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Adult Students' Experiences of a Flipped Mathematics Classroom

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Abstract

The flipped classroom is a flexible blended learning model that is growing in popularity due to the emergent accessibility to online content delivery technology. By delivering content outside of class time asynchronously, teachers are able to dedicate their face to face class time for student-centred teaching approaches. The flexibility in implementation of a flipped classroom allows for a diversity in student experiences. The study presented in this paper uses qualitative methods of analytic induction to conduct a case analysis on survey and interview data collected from students participating in a flipped adult mathematics upgrading course at an urban Canadian university near Vancouver, BC. The key phenomenon of interest in the study is how adult students experience a flipped mathematics classroom. Of secondary interest is how factors such as autonomy and goals interrelate with these experiences. It is found that flipped classrooms can bifurcate into two types of student interaction: completely engaged and self-paced. Key interrelated factors in this bifurcation include adoption of cognitive autonomy support, goal orientation, and attendance.

Keywords: flipped classroom, autonomy, goals, classroom experience

Introduction

Empowering adults to learn mathematics, especially when they have encountered low mathematical performance in their past and have returned to the subject for the key purpose of obtaining high school prerequisites required towards a new career path, can be very challenging. The underlying goal of this study is motivated by the desire to enrich the experiences of this population of adult learners by providing them with a student-centred learning environment, which differs from the dominant teacher-centred learning environments they were most likely exposed to in their public school experiences.

In a teacher-centred learning environment, the focus is on pursuing the teacher's agenda, which is not directly related to emergent student learning needs. In contrast, student-centred learning approaches focus on the student and their learning journey. The notion of a student-centred learning environment is rooted in constructivism and embraces student agency. Knowledge is actively constructed by the learner rather than imparted by the teacher, and "goals are negotiated and selected by the learners" (Elen, Clarebout, Léonard, & Lowyck, 2007, p. 107). In this research, Elen et al.'s (2007) transactional view of student-centred learning is adopted, where there is a "continuous interchange between students' and teachers' responsibilities and tasks" (p. 108). The key premise is that the teacher observes student interactions and adapts teaching interventions accordingly to student needs.

Overall, student-centred approaches have been found more effective than teacher-centred ones (Åkerlind, 2003; Barr & Tagg, 1995; Grubb & Associates, 1999; Grubb & Cox, 2003; Kember & Gow, 1994; Prosser & Trigwell, 1999). However, creating student-centred learning environments can be challenging for teachers, especially in mathematics, where curriculum constraints are demanding. Wang (2011) notes that "student-centred teaching tends to be more time-consuming and unpredictable than whole-class lecturing" and that "teachers working under a fixed curriculum and schedule are inclined to organize the class in a more teacher-centred manner to secure the completion of required tasks" (p. 157). In an effort to relieve these tensions between allowing for student-centred learning practices and maintaining adherence to the curriculum, educators have become drawn to the affordances provided by increasingly accessible technologies to deliver content asynchronously out of class time while dedicating class time for student learning. Bergmann and Sams (2012) have coined the phrase 'flipped classroom' in reference to this teaching approach.

The concept of reversing content delivery and practice time is not a new phenomenon in education, but the increasing accessibility to technology that allows teachers to create their own content videos and the improved ability available for teachers to share their teaching practices to a wider audience online have contributed to the increasing popularity of the flipped classroom model. Media outlets such as USA Today (Toppo, 2011), Washington Post (Strauss, 2012), and CNN (Green, 2012) have covered experiences and opinions regarding the flipped classroom. However, research based literature pertaining to flipped classrooms is still limited. Several studies report increased student achievement in flipped classrooms (Day & Foley, 2006; Green, 2011; Johnson, 2013; Kirch, 2012; Mussallam, 2010), but few of them relate directly to a mathematics context, let alone the adult population.

The most notable studies within a mathematics context focus on student perceptions. One of these studies looks at an undergraduate level statistics course (Strayer, 2008) and the other looks at a set of high school level mathematics classes (Johnson, 2013). Strayer (2008) compares student responses from a flipped classroom version of an undergraduate statistics course with that of a traditional classroom version of the same course. He finds that students in a flipped classroom can experience higher levels of innovation and cooperation than those in a traditional classroom but that they can also experience feelings of unsettledness due to the unpredictability of class time. Students in the flipped classroom can also find the learning model difficult to accustom to if they are used to a traditional approach. In contrast, Johnson (2013) finds that high school mathematics students experience the flipped classroom approach more positively. His students evidence enjoyment from classroom learning activities, frequent interaction with teacher and peers, and a reduction in time necessary to spend on homework outside of class time. Johnson (2013) also finds evidence of improvement in students' perceptions of engagement, communication, and understanding. The varying and almost contradictory results in these studies may in part be attributed to various methods of implementation and a difference in student population. Based on these two small-scale studies, one could conclude that adult learners may have a more difficult time adjusting to the teaching approach than high school students. However, the evidence for such an argument is not substantive enough and needs further exploration.

More importantly, there is no single method of implementation of a flipped classroom, and just like with any student-centred teaching approach, its success rests on a teacher's pedagogical sensitivity and ability to adapt to student needs. Although student-centred approaches are desirable, they are not always easy to carry out. The flipped classroom approach provides teachers who want to evolve their classes into student-centred learning environments with the option to deliver direct instruction outside of class time, leaving time during class for student-centred tactics. Flipped Learning Network (2014) has defined flipped learning as "a pedagogical approach in which direct instruction moves from the group learning space to the individual learning space, and the resulting group space is transformed into a dynamic, interactive learning environment where the educator guides students as they apply concepts and engage creatively in the subject matter" (para. 2). They claim that flipped classrooms can lead to flipped learning through a flexible environment, a rich learning culture, intentional content, and a professional educator, but that a flipped classroom in itself does not promise flipped learning. Rather, a flipped classroom offers teachers a means with which to employ student-centred approaches. Kachka (2012) notes that "the increase of teacher-student interaction during class time is what characterizes [the flipped classroom model's] success" (para. 6). Student-centredness within a flipped classroom, by its nature, affords student autonomy over learning, and is closely tied with factors such as goals, self-efficacy, and anxiety.

The research presented in this paper is therefore motivated by the question of how adult mathematics students experience a student-centred flipped classroom environment that offers opportunities for student autonomy over learning in the context of an adult mathematics high school level upgrading course at the University of the Fraser Valley in British Columbia, Canada. A secondary question that guides the study pertains to how factors such as goals, self-efficacy, and anxiety are interrelated with adult student experiences in this classroom. For purposes of brevity, only factors of autonomy and goals are detailed in this paper. In what follows, key literature, background, results, and conclusions from the study are overviewed as pertaining to autonomy and goals. Possible implications for the field of mathematics education are also discussed.

Key literature

Analysis of the context of this study is informed by literature on the provision of autonomy support. Student experiences in the context are examined in relation to literature on goal orientation. These theories are briefly overviewed in order to ground the results.

Autonomy

Student-centred learning environments by nature allow for student autonomy. In general, autonomy is viewed as the availability of choice, which is evident in Black and Deci's (2000) definition: autonomy is supported by providing students with "pertinent information and opportunities for choice, while minimizing the use of pressures and demands" (p. 742). Studies have shown that students of autonomy supportive teachers experience more classroom engagement, positive emotion, self-esteem, creativity, intrinsic motivation, psychological well-being, persistence in school, academic achievement, and conceptual understanding (Assor, Kaplan, & Roth, 2002; Benware & Deci, 1984; Deci & Ryan, 1985, 1987; Deci, Nezlek, & Sheinman, 1981; Hardre & Reeve, 2003; Koestner & Ryan, 1984; Reeve & Jang, 2006; Reeve, 2009; Ryan & Grolnick, 1986; Vallerand, Fortier, & Guay, 1997). Therefore, it is important to consider the role of autonomy and its implications for mathematics classrooms.

Although positive effects are associated with autonomy in classrooms, it is important to emphasize that autonomy cannot simply be provided, it needs to be supported. Autonomy supportive teaching should "adopt the students' perspective, welcome students' thoughts, feelings, and behaviours, and support students' motivational development and capacity for autonomous self-regulation" (Reeve, 2009, p. 162). Stefanou, Perencevich, DiCinto and Turner (2004) classify autonomy support into three dimensions: *organizational* autonomy support, *procedural* autonomy support, and *cognitive* autonomy support.

Organizational autonomy support allows students to control their environment by directing them to choose classroom rules, the pace at which they learn, due dates which they set, students with whom they work, and ways in which they are evaluated. Meanwhile, *procedural* autonomy support allows students to control the form in which they present their work by inciting them to choose materials they use for a project, the ways in which they display work, and the ways in which their materials are handled. Finally, *cognitive* autonomy support allows for students to control their learning by encouraging them to generate their own distinct solutions, justify their solutions according to mathematical principles, evaluate their own work, evaluate work of their peers, discuss multiple approaches, debate ideas freely, ask questions, and formulate personal goals.

Stefanou et al. (2004) argue that although organizational or procedural autonomy support may be necessary, it may be insufficient in maximizing motivation and engagement. They claim that cognitive autonomy support is the most essential type of autonomy support in order for positive educational benefits such as motivation and engagement to occur. Although Stefanou et al. (2004) do not clearly indicate whether organizational and procedural dimensions are best structured or left autonomous, a study conducted by Jang et al. (2010) suggests that student engagement can be more prominently observed when a learning environment has higher levels of structure (i.e. structured organizational and procedural dimensions) as long as students are provided with high levels of cