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Design Research on Mathematics Education: Investigating The Progress of Indonesian Fifth Grade Students' Learning on Multiplication of Fractions With Natural Numbers

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Abstract

This study aimed at investigating the progress of students' learning on multiplication fractions with natural numbers through the five activity levels based on Realistic Mathematics Education (RME) approach proposed by Streefland. Design research was chosen to achieve this research goal. In design research, the Hypothetical Learning Trajectory (HLT) plays important role as a design and research instrument. This HLT tested to thirty-seven students of grade five primary school (i.e. SDN 179 Palembang).

The result of the classroom practices showed that measurement (length) activity could stimulate students' to produce fractions as the first level in learning multiplication of fractions with natural numbers. Furthermore, strategies and tools used by the students in partitioning gradually be developed into a more formal mathematics in which number line be used as the model of measuring situation and the model for more formal reasoning. The number line then could bring the students to the last activity level, namely on the way to rules for multiplying fractions with natural numbers. Based on this findings, it is suggested that Streefland's five activity levels can be used as a guideline in learning multiplication of fractions with natural numbers in which the learning process become a more progressive learning.

Keywords: multiplication fractions with natural numbers, measurement (length) activity, design research, Hypothetical Learning Trajectory, model of, model for.

Abstrak

Penelitian ini bertujuan untuk mengivestigasi kemajuan belajar siswa dalam mempelajari materi perkalian pecahan denagn bilangan bulat melalui lima tingkatan aktivitas berdasarkan Realistic Mathematics Education (RME) yang diusulkan oleh Streefland. Design Research dipilih untuk mencapai tujuan penelitian ini. Dalam design research, lintasan belajar (Hypothetical Learning Trajectory) memegang peranan penting sebagai desain dan instrumen penelitian. Lintasan belajar ini diujikan pada 37 siswa kelas lima Sekolah Dasar (yaitu, SDN 179 Palembang). Hasil penelitian menunjukkan bahwa aktivitas pengukuran panjang dapat menstimulasi pengetahuan informal siswa dalam mempartisi untuk menghasilkan pecahan sebagai level pertama dalam tahapan pembelajaran perkalian pecahan dengan bilangan bulat. Selanjutnya, strategi-strategi dan alat yang digunakan oleh siswa dalam mempartisi secara bertahap dikembangkan menjadi matematika yang lebih formal dimana garis bilangan digunakan sebagai model dari (model of) situasi pengukuran dan model untuk (model for) penalaran yang lebih formal. Garis bilangan dapat membawa siswa menuju tingkat aktivitas akhir, yaitu dalam perjalanan menuju aturan perkalian pecahan dengan bilangan bulat. Berdasarkan temuan-temuan yang didapatkan, dapat disimpulkan bahwa pembelajaran siswa mengenai materi perkalian pecahan dengan bilangan bulat dimana proses belajar lebih progresif berkembang melalui tingkatan yang berbeda-beda.

Kata kunci: perkalian pecahan dengan bilangan bulat, aktivitas pengukuran panjang, design research, lintasan belajar.

Introduction

Researches have identified major problems with current learning methods for teaching fractions. The first dealt with a syntactic (rules) rather than semantic (meaning) emphasis of learning rational numbers, where the learning processes often emphasize technical procedures in doing fraction operations at the expense of developing a strong sense in students of the meaning of rational numbers (Moss & Case, 1999). This problem leads to algorithmically-based mistakes, which result when an algorithm is viewed as a meaningless series of steps so that students often forget some of these steps or change them in ways that lead to errors (Freiman & Volkov, 2004).

In learning multiplication by fractions in Indonesia, most of the students are required to master the procedures and algorithms. They just need to memorize formulas and tricks in calculation to solve the problems. However, we do not know whether the students know and understand the meaning of the procedures and algorithms lay behind it.

Secondly, one of the reasons points out as to why the mathematical notion of fractions is systematically misinterpreted because fractions are not consistent with the counting principles that apply to natural numbers to which students often relate (Stafylidou & Vosniadou, 2004). Focus on the multiplication in counting principles, in multiplying natural numbers, the product is larger than the factor. On the other hand, in multiplying fractions, the product may either be higher or lower than the factors. The fact that multiplication by fractions does not increase the value of the product might confuse those who remember the definition of multiplication presented earlier for natural numbers.

Considering the two aforementioned issues, it seems to be necessary to remodel mathematics teaching and learning, especially in domain multiplication fractions with natural numbers. Therefore, we conducted a design research that developed a sequence of activities referred to Realistic Mathematics Education (RME) approach.

According to Streefland (1991), who develops learning by using RME approach, there are five activity levels in learning operation with fractions, namely producing fractions, generating equivalencies, operating through mediating quantity, doing one's own productions and on the way to rules for the operations with fractions. In this study, we designed the Hypothetical Learning Trajectory (HLT) as a research instrument containing a sequence of activities. The five activity levels was used as a guideline to design combined with RME approach. We proposed the use measurement (length) as the contextual situation of the activities to support students' learning. The context of running route was used as a starting point to introduce the number line which could be used as a helpful tool to solve problems related to fractions and natural numbers.

The aim of the research was to investigate the progress of students' learning on multiplication of fractions with natural numbers through the five activity levels proposed by Streefland. Therefore, the research question was formulated as follows.

"*How does students' learning on multiplication of fractions with natural numbers progress through the five activity levels proposed by Streefland based on RME approach?*"

By conducting this research, it is expected that this five activity levels could help students to reach a more progressive learning start from concrete level to a more formal mathematics in learning multiplication of fractions with natural numbers. In addition, RME can be used as an approach to teach mathematics.

Theoretical Framework

Multiplication by Fractions

Streefland (1991) suggested five activity levels in learning operation with fractions, namely producing fractions, generating equivalencies, operating through mediating quantity, doing one's own productions and on the way to rules for the operations with fractions. In this sub chapter, we focused on the five activity levels that are offered by Streefland, more specific to our main domain which is multiplication by fractions. The levels are described as follows.

1. *Producing fractions.*

   The activities here are concentrated in providing contexts at the concrete level. In order to solve all the problems, fractions material is produced by means of estimation and varied distribution. Divergent contexts and processes are explored which could produce fractions, such as fair sharing, division, measurement (length), making mixtures, combining and applying recipes. By linking to the involved magnitudes and varying the objects, the notion of a fraction in operator became clear. For instance, a certain condition, when applies to partition a certain length, will naturally cause a variety of solutions with accompanying notations of fractions.

2. *Generating equivalencies.*

   After students has experienced with notating fractions of their own fractions production, the learning process will continue to generate equivalencies as students are asked to determine fractions in the same position. Equivalencies occur when the distribution problem is given, for instance, the case of partitioning a certain length into eight parts. The equivalencies may occur when students were asked about the relation between $\frac{5}{8}$ and $\frac{1}{8}$.

   The multiplicative reasoning of fractions within fractions equivalency also refers to two definitions, namely fractions involve *between* – and *within – multiplicative relations* (Vanhille & Baroody, 2002). Between-multiplicative relation refers to the relation between the numerators and the denominators of equivalent fractions. If two fractions are equivalent, the ratio between the numerators is the same as the ratio between the denominators. Within-multiplicative relation refers to the relation between the numerator and the denominator of a fraction. For example, in $\frac{3}{4}$ the numerator, 3, is $\frac{3}{4}$ of the denominator 4 (i.e., $3=\frac{3}{4}\times 4$); and the denominator, 4, is $\frac{4}{3}$ of 3 (i.e., $4=\frac{4}{3}\times 3$). All equivalent fractions share the same within-fraction multiplicative relation.

3. *Operating through mediating quantity.*

   To lead to the idea of fractions as operator, we can involve the length to a given unit. The fraction which at the first is described as part of a whole relationship now become a fraction *in* an operator. Based on Fosnot and Dolk (2002), this concept is important because it will connect to the idea of double number line. Taber (1992) suggested that instruction of multiplication with fractions shall relate to multiplication with natural numbers while reconceptualizing students' understanding of natural numbers multiplication to include fractions as multipliers.

