • Understanding the problem is important to help the children solve the problem. • Giving feedback indicate children's enthusiasm and involvement in this contextual situation.
2. Creating the Structures of Candles	• Ask children to arrange 5 candles on a cake. • Ask them to discuss it with their partner, beside them, before they show their arrangement. • After they make an arrangement, ask them to draw the each configuration they made on a square paper. • When children in work, give them guidance by posing the questions, so that they can find as many as structures of candles with their own way.	To help developing spatial thinking of young children by structuring 5 candles on a cake. Give them chance to look for many different ways to structure a number of objects.	Let children decide the position themselves. **However,** you can pose the question as guidance in this order: 1. Now, discuss with your partner beside you, how to arrange these 5 candles on this cake! Who wants to be the first? 2. Can you make other arrangements of 5 candles? Any other way to put these candles? 3. How about the other group? Do you find out another way to arrange these candles?	• By understanding the problem, children will be able to: Making unstructured arrangements of five objects which lead them to making structured arrangements of five objects, so that they can compare between unstructured and structured arrangements.
3. Finding the Easiest Structures	• There will be many different structures of 5 candles, e.g structures of dice, a line, etc . • Ask them question to find out which is the easiest structure that they can easily recognize for structured arrangements. • Children can give their ideas about it. • Bring them to get an agreement for the easiest structure, e.g structure 5 in dice. • Ask them to draw their finding (the easiest structure of 5 candles) on a paper, so that every child surely knows what they learn today.	To strengthen children's spatial thinking in structuring objects To help children establish number sense in determining quantities.	Teacher can pose this question to guide children: "How can you easily recognize the number of candles without count it? I mean, can you make very nice and good arrangement of the candles so that when the other person comes to our class and see this cake, he/she can directly recognize these 5 candles without count it?"	• After comparing between unstructured and structured objects, children will be able to: Comparing structured arrangements of five objects for applying the ability to recognize familiar structured arrangement, as the easiest structure for determining quantities.

METHODS

Participants

Six pre school children in Honey Bee Pre School Surabaya were participated in this research. They were separated in three groups:

Group 1: Randy (3.5 years old) and Rayya (2.5 years old)

Group 2: Keisha (3 years old) and Reiko (2.5 years old)

Group 3: Bryan (2 years old) and Keitaro (2 years old)

The teacher classified them based on their age and gender. Each group was in guided of a teacher assistant.

Materials and procedure

As the main goal of this study, we designed one day activities for the children to investigate their ability on making structured arrangement that easy for them to determine the quantities of candles. To make it more interesting for them, we used real birthday cakes instead of paper-and-pencil stuff to create their own arrangement of five candles. Furthermore, the task would be challenging for them to arrange the candles structurally. However, in the middle of the task we asked them to record their arrangement on a piece of paper. This task was provided to examine their ability of symbolizing real situations. Of course, the guidance of teachers is more given for these very young children.

The activities were inspired from literature of van de Heuvel-Panhuizen (2001). Considering of level one at the pre school level in early number sense, the level of context-bound counting-and-calculating, the tasks were posed would be expected meaningful for them with such two questions, namely: “How many candles?” and “How old?” or “How many years old?”.

The study was performed in three sections. Those sections described following path of the HLT. As mentioned in our HLT, each section brings some essential features that are as our expectations. We would look into children’s arrangement and their actions during the activities rather than their ability of reasoning. Therefore, the result will be analyzed qualitatively.

In order to keep validity of the result, we elaborated three views of observations. They were an observer of teacher experiment (to keep teacher’s actions stay in our HLT), an observer of class experiment (to observe students activities both in observer point of view and teacher point of view), and the video recorder (to explore other verbal and non verbal signs, both from teachers and from children).

RESULTS AND ANALYSIS

Before going detail of our analysis, generally we are going to examine general situation and compare it with our design. From the video recorder and observation of teacher experiment, we saw that some of the whole class experiments were going out of our design. Table 1 shows us some the summary of differences between our expectations in our design and the class experiment:

Our design	Class experiment
• Giving story for bringing the context • Give a square-shape-cake in front of class • The first group making arrangement in front of the class • Making record on paper • Other groups giving comments • Other groups making their own arrangement in front of the class • Repeat and continue the process until getting structured arrangement • Comparing the structures	• Giving story for bringing the context • Give a circle-shape-cake for each group • Each group working separately in guiding of a teacher assistant • Making record on paper • Teacher made structured arrangement • Comparing the structures

Table 1. Differences between the design and the class experiment

Realizing that difference given treatments would be giving result that out of our expectations, we tried to analyze children's behavior and their actions during the activities. Using actor point of views, we recognized that the teacher tried hard to apply our design and made some improvisations that they thought more suitable for their children. The observer gave comments in the observation sheet about the teacher:

- The teacher has done all she could have. She told the story and explained the instructions before she looked around to each table and helped the children made clear of the instructions like putting the candles, counting the candles, and changing the broken candles.
- Since a boy seems prefer walking around to sitting down, the teacher seems to have difficulties to manage them, with the help of other teacher she then could put the class under control.
- The other teachers helped each group to draw in the paper.

