academicJournals

Vol. 10(16), pp. 2352-2361, 23 August, 2015
DOI: 10.5897/ERR2015.2329
Article Number: 57FA65D55048
ISSN 1990-3839

http://www.academicjournals.org/ERR

Educational Research and Reviews

Full Length Research Paper

An examination in Turkey: Error analysis of Mathematics students on group theory

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Received 04 June, 2015; Accepted 19 August, 2015

The aim of this study is to analyze the mistakes that have been made in the group theory underlying the algebra mathematics. The 100 students taking algebra math 1 class and studying at the 2nd grade at a state university in Istanbul participated in this study. The related findings were prepared as a classical exam of 6 questions which have been presented by 3 academic members working at the same university and these questions were presented to the students accordingly. After findings were put into codes as Correct/Wrong answers, solutions coded as "Wrong" were analyzed according to the content analysis method. Classifying the mistakes made in the solutions by students, suggestions about how to rectify these mistakes were carefully offered.

Key words: Math Training, Group Theory, Learning Disabilities.

INTRODUCTION

One of the meanings of the mathematical thinking is of course abstract thinking. Lacking the amount of the individuals who have the ability of abstract thinking may result in a problem while trying to keep up with a contemporary world (Cucen and Erturk, 2008). The lesson of the mathematics is the most important basic lesson inside the developed societies who have already discovered this detail (Kahramaner and Kahramaner, 2002).

The mathematical branches of the universities consist of analysis, algebra, numbers theory, geometry, applied mathematics, topology, basics of mathematics and the departments of the mathematical logic. Algebra lessons contain just one part of the basic capabilities which have been expected from the students studying at the mathematics department. That is, the student who graduated from mathematics section has already obtained the basic knowledge of algebra. Because, it is thought that algebra is a basic bridge while accessing to the higher level of the mathematics (James, 2000). Algebra lesson has some missions, such as finding the common features of the algebraic structures, trying to find extra results from these results and making classifying operations on these structures. As a result of this, the algebra which has been instructed at the universities has been called as "Abstract Algebra" by some researchers as well as some educators. The first subject of the algebra instructed at the university is the theory of group. The problem in understanding of the situations of being a group has been resulting in another problem during the conception of "ring" and "substance" situations of the next structures. The first concept, which has to be known just because of the fact that these concepts have been correlated with each other, is to make a decision whether a set is a group or

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not.

Definition of the Group Theory: There has to be a set, which is different from the empty set and an operation which has to be defined on this set, in order to compose a group. In addition, the below detailed three features have also to be provided:

1. The first feature in order to compose a group is associative feature, i.e. for each x, y, z $\in$ G
$x \star (y \star z) = (x \star y) \star z$
2. A group has to have an effect-less element in order to be called as a group.
By means of calling e as an effect-less element, for each x $\in$ G, the equation is provided:

$e \star x = x \star e = x$
3. A y element exists in each group, (Nesin, 2014):
$x \star y = y \star x = e$

Identify learning disabilities and reasons

Knowing the difficulties confronted by students on any subject is an important first-step for the researches on learning. Synthesizing such data with the latter studies and establishing a connection with it, will be regarded as a crucial base on regulating the following curriculums and composing the teaching model (Rasmussen, 1997). Yetkin (2003) stated that improving cognition at math is an important but a difficult aim. He stated that being aware of the learning disabilities of the students and the reasons for these disabilities in order to design a teaching method and rectify them accordingly are the most important steps for achieving this aim.

By examining the studies for revealing the learning disabilities on different subjects of math according to the findings of the studies of Baker (1996), Moore (1994), Tall (1993), Tall & Razali (1993), Artigue (1990), Weber (2001), Ersoy & Erbas (2005) and Durmus (2004), the reasons of learning disabilities can be collected under these titles:

Concepts Being in Abstract Structure; Concepts being subtract means students being unable to think abstractly, unable to interpret verbal expressions (cannot formulate mathematically).

Deficiencies in Mathematics Education; Deficiencies in transferring main concepts to the students inadequately, their definitions, their images in mind, their usage, rote learning, lack of motivation, being unable to inform the students with the knowledge why mathematical operations are needed and where they will use it.

Student's Deficiencies in Proof and Axiomatic Method; Student's deficiency at the level of readiness, commencing in proof (perception ways of math and proof), students' ways which they use in proof, being in the form of sampling rather than being conception, students' awareness of knowledge essential for proofing and still having difficulty in the proof.

In addition to the above reasons, let us touch on briefly the research of Weber (2001): In his study, which he made to reveal the deficiencies of students' learning while they are creating any proof of theorem in the field of abstract algebra, he asked each student who joined to the research, to prove 7 theorems about group homomorphism by expressing their thoughts vocally. Each of the proof trials was coded as;

1. Correct (participant created a valid proof)
2. Failure to contact knowledge (participant has the required knowledge but he cannot apply this knowledge into creating a proof)
3. Inadequate knowledge (participant does not have the required knowledge to create any proof of theorem)
4. Logical error (participant has a faith in creating a proof but the proof is invalid)

At the end of his study, he found out that having the perception and knowledge which form a proof in group homomorphism is not adequate to create proof. Especially, most of the university students are often being aware of the knowledge needed to prove an expression, but however, their deficiency in creating a proof was still overwhelming.

But Barnett (1999) took a different way to determine the learning deficiencies in his research and he pointed out that this way is much more measurable. Barnett (1999) emphasized on the facts in his study that the best way to determine what the students understand is asking True/False questions. That is, the aim of these questions is to draw an attention to the important features of the concept itself rather than misleading students. He expressed that written explanations to True/False questions are much more important in determining students' mistakes and learning disabilities than the answers to the multiple choice questions which are answered with just one choice. Barnett also emphasized on the fact that an inadequate explanation for a right answer would be much less reliable than a good explanation for a wrong answer.

METHOD

Research design

The related study has been carried out with a descriptive survey model among qualitative research models that have been expected to be examined variably within its own borders of a current situation.

Research population and sample

A group of 100 students at the 2nd class, who studied Algebra Math-1 at the Department of Mathematics in the Faculty of Arts and Sciences of a state university in Istanbul for the 2011-2012 academic years, constituted the population of the research itself. These students had been graduated from a high school formerly and studied at the 2nd class of mathematics department. Algebra Math-1 is taught in the 2nd class (third semester) and their knowledge related to the group theory was the same as the one they have obtained from their Algebra Math-1 course accordingly.

Data collection tool

The required data is collected by researchers by means of applying the below items prepared by 3 academic members, who are expert in their field, teaching at the same university and dealing with the students who study Algebra Math.

1. Do the following sets compose of a group according to the processes defined over them. Explain the reasons.

(a) $(\mathbb{R}^{+},*)$; a*b = $\sqrt{ab}$
(b) $(\mathbb{R}^{-},*)$; a*b = $\sqrt{ab}$
(c) $(\mathbb{R}^{+},*)$; a*b = $\frac{a}{b}$
(d) $(\mathbb{R}^{-},*)$; a*b = $\frac{a}{b}$

2. Which of the following is true and which of the following is false? Explain the reasons.

a-) Empty set is a group.
b-) There is only one solution to the equation defined as a*x*b=c in a group.
c-) A finite goup which has maximum 3 items is an Abelian group.
d-) Each social group is also a group under the multiplication.
e-) Each group has minimum 2 subgroups.

3. Show that the group equalizing $\forall a \in$ G for $\boldsymbol{a^2}$ **= e** is an Abelian group.
4. There are 8 groups given below. Sort these groups by subgroup relations in a particular way that no groups remain out.

G_1=($\mathbb{Z}$,+)
G_2=(12$\mathbb{Z}$,+)
G_3=($\mathbb{Q}^{+}$, .)
G_4=($\mathbb{R}$,+)
G_5=($\mathbb{R}^{+}$, .)
G_6={$\prod^{n}|n \in \mathbb{Z}$}, under multiplication
G_7=(3$\mathbb{Z}$, +)
G_8={$6^{n}|n \in \mathbb{Z}$}, under multipication

5. Groups given below;

a-) Search whether it is subgroup of $(\mathbb{C},+)$group or not.
i. $\prod\mathbb{Q}$
ii. $\{\prod^{n}|n \in \mathbb{Z}\}$
b-) Search whether it is subgroup of ($\mathbb{C}-\{0\}$, **.)** group or not.
i. including i$\mathbb{R}$; i$\mathbb{R}$ ={ $ia \mid a \in \mathbb{R}, i^2=-1$ }
ii. $\{\prod^{n}|n \in \mathbb{Z}\}$