4. *Doing one's own productions.*

   At this moment, we cannot put high expectation that the students will come up with their own production. Therefore, questions which can provoke them are needed at this level. Multiplication strategies for fractions can be built upon this equivalence. For instance, the decompositions of $\frac{5}{8}$ 5 8 can be carried out in with the aid of unit by unit division, which is in the meantime becoming a more standard procedure: $\frac{5}{8}=5\times\frac{1}{8}=\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$. At this moment, students can also explore multiplication fractions as repeated addition. Connected with the third level, when natural number involves in the multiplication fractions, the use of fractions as multipliers is important. For instance, when students are asked to find five-eighth of 6 kilometers which can be written in more formal mathematics as $\frac{5}{8}\times 6$. At this level, progression means that the students are able to solve problems in a more and more refined manner at the symbolic level.

5. *On the way to rules for the operations with fractions.*

   Within mini lesson which include fractions as multipliers, the students reflect on the rules for the multiplication by fractions operations which may be in force here. The transition to more formal fractions is preceded by stimulating students to contribute their own informal ways of working. Therefore, in the mini lesson, it is possible to bring about this transition to application of more formal rules.

Realistic Mathematics Education (RME)

The process of designing a sequence of instructional activities that started with contextual situation in this study is inspired by five tenets in Realistic Mathematics Education as a combination of Van Hiele's three levels, Freudenthal's didactical phenomenology and Treffer's progressive mathematization (Treffers, 1991). The descriptions are as follows.

1. *The use of contextual problems.*

   Contextual problems figure as applications and as starting points from which the intended mathematics can come out. The mathematical activity is not started from a formal level as students usually face with, but from a situation that is experientially real for students. Consequently, this study used the running race route as the context in which the students could act and reason to the given problems.

2. *The use of models or bridging by vertical instruments.*

   Broad attention is paid to the development models, schemas and symbolizations rather than being offered the rule or formal mathematics right away. Students' informal knowledge as a result of students' experience in making partitioning using tools (i.e., yarn) needed to be developed into formal knowledge of fractions which would lead to the idea of equivalent fractions when the notation of the result of partitioning were put on the string of yarn. The use of string of yarn here was as a bridge to the number line model which was in more abstract level.

3. *The use of students' own creations and contributions.*

   The biggest contributions to the learning process are coming from student's own constructions which lead them from their own informal to the more standard formal methods. Students' strategies and solutions can be used to develop the next learning process. The use of string of yarn served as the base of the emergence model of number line.

4. *The interactive character of the teaching process or interactivity.*

   The explicit negotiation, intervention, discussion, cooperation and evaluation among students and teachers are essential elements in a constructive learning process in which the students' informal strategies are used to attain the formal ones. Through discussions about running race problems in each day which were designed in continuity story, students could communicate their works and thoughts in the social interaction emerging in the classroom.

5. *The intertwining of various mathematics strands or units.*

   From the beginning of the learning process, the learning activities of fractions are intertwined with proportion. This means that explanation of the unifying relationship between, for instance, equivalent fractions and proportion was not kept until the very end of the learning process. Moreover, learning multiplication by fractions within measurement (length) activity would also support the development of students' skills and ability in the domain of geometry.

Emergent Modeling

Gravemeijer elaborated the *model of* and *model for* distinction by identifying four general types of activity (Gravemeijer, 1994). The implementation of the four general types of activities in this study is described as follows.

1. *Situational activity*, in which interpretations and solutions depend on the understanding of how to act and to reason in the setting. The setting in this present study was the context of running race route. In this level, within the problem of "*locating flags and water posts on the running route*" with the same distance, students would explore their informal knowledge of partitioning when they were asked to divide a certain length into some equal parts.
2. *Referential activity*, in which *model of* refer to activity in the setting described in instructional activities. Students' activities might be considered referential when they were

initially use tools (yarn) as a representation of running race route. In this study, the activity of "*notating fractions in the empty fractions cards and putting fractions cards on the string of yarn*" also served as referential activity in which students produced their own fractions to represent their way in making construction of partitioning. In this activity, the number line as a representation of string of yarn became the *model of* measuring situation.

3. *General activity,* in which *model for* refer to a framework of mathematical relations. The *model for* more mathematical reasoning in this present study was the reasoning of relationship among fractions i.e. the equivalent fractions through the use of number line. In this activity, number line was introduced as a generalization tool of string of yarn. Students were asked to describe the relation among fractions which they could see from their own fractions production. In addition, the activity of "*determining who is running farther*" which the length of the running race route was involved would lead students to the idea of fractions as operator. At this moment, the double number line could be used as a helpful tool to find the distance that could be covered by two runners where their locations were known with the help of flags and water posts.
4. *Formal mathematical reasoning* which is no longer dependent on the support of *model for* mathematical activity. The focus of the discussion moves to more specific characteristics of models related to the concepts of equivalent fractions, fractions as operator and fractions as multipliers.

Research Methodology

Design Research

The type of research that we used was design research (Gravemeijer & Cobb, 2006). Design research consists of three phases, namely developing a preliminary design, conducting pilot and teaching experiments, and carrying out a retrospective analysis (Gravemeijer, 2004; Bakker, 2004).

In this study, we designed Hypothetical Learning Trajectory (HLT) as a design and research instrument. During the preliminary design, HLT guided the design of instructional materials that had to be developed or adapted. During pilot and teaching experiments, the HLT functioned as a guideline for the teacher and researcher what to focus on in teaching, interviewing, and observing. During the retrospective analysis, HLT functioned as guideline in determining what the researcher should focus on in the analysis (Bakker, 2004).

Thirty-seven students (i.e., 5 students in the pilot experiment and 32 students in the teaching experiment) and a teacher of grade five in an Indonesian primary school in Palembang –

Indonesia, SDN 179 Palembang, were involved in this research. The data collected in this research were interviews with the teacher and the students, classroom observations including field notes, and students' works. After we collected all data, we analyzed these data in the retrospective analysis. Finally, we made conclusions based on the retrospective analysis. These conclusions focused on answering the research question. We also gave recommendations for mathematics educational practice in Indonesia and for further research.

Hypothetical Learning Trajectory (HLT)

To investigate the progress of students' learning on multiplication of fractions with natural numbers, we designed a sequence of activities which consists of six activities. The hypothetical learning trajectory was elaborated based on the five activity levels proposed by Streefland based on RME approach which became the instructional activities as follows.

1. Producing fractions
   a. Activity 1: locating flags and water posts on the running route
      Goal: students make a construction of partitioning, part of a whole.
   b. Activity 2: notating fractions in the empty fractions cards, putting the fraction cards on the string of yarn, describing the relations among fractions.
      Goals: students symbolize the result of partitioning and show it on the string of yarn; students will describe the relations among fractions such as equivalent fractions.
2. Generating equivalencies
   Activity 3: math congress 1
   Goals: students will share their ideas and their experiences in partitioning the track, symbolize the result of partitioning, and describe the relations among fractions i.e. equivalent fractions; students will construct multiplicative reasoning within equivalent fractions.
3. Operating through mediating quantity
   Activity 4: determining who is running farther
   Goals: students will compare fractions within a certain length; students informally use fractions as multipliers.
4. Doing one's own production
   Activity 5: math congress 2
   Goals: students will share their ideas and their experiences in informally using fractions as operator; students will discuss several big ideas which will appear in the discussion.

5. On the way to rules for multiplying fractions with natural numbers
   Activity 6: minilesson - fractions as operator
   Goals: students are able to make their own word problem from the string of number in this minilesson; students are given opportunity to use their strategies in solving the problems from the previous activity.

We conjectured that the sequence of activities above could promote the students to gain more insight in multiplying fractions with natural numbers. It is hypothesized that the five activity levels proposed by Streefland could help students to reach a more progressive learning start from concrete level to a more formal mathematics.

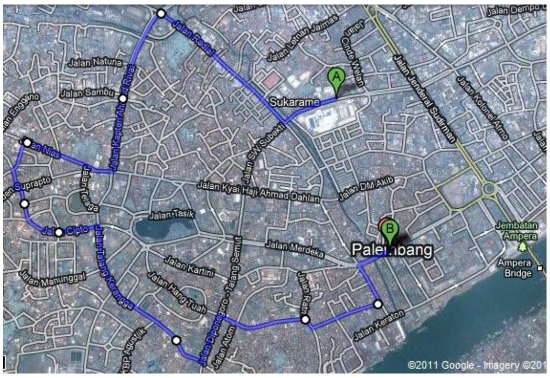
Results and Discussions

Based on the evidence both from pilot and teaching experiment, it was obvious that students' learning progress through the five activity levels based on RME approach proposed by Streefland. Below, we will describe and explain the analysis of the result of our designed experiment based on the Streefland's five activity levels.

Producing Fractions

As mentioned in the first tenets of Realistic Mathematics Education, contextual problems figured as applications and as starting points from which the intended mathematics could come out. For that reason, the running race route context was chosen as the context in which the students could produce fractions by their selves within measurement (length) activity.
This activity with the help of yarn could provoke students in producing their own fractions. Starting from the activity of "*locating flags and water posts on the running route*" (activity 1), the students were used their informal knowledge of *partitioning* by the help of yarn to measure the total length of the running route. The problem in this activity is as follows.

> *To prepare running competition in the celebration of Indonesian Independence Day, Ari and Bimo practice their running skills. They plan to run from Palembang Indah Mall (point A) to Palembang district office (point B) following the running route (see picture below). Eight flags and 6 water posts are stored on the track to know the position where Ari and Bimo will stop. Flags are placed on the running route with the same distance. The water posts also are placed on the running route with the same distance. The last flag and the last water post are stored at the finish line (in front of Palembang district office).*



Running route taken from http://maps.google.com/

Note: the picture was printed in A4 size.

The fractions could be produced when the students were asked to notate the result of partitioning. In the activity of notating fractions (activity 2), students came to the idea of an eighth as one part of eight parts when the teacher posed question: '*if this yarn divided into eight parts, what fraction each part?'*. The students then were asked to give fraction notation to each portioned part. Some students used unit fractions (e.g., $\frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}$) and others used non unit fractions (e.g., $\frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}, \frac{8}{8}$). The students have no difficulties in notating fractions since they had already learnt fractions in grade 3. However, the differences of students' answers about unit and non-unit fractions would be realized by the students that they would need non-unit fractions to grasp the next level, namely generating equivalencies.

Generating Equivalencies

To generate equivalencies, number line was required as a model. The idea of number line appeared when the students were asked to draw the representation of yarn and fractions cards hung on it (activity 2). Due to the form of yarn which is thin, it led the students to draw a line as representation of a string of yarn. This line later named as number line. Moreover, this number line was called as number line of fractions when students realized the existence of fractions in the number line. Connected to the second tenet of RME, namely the use of models or bridging by vertical instruments, the use of string of yarn here served as the base of the *emergence model* of number line. The number line became the *model of* measuring situation.

The number line was proven as a powerful model to encourage the students in generating equivalencies of fractions. Through generating equivalencies, the students could relate equivalent fractions and the relation among fractions with the idea of multiplication of fractions.

As stated in VanHille & Baroody (2002), teacher could focus students' attention on multiplicative reasoning of fractions as they taught equivalent fractions. If two fractions were equivalent, the ratio between the numerators was the same as the ratio between the denominators. From students' drawing of number line as a representation of yarn, it was found there were two pairs of fractions which were in the same position, namely $\frac{3}{6}$ with $\frac{4}{8}$ and $\frac{6}{6}$ with $\frac{8}{8}$. To proof these two pairs of fractions were equal, the idea of simplifying fractions was used. At this phase, the students developed their multiplicative reasoning of fractions through equivalent fractions which can be seen from fractions in the same positions on the number line.

Moreover, the number line also led the students in learning multiplication of fractions when they were asked to find the relation between fractions. By discussing the use of the word *"jumps"* in the math congress 1 (activity 3), the students came to the idea of multiplication of fractions as repeated addition of fractions. For instance, through the problem of finding the relation between $\frac{1}{8}$-jumps and $\frac{5}{8}$ on the number line, students could see that there were five jumps of $\frac{1}{8}$-jumps from zero point to $\frac{5}{8}$. Then they related this with the definition of multiplication as repeated addition. Furthermore, it was written in more formal mathematical notation as $5\times\frac{1}{8}$.

Operating through Mediating Quantity

Based on the explanation in the third activity level, to lead to the idea of fractions as operator, we involved the length to a given unit. Through the activity of "*determining who is running farther*" (activity 4), it was expected that the students could compare fractions within a certain length and informally use fractions as multipliers. Problem in this activity still related to the story in the first activity.

> *After all flags and water posts are in its position, Ari and Bimo start their training. They know the track length from Palembang Indah Mall to Palembang municipality office is 6 kilometers. After running for a while, Bimo decides to stop because he is exhausted. He stop at the fifth flag. Ari also decides to stop at the fourth water post. How many kilometers have Bimo and Ari run? Explain your answer!*

Continuing from the number line which had emerged from the previous activity as a *model of* measuring situation, in this activity, a natural number was involved on the number line as the distance of the running route. Fractions on the number line transformed from fractions as part of a whole into fractions as operator. By connecting students' strategies in partitioning the route, the use of double number line could be introduced as a helpful model to find the length of certain fractions.

Next, the students came to the idea of multiplication of fractions by natural numbers when they were asked to find $\frac{5}{8}$ of 6 kilometers and $\frac{4}{6}$ of 6 kilometers. For instance, we took the problem of $\frac{5}{8}$ of 6 kilometers. The idea of multiplication of fractions as repeated addition of fractions appeared when the students added the length of $\frac{1}{8}$ which was $\frac{3}{4}$ kilometers as many as five times because it took five jumps from zero point to $\frac{5}{8}$. Then it was written into a more formal mathematics operation as $\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}$. From this repeated addition of fractions, the students then related it with the idea of multiplication natural numbers by fractions which then was written as $5\times\frac{3}{4}$. In addition to students success in linking the issue with the idea of multiplication natural numbers by fractions, through discussing a group's strategy in solving problem $\frac{4}{6}$ of 6 kilometers which then was written in mathematical notation as $\frac{4}{6}\times 6$, the students began to transform the word '*of*' into mathematical notation '×'. Most students succeeded in linking the problem of finding the length of certain fractions with multiplication fractions by natural numbers.

Doing One's Own Production

At this level, progression meant that the students were able to solve problems in a more and more refined manner at the symbolic level. As mentioned in the third tenet of RME, the biggest contributions to the learning process were coming from *students' own creations and contributions* which led them from their own informal to the more standard formal methods. Students' strategies and solutions could be used to develop the next learning process.

As a continuation of "*determining who is running farther activity*", math congress 2 (activity 5) was held in the next meeting. In this math congress, teacher tried to bring the students into a discussion by asking two groups which had different strategies to present their solution on the board. They were asked to write down their strategy in finding $\frac{4}{6}$ of 6 kilometers.

The representative of the first group, tried to explain how they got $4\times\frac{3}{3}$ as the answer of $\frac{4}{6}$ of 6 kilometers. This group used double number line as a tool to get an overview of the story. They got 4 from the number of jumps from the starting point to the point where Ari stopped (i.e.,$\frac{4}{6}$). They got $\frac{3}{3}$ from 3 kilometers divided by 3 parts (from starting point to point $\frac{3}{6}$). The result was $\frac{12}{3}$ which described again as repeated addition of $\frac{3}{3}$ five times, or can be written as $\frac{3}{3}+\frac{3}{3}+\frac{3}{3}+\frac{3}{3}+\frac{3}{3}$. In this phase students started to use double number line model as a *model for* more mathematical reasoning.

The discussion continued by exploring another group's solution. This group came to the idea of multiplying $\frac{4}{6}$ by 6, or can be written as $\frac{4}{6}\times 6$, to find $\frac{4}{6}$ of 6 kilometers. They found the answer of $\frac{4}{6}$ of 6 kilometers by multiplying 4 by 6 kilometers then divided it by 6 (the denominator of $\frac{4}{6}$). From the conversation with one of students in this group, she connected the result of $5\times\frac{1}{8}$ which produced $\frac{5}{8}$ from the previous activity about the idea of '*jumps*'. To multiply 5 by $\frac{1}{8}$, she only needed to multiply 5 by 1 then divided it by 8 (the denominator of $\frac{1}{8}$). Therefore, by using the same strategy, she found the answer of $\frac{4}{6}$ of 6 kilometers.

After comparing two strategies, the students then realized that by using different strategies, it produced the same result. This fact gave impact to the students to choose a more effiecient way to solve problem involving multiplication of fractions by natural numbers. Almost all students preferred to use the second group's idea to solve problem in the mini lesson (activity 6) in which multiplication of fractions by natural numbers was presented in more formal way. The use of student's contribution as the *model for* more formal reasoning showed that *general level* of modeling has been attained by the students.

On the Way to Rules for Multiplication of Fractions with Natural Numbers

In the formal, level students' reasoning with conventional symbolizations started to be independent from the support of models for mathematical activity. The last level of emergent modeling, the formal level, the focus of discussion move to more specific characteristics of models related to the concept of equivalent of fractions and multiplication fractions with natural numbers.

Throughout mini lesson (activity 6) which included fractions as operator, the students reflected on the rule for the multiplication fractions with natural numbers. The transition to a more formal fractions was preceded by stimulating students to contribute their own informal ways of working which led by students' idea about the rule in multiplying fraction with natural number. However, this activity level had not reach the level of generalizing rules for multiplication of fractions with natural numbers. As stated in the title of this level, '*on the way*', the students were still on the process leading to generalizing rules. Therefore, they need more practices in solving problems related to multiplication of fractions with natural numbers.

Conclusion

In conclusion, this research has shown students learning about multiplication fractions with natural numbers by conducting the five activity levels proposed by Streefland (1991) as the steps of learning with fractions. In this research, some ideas and concepts from RME theory has underpinned the design of activities. The context used was about measurement (length) activity and we found that this is a good context that has allowed students to structure and to mathematize following the five activity levels.

In the first level, students started to produce their own fractions and recalled back their knowledge about the meaning of fractions as part of a whole. Starting from a situation, students can cross the border to mathematics on their own, by learning to structure, arrange, symbolize, visualize, and much more. Related to the first tenet of RME, namely *the use of contextual problems*, the measurement (length) activity serves as the *source* for the mathematics to be produced.

In the second and third levels, students were generating equivalencies and operating through mediating quantity. Problem situation in which the students drew the number line as a representation of string of yarn had a strongly generative nature. Connected to the second tenet of RME, namely *the use of models*, the use of string of yarn here was as a bridge to the number line model which was in more abstract level. At this phase, through using the material, the students are given the opportunity to *actively contribute* to their own learning process. They will themselves become constructors – producers of their own mathematics.

The fourth and fifth levels gave opportunity to the students to use their strategies that can be used to develop the next learning process. Connected to the third tenet of RME, namely *the use of students' own creations and contributions*, the use of number line as a *model of* measuring situation transformed into a *model for* more formal reasoning. This transformation is another important learning moment for students where they can use the model to move from concrete context to a more formal mathematics.

In this study, we also included the moment in which the students could share their thinking and strategies in solving the problem in so called *math congress*. Connected to the fourth tenet of RME, namely *interactivity*, the classroom settings was elaborated where the students could share their thinking in the class discussion. Teacher played an important role in orchestrating the flow of the discussion.

From the explanation above about the five activity levels and the connection with RME approach, it was obvious that this study has taken an important initiative to improve education in Indonesia. Realistic Mathematics Education (RME) can be used as an approach to teach mathematics, or in specific, related to this study, in the topic of multiplication of fractions with natural numbers. It is suggested that if we want to established 'mathematics for all', we should set priorities for all students including the low-achievers. Students who cannot learn formal mathematics should be made to feel welcome, since they have a right to experience mathematics on a level they can understand and use in daily life.

Considering the last tenet in RME, namely *intertwinement*, some activities used in this research could be developed to reach other mathematical topics by intertwining with other mathematics topics. Another mathematics topic that is taught in grade five is about *proportion*. We found the close relation between proportions and fractions during the learning process. The lines of learning for proportions and for fractions were tightly intertwined. There are also, of course, even more intersections with all kinds of other lines of learning, such as those for division, for practicing the basic operations, measuring, decimal numbers, scale, percentages and probability. Therefore, the suggestion for the next research is about the topics mentioned above.

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