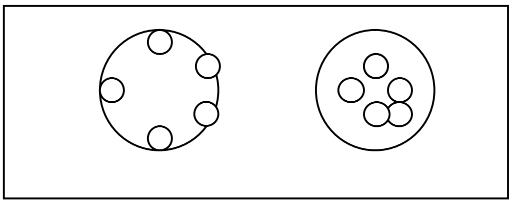
Most of the time, she thought that mathematics was only about counting and number. It was difficult for them to be a mathematician as well as to be a mathematics educator in the same time, so that to realize that structuring objects are also part of mathematics is a hard work for them. Besides, neither controlling such very young children nor leading them to grasp knowledge is an easy job.

Meanwhile, we saw that the children did the tasks and struggled so much in determination of the quantities. The teacher wrote addition note in observation sheet about their struggling of the tasks:

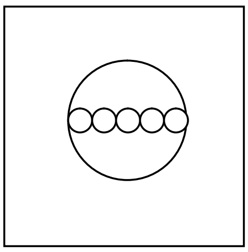
- A group was more interested in the cakes. Another group was more interested in breaking the candles.
- They shared their ideas by doing actions.
- In this age, 2-3.5 years old children, they are still individualist, so they are still in the stage giving attention and following instructions.

However, we believe something happened in their mind during the activities, something that related with their mathematical thinking. They used to count one by one when they were asked "how many?" Therefore, asking them directly determining quantities from their arrangement is more challenging for them.

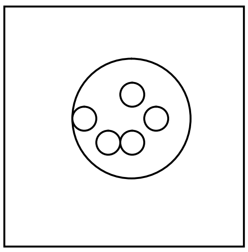
Some findings of those young children's acting were evidence for mathematical thinking in their minds. Each group created different arrangement. Picture 1, 2 and 3 show their arrangements.



Picture 1. Randy and Rayya's arrangement



Picture 2. Keisha and Reiko's arrangement



Picture 3. Bryan and Keitaro's arrangement

From Picture 1, we can see that Randy and Rayya firstly arranged the candles following the side of the circle-shape-cake. One could argue that the shape of the cake influenced these children to arrange their candles. Otherwise they changed candles positions and made structured arrangement.

Moreover, from the video recorder, children in this group were not able yet to determine number of candles based on their own arrangement without counting.

T : Rayya, how many candles on your cake?

Rayya : Two… (*without looking at his cake*)

T : Is that right? Two?

Rayya : Yes, that’s right

(*Meanwhile, group 3 counted their candles one by one with helping of the assistant teacher*)

Rayya (2.5 years old) shouted two (without staring at the candles) as the teacher asked how many candles on their cake. Mostly, young children at this age answer teacher’s question in order to show their existences instead of giving the right answer. It explains that very young children mostly learn about social behavior as a part of a community. Expanding such knowledge is not essential for them as long as they can attract teacher’s attention with their answers.

Another finding of this first group is about their ability to make pictorial representation of the real situation. Video recorder shows when they asked to record their arrangement on a paper, both of them presented different actions. Rayya (2.5 years old) did not make a drawing (see picture 5). Our conjecture for this boy is at this age (2.5 years old), children find difficulties to make pictorial representation of the situations. He saw a circle-shape cake and the candles but the teacher asked him to draw circles representing the candles on a square-shape paper representing a circle-shape cake. This situation was confusing for him and for most other children up to this age (2.5 years old). The words ‘circle’ and ‘representation’ that the teacher used did not give any meaning for him.

Meanwhile, Randy tried to make his own drawing, instead of making circles to represent the candles (as what the teacher asked to him), he made a shape that more or less similar with the candles’ shape (see picture 4) and he drew them around the side of the paper (like his arrangement).

Picture 4. Randy's drawing

Picture 5. Rayya's drawing

This group, however, showed some abilities in mathematical thinking. During the activities, they were able to decide on where they placed the candles by their own way.

T : Let's place the candles, where will you place them so it will look nice?

Randy : I want to put it here… (*Point to a position on the cake*)

T : Okay, do that…

Rayya, put the candle on the white area (*she asked Rayya by pointing to a position on the cake*)

Rayya : (*placing the candle not on where the teacher asked but on another place that made a line with two previous placed candles*)

It seems Randy could argue and show that he had his own idea to think of nice arrangement. For Rayya, even though he did not talk so much, his acting showed that he also thought when he would put the candles. He avoided teacher's help when the teacher pointed on a place. He placed the candles on other places by his own way.