6. Including a * $b=\boldsymbol{a^b}$ is the set of odd integers a group under * operation? Explain please.

a-) Search whether it is subgroup of $(\mathbb{C},+)$group or not.
i. $\prod\mathbb{Q}$
ii. $\{\prod^{n}|n \in \mathbb{Z}\}$
b-) Search whether it is subgroup of ($\mathbb{C}-\{0\}$, **.)** group or not.
i. including i$\mathbb{R}$; i$\mathbb{R}$ ={ $ia \mid a \in \mathbb{R}, i^2=-1$ }
ii. $\{\prod^{n}|n \in \mathbb{Z}\}$

Analysis of data

When the data were analyzed, students' answers were coded as Right or Wrong. Then examining the wrong answers was performed by content analysis method. According to this action, it was categorized referring to common denominator of students' mistakes.

FINDINGS

1st and 2nd Question's Analysis: Determining whether it forms a group by checking the group axioms.

'Do the following sets compose a group according to the processes defined over them? Explain the reasons.'

We wanted to survey whether the students learnt the group axioms in this question. We observed that most of the students memorized the group axioms (closure, associativity, neutral element, inverse element) but they could not analyze them.

Option a: $(\mathbb{R}^{+},*)$; a*b = $\sqrt{ab}$

In this question, students said that $\mathbb{R}^{+}$ was closed under * operation, as $\forall$ a,b $\in \mathbb{R}^{+}$ for a*b = $\sqrt{ab} \in \mathbb{R}^{+}$ but, it did not prove associativity as $\forall$ a,b,c $\in \mathbb{R}^{+}$ for (a*b)*c ≠ a*(b*c). Therefore, it was not a group (Table 1).

We categorized the mistakes into two groups on this question. In the 1st Category, students listed the group axioms but they accepted them "Correct" without sufficiently analyzing it. In short, they accepted that associativity was proved without analyzing. In the 2nd Category, while they were examining the associativity ,students controlled whether $\forall$ a,b,c $\in \mathbb{R}^{+}$ for (a*b)*c $\in \mathbb{R}^{+}$ instead of proving $\forall$ a,b,c $\in \mathbb{R}^{+}$ for (a*b)*c = a*(b*c). The result we found is that the students could not comprehend the required associativity.

The 1st category of students' answer

1. Do the following sets compose of a group according to the processes defined over them? Explain the reasons.

a) $(\mathbb{R}^{+},*)$; a*b = $\sqrt{ab}$ composes of a group
I) $\forall$ a,b $\in \mathbb{R}^{+}$ for $\sqrt{ab} \in \mathbb{R}^{+}$ provides closure
II) provides associativity
III) there is inverse element since O is not involved in set $\mathbb{R}^{+}$
IV) "t" is unit a element

The 2nd category of students' answer

1. Do the following sets compose of a group according to the processes defined over them? Explain the reasons.

Table 1. Performance of the students for the 1st question with the option "a".

No of Question	Percentage of correct-answered question	Percentage of unanswered question	Percentage of wrong-answered question	Percentage of wrong category 1	Percentage of wrong category 2
1. a.	10	10	80	62	18

Table 2. Performance of the students in the 1st question with the option "b"

No of question	Percentage of correct-answered question	Percentage of unanswered question	Percentage of wrong-answered question
1. b.	42	12	46

Table 3. Performance of the students in the 1st question with the option "c".

No of question	Percentage of correct-answered question	Percentage of unanswered question	Percentage of wrong-answered question	Percentage of wrong category 1	Percentage of wrong category 2
1. c.	14	12	74	56	6

a) $(\mathbb{R}^+,*)$; a*b = $\sqrt{ab}$

Option b: $(\mathbb{R}^-,*)$; a*b = $\sqrt{ab}$

Students should have stated at this question that the set could not have been in the group since the given set was not close under this operation. But, most of them accepted that this set proved the closure axiom according to this operation and they started to examine the other axioms. Mistake which was made was common (Table 2).

Students' answer

1. Do the following sets compose of a group according to the processes defined over them? Explain the reasons.

b) $(\mathbb{R}^-,*)$; a*b = $\sqrt{ab}$→ is a group (a,b ∈ $\mathbb{R}^-$ when ab> O ∈ $\mathbb{R}^-$)
Option c: $(\mathbb{R}^+,*)$; a*b = $\frac{a}{b}$

Since this question is similar to the option a, mistakes made were similar to each other. Therefore, mistakes made accumulate upon the same category. In the 1st Category, students listed the group axioms but they accepted them correct without analyzing it. In the 2nd Category, there were mistakes proceeded from lacking of learning the required associativity (Table 3).

1st category of students' answer

1. Do the following sets compose of a group according to the processes defined over them? Explain the reasons.

$(\mathbb{R}^+,*)$; a*b = $\frac{a}{b}$Set features are provided, it should be b =0
Not problem for $\mathbb{R}^+$

2nd category of students' answer

1. Do the following sets compose of a group according to the processes defined over them? Explain the reasons.

c) $(\mathbb{R}^+,*)$; a*b = $\frac{a}{b}$ 1) ∀ a,b ∈ $\mathbb{R}^+$for a*b = $\frac{a}{b}$ ∈ $\mathbb{R}^+$
2)∀ a,b ∈ $\mathbb{R}^+$ for (a*b)*c =($\frac{a}{b}$*c) = $\frac{a}{bc}$ ∈ $\mathbb{R}^+$
3) ∀ a,b ∈ $\mathbb{R}^+$for c∈ $\mathbb{R}^+$ should be found as c∈ $\mathbb{R}^+$ a*a=a
a*c= $\frac{a}{c}$ =a a=2∈ $\mathbb{R}^+$
Since a*a=a =……???? not group

Option d: $(\mathbb{R}^-,*)$; a*b = $\frac{a}{b}$

There is only one type of mistake at this question, which resembles to the option b regarding to the error analysis, is that it is examining the other group axioms before looking closure axioms (Table 4).

Students' answer

1. Do the following sets compose of a group according to the processes defined over them? Explain the reasons.

$(\mathbb{R}^-,*)$; a*b = $\frac{a}{b}$ is not a group, it has not got any unit

Table 4. Performance of the students in the 1st question with the option “d”.

No of question	Percentage of correct-answered question	Percentage of unanswered question	Percentage of wrong-answered question
1. d.	50	20	30

Table 5. Performance of the students in the 2nd question with the option “a”.

No of Question	Percentage of correct-answered question	Percentage of unanswered question	Percentage of wrong-answered question
2. a.	68	16	16

Table 6. Performance of the students in the 2nd question with the option “b”.

No of question	Percentage of correct-answered question	Percentage of unanswered question	Percentage of wrong-answered question
2. b.	44	44	12

Table 7. Performance of the Students in the 2nd Question with the option “c”.

No of Question	Percentage of correct-answered question	Percentage of unanswered question	Percentage of wrong-answered question
2. c.	22	46	32

element.

2nd Question Analysis: True/False questions

Which of the following is true,and which of them is false? Explain the reasons.
a-) Empty set is a group.

We wanted to measure whether students know that a set must include at least neutral element to be a group. Mistakes made were resulted from ignoring this condition (Table 5).

b-) There is only one solution to equation defined as a*x*b=c in a group.

Since each binary operation is sufficiently defined, equation given has one solution. Students generally gave right answers to this question, but none of them successfully explained its reason (Table 6).

c-) A finite group which has maximum 3 items is an Abelian group.

We wanted students to make comments on the set given overtly (Table 7). Since a group needs to include at least the neutral element by definition; we wanted them to see each items as an Abelian as follows:

1- element group {e};
2- element group {e, a };
3- element group {e, a, a^{-1}}

We have assumed that the students who gave correct answers could give coincidental answers since there was not any satisfactory explanation.

d-) Each social group is also a group under multiplication.

Each set can be a group or not according to the operation defined over it. Being additive does not require being multiplicative or vice versa. Most of the students answered the question itself correct (Table 8).

e-) Each group has minimum 2 subgroups.

Most of the students stated that a group had at least two subgroups and they were trivial subgroups ({e} itself). The group itself could be {e} was ignored. In these circumstances, the subgroup of the group is only {e} and there is just one (Table 9).

Table 8. Performance of the Students in the 2nd Question with the option "d"

No of Question	Percentage of correct-answered question	Percentage of unanswered question	Percentage of wrong-answered question
2. d.	74	22	4

Table 9. Performance of the students for the 2nd question with the option "e".

No of Question	Percentage of correct-answered question	Percentage of unanswered question	Percentage of wrong-answered question
2. e.	50	24	26

Table 10. Performance of the students for the 3rd question.

No of Question	Percentage of correct-answered question	Percentage of unanswered question	Percentage of wrong-answered question	Percentage of wrong category 1	Percentage of wrong category 2
3	34	28	38	12	30

Students' answer

2- a-) T_ Empty set is a group. -> provides closure, associativity -> group itself, inverse itself

b-) F_ There is only one solution to equation defined as a*x*b=c in a group.

c-) F_ A finite group which has maximum 3 items, is an Abelian group.

d-) F_ Each social group is also a group under multiplication.

e-) T_ Each group has minimum 2 subgroups. 1- Empty set 2- Itself

3rd Question Analysis: $\forall a \in$ G for a^2 = e is an Abelian group

We wanted to measure students' approaches to the proof questions at this question and we categorized their mistakes into 2 groups in general (Table 10). In the 1st category, students reached this statement by looking at the statement they need to gain it as a result of proof. Proving as it is is out of question. In the 2nd Category students did some operations ignoring even what they had to prove after they had started to prove. While analyzing the error analysis of this question, their lack of knowledge has been observed in terms of not using the given data appropriately and we have reached to right conclusion by this way.

1st category students' answer

3-Show that the group equalising $\forall a \in$ G for a^2 = e is an Abelian group.

for $\forall a$,b ∈ G if ab=ba it is Abelian

aab=aba

b=aba

ba=abaa

ba=ab

2nd Category Student Answer

3- Show that the group equalising $\forall a \in$ G for $\boldsymbol{a^2}$ **= e** is an Abelian group.

a and b

a^2 =eb^2 = e

ab = ba

e = e

4th Question's Analysis: Ordering of Groups

3- There are 8 groups given below. Sort these groups by subgroup relations in such a way that no groups remain out.

G_1=($\mathbb{Z}$,+)

G_2=(12$\mathbb{Z}$,+)

G_3=($\mathbb{Q}^+$, .)

G_4=($\mathbb{R}$,+)

G_5=($\mathbb{R}^+$, .)

G_6={$\prod^n$|$n \in \mathbb{Z}$}, under multiplication

G_7=(3$\mathbb{Z}$, +)

G_8={6^n|$n \in \mathbb{Z}$}, under multiplication

We wanted to see students to show the groups as S< G under the same operation, when the condition of S⊂G was proved. Mistakes, which were often made, were just because of intensifying on two categories. In the 1st

Table 11. Performance of the students in the 4th question.

No of Question	Percentage of correct-answered question	Percentage of unanswered question	Percentage of wrong-answered question	Percentage of wrong category 1	Percentage of wrong category 2
4	4	50	46	30	20

Table 12. Performance of the students in the 5th question with the option "a.i".

No of Question	Percentage of correct-answered question	Percentage of unanswered question	Percentage of wrong-answered question
5. a. i.	**6**	**84**	**10**

Category, students emphasized on subgroup connection between multiplicative and additive groups, but such an equation was not possible. In the 2nd Category students compared the G_6 and G_8 groups which were not each other's subgroup according to the subgroup relation. These groups could not be each other's subgroups by definition (Table 11).

1st Category students answer

3- There are 8 groups given below. Sort these groups by subgroup relations in such a way that no groups remain out.

G_1=($\mathbb{Z}$,+)
G_2=(12$\mathbb{Z}$,+)
G_3=($\mathbb{Q}^+$, .)
G_4=($\mathbb{R}$,+)
G_4>G_5>G_3>G_6
G_5=($\mathbb{R}^+$, .)
G_6={$\prod^n | n \in \mathbb{Z}$}, under multipication
G_1>G_7>G_2>G_8
G_7=(3$\mathbb{Z}$, +)
G_8={$6^n | n \in \mathbb{Z}$}, under multipication

2nd Category Student Answer

There are 8 groups given below. Sort these groups by subgroup relations in such a way that no groups remain out.

G_1=($\mathbb{Z}$,+)
G_2=(12$\mathbb{Z}$,+)
G_2>G_7>G_1>G_4
G_3=($\mathbb{Q}^+$, .)
G_4=($\mathbb{R}$,+)
G_8>G_6>G_5>G_3
G_5=($\mathbb{R}^+$, .)
G_6={$\prod^n | n \in \mathbb{Z}$}, under multiplication
G_7=(3$\mathbb{Z}$, +)
G_8={$6^n | n \in \mathbb{Z}$}, under multipication

Analysis of Question 5: Determining subgroup

We wanted the students to examine the condition of being subgroup ($a.b^{-1}$). They had to be carefull about the element they chose from the additive and multiplicative groups as they examine that condition (*a-b* for additive group; $a.b^{-1}$ for multiplicative group)

a-) Search whether it is subgroup of ($\mathbb{C}$, +)group or not;

Option i : .$\prod\mathbb{Q}$

Students fell into an error which was mentioned above (Table 12), thus they examined *a-b* instead of a.b^{-1}.

Students' answer

a-) Search whether it is a subgroup of ($\mathbb{C}$, +)group.
if a.b^{-1} S<G let's look
i. $\prod\mathbb{Q}$ =$\prod\frac{a}{b}$.($\prod\frac{c}{d})^{-1}$=……………………???
because no T , not subgroup
Option ii: {$\prod^n | n \in \mathbb{Z}$}
Some of the mistakes made on this question are of the type of option i again. Another common mistake was choosing the elements to be examined correctly, but claiming $\prod^k - \prod^l$ expression was the element of the set which was previously given (Table 13).

Students' answer

Search whether it is a subgroup of ($\mathbb{C}$, +)group.

i. s a-b ∈ S when aı b∈ S?
ii.S{$\prod^n | n \in \mathbb{Z}$}$\prod^k$,$\prod^l$ $\in S$
k>l $\prod^k - \prod^l$ = $\prod^l$ [$\prod^{k-l}$ -1] ∈ S l>k $\prod^k -$ $\prod^l$ =$\prod^k$(**1**−$\prod^{l-k}$) $\in S$ → subgroup

b- Search whether it is a subgroup of ($\mathbb{C}$ −{0}, .) group.
Option i: i$\mathbb{R}$

Table 13. Performance of the students in the 5th question with the option "a.ii".

No of question	Percentage of correct question	Percentage of unanswered question	Percentage of wrong-answered question
5. a. ii.	2	88	10

Table 14. Performance of the students in the 5th question with the option "b.i"

No of Question	Percentage of Correct-Answered Question	Percentage of Unanswered Question	Percentage of Wrong-Answered Question	Percentage of Wrong Category 1	Percentage of Wrong Category 2
5. b. i.	2	82	10	4	4

Table 15. Performance of the students in the 5th question with the option "b.ii"

No of Question	Percentage of Correct-Answered Question	Percentage of Unanswered Question	Percentage of Wrong-Answered Question
5. b. ii.	8	92	0

Table 16. Performance of students in the 6th question.

No of Question	Percentage of Correct-Answered Question	Percentage of Unanswered Question	Percentage of Wrong-Answered Question
6	2	88	10

We collected the mistakes made on this question under two titles. 1st Category was consisted of the mistakes arising from not to able to find the reverse of an element taken from the set according to the multiplication (Table 14). 2nd Category consisted of the false results arising from finding the multiplication reverse of an element taken from the set but it was not to be able to write $a.b^{-1}$ as -1 instead of i^2 in the expression given as a result to operation $a.b^{-1}$.

1st Category Students' Answer

5. including i$\mathbb{R}$; i$\mathbb{R}$ =$\{ ia \mid a \in \mathbb{R}, i^2 = -1 \}$
Search whether it is a subgroup of $(\mathbb{C}, +)$group.

2nd Category Student Answer

b- Search whether it is a subgroup of $(\mathbb{C}, +)$group.
i - Including i$\mathbb{R}$; i$\mathbb{R}$ =$\{ ia \mid a \in \mathbb{R}, i^2 = -1 \}$
x,y ∈i$\mathbb{R}$ x.$y^1 \in \mathbb{R}$ i.a $(\frac{i}{b}) = -i^2 . \frac{a}{b}$ ∈i$\mathbb{R}$
Option ii : $\{\prod^{\boldsymbol{n}} | \boldsymbol{n} \in \mathbb{Z}\}$
Students who answered this question did not make any mistake (Table 15).

Analysis of 6th Question: Whether $a * b = \boldsymbol{a^b}$ **is a group.**

Including $a * b = \boldsymbol{a^b}$, is the set of odd integers a group under * operation? Explain please.

We wanted to see to be shown closure axiom was not proved since $a^b \notin T$ for each ∀ *a*, b ∈ℤ as the odd integers set is *T*= {….,-3,-1,1,3,….} . But most of the students stated that the closure was proved, because they assumed the T set as it is the only odd positive integers set (Table 16).

Students' answer

3- Including $a * b = \boldsymbol{a^b}$, is theset of odd integers a group under * operation? Explain please.
{….,-3,-1,1,3,….} =T 3x5 =3^5 = T
1x3 =1^3= 1 =T→ 5x7=5^7 ⟸
1x5 =1^5 = 1 =T→ $(odd)^{odd}$ = odd odd.odd=odd
Since it is close,* is a group

DISCUSSION

While students were expected to show the given operation as well-defined, they passed on to decision making process without any question. It could be said that the students preferred to copy rather than to think abstractly when we consider that they attended to the university as a result of test exam, i.e. the central exam system (Soylu and Isik, 2008). It sounds believable that they could have just memorized the rules of theory without internalizing the descriptions. Trying to proving the group axioms without thinking on descriptions is a sign of rote learning based education system. Whether a cognitive teaching has been done on the algebraic structures or not has not been known. Unless we internalize the meanings of the concepts covered with the different learning methods, mastering on a subject by rote will come into the question. Using computer programmes, e.g. computer algebra system CBS), will provide convenience but, are there any academic members applying to the computer programmes or how are their perspectives to these embodying processes? Doing a scientific research by academic members about this matter, their opinions and their approaches could be significantly useful.

The questions, which measure whether definitions and features of algebraic structures are learnt, are generally measure proving, reasoning, and discernment ability. Individuals experience many problems in their daily life and they think mathematically to solve their problems. Actions like explaining a proposition, saying why it is right or wrong and choosing and using different logical thinking ways and proving types, present individual's ability on mathematical thinking. In this sense, the students of the mathematics department are supposed to use their ability of mathematical thinking and to let the operations they do make sense. Mistakes made by students, who participated in the study, came up as a result of either misunderstanding the conditions of group theory or examining these conditions wrong. Some challenges could be experienced during the learning process but the matter is to identify them correctly and to enhance various methods to deal with them. Having difficulties at the learning abstract concepts is the most important one. Students can apply to rote learning in order to overcome this difficulty, but they can have difficulty in practice at this time. For example, student lists the axioms (closure, associativity, inverse element, neutral element) while controlling the set whether it is group or not, but he or she makes the operation supposing that the set is closed. In other words, student cannot practice what he or she memorized or could not know what to do in other cases. We have been thinking the fact that this problem traces to the gaps of education which was received in both high school and university years. Students' infrastructure they set up with math training, which they had during their education life up to attending university, has inadequate mathematics they meet at the university. Because throughout their primary and secondary school years, they learn mathematics with its operational aspect, i.e. they assume that the success at this lesson to be able to perform the operations without using calculator and dealing with just practical solutions in the math exams. However, they meet theoretical mathematics after the gradation from a high school before the college and as a natural result, they are afraid of another learning difficulty, which we thought it arises from the same reasons, is the one which is dealing with proving the theories. While it is rehearsing as if definitions and proofs have no significance at secondary education, the theoretical side of the mathematics is at the forefront at the university, especially at the Algebra Math-1 Class. Students even do not know how to study for this lesson and they are having enormous learning difficulties. Our suggestion to minimize this wavers during this gradation process is to lecture the abstract mathematics, such as logic, proving methods before Linear Algebra and Mathematics Analysis I class, in which the main subjects of the theoretical mathematics have been taken into account.

Conceptual learning has much higher degree of importance in the mathematics education for the students who study at the mathematics department. Unless the students can successfully comprehend algebraic definitions, concepts and structures, they will try to memorize these phenomenons (Soylu and Isik, 2008).

The related suggestions to overcome these above mentioned learning difficulties can be listed as follows by taking the studies of Woerner (1980), Harel (1989), Haddad (1999) and Tatar and Dikici (2014) into consideration; computer programmes can be used, visualization can be referred, appropriate materials can be used, classroom tasks can be carried out, and teaching system can be redesigned in the direction of learning difficulties.

Conflict of Interests

The authors have not declared any conflicts of interest.

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