The second group had better understanding of the given tasks. From picture 2, we can see that they arranged the candles in a line. The video recorder shows they well made pictorial representation of their arrangement and they understood each given task. Interesting finding of this group appears when the teacher asked Keisha to guess how many candles on the cake that the teacher made.

(*The teacher showed her structured arrangement and posed question*)

T : Keisha, how many candles here?

Keisha : (*point on the candles and start counting one by one*) One…

T : (*Interrupt her*) No, no… you're not allowed to count them

Keisha : (*Silent and a bit confused*)

AT : Look at your own arrangement, please.

Keisha : (*Look at her own arrangement, a line arrangement, and look back at the teacher's arrangement, then directly give the answer*) Five!

From this evidence, we can make some probabilities: first, she could determine the quantities only by looking at her own arrangement. It means her own arrangement is structured arrangement for her to easily determine quantities. Second, she realized that the candles on her own cake were five. Therefore, when the teacher assistant asked her to look at her own cake, she quickly considered that teacher's candles had a same number with hers. Third, she was good in quick counting. When she looked at her own candles, it gave her time to count the candles without tagging. Even though the first probability is near with our expectation for this study, we can not ignore two other probabilities.

Last, the third group showed less understanding of the tasks. Some of our analyses for these two children are:

- From the beginning it seem the task did not meaningful enough for them
- They arranged the candles un-structurally. It means for them the task was only placing the candles on the cake. They still do not understand the word "nice arrangement" is.
- They could not conclude the nice structure nor record their arrangement.

As we mentioned before, very young children at this age (2 years old) mostly find their selves learning as a part of social community. Such formal knowledge is less learnt by them. However, a little evidence appeared to show their mathematical thinking. Whenever the teacher asked how many candles on their cakes, they always counted one by one. It seems that the arrangement they made was not helpful enough for them to easily determine the quantities. One could argue that it is because they did not make structured arrangement. Nevertheless, in the actor point of view, the task to make nice arrangement seems not having any relation with determining quantities for them.

Some important points for the observation of children activities appear to represent these two boys:

> They only understood that they had to arrange the candles, so as all candles were already in arrangement, they seem ignored to change their arrangement. They argued that arrangement was already nice so that they did not need to change it or they thought that the task was over.

As the teacher asked them to make pictorial representation of candles' position, they were not able to do that. Based on observation, our conjectures for these boys are same with Rayya, He saw a circle-shape cake and the candles but the teacher asked him to draw circles representing the candles on a square-shape paper representing a circle-shape cake. This situation was confusing for them and for most other children up to this age (2.5 years old). The words 'circle' and 'representation' that the teacher used did not give any meaning for him. Those words did not evoke in act. Therefore, the most meaningful task for them in this study was only placing the candles.

CONCLUSION

As other results of many studies, the conclusion can not be drawn generally to contribute for future researches. In such cases, children's differences on thinking and the given treatments also give specific influence for the limitation of the results. The objects of this study, then, provide some insights into how arranging objects and determining quantities probably have relation with children's mathematical thinking.

Even though the result of this study can not explicitly answer the research question, we found some interesting findings in this study. 2-to 3.5 year old children can decide to make their own arrangement. They are able to make arrangement, thus, they can not easily find such structured arrangements. These children also perform their mathematical skills in counting and determining quantities. They found that 'how many' question leads them to cardinality and 'how old' question gives more meaningful context of numbers.

For 2-to 2.5 year old children, making pictorial representation is difficult task for them, either to determine the quantities by only looking at such arrangement. They can not realize the connection between the arrangements with the ability in easily determining quantities.

For older children, between 2.5 and 3.5 years old, the word 'nice arrangement' gives a clear meaning for them. They are able to make their own structured arrangement. Limitation for the definition of structured arrangements as structures, though, gives different meaning for them. It means their own arrangement, however it was, is a structure for them, something that gives help for them to easily determine quantities.

Considering further of our goals, we found achievement of our mathematical goals. They could discover their own arrangement of five candles and also could find that their own arrangement as the easiest structure that they can use to easily determine quantities (for 2.5-to 3.5 year old children). Didactically, they attained to find solution, in this case such arrangement, on their own, but they could not be able to reasoning and having agreement yet.

This study has taken an important initiative for further research. The view of psychological approach to look deeper into insight of children in this age will be very interesting to be examined, as well as the difference of children thinking based on the gender, because one little finding in this study showed that even though Rayya and Reiko, for instance, have the same age (2.5 years old), but their ability to understand the task, to structure the objects and to determine the quantities is big different. This should contribute to a better understanding of the development pre-school children's mathematical thinking.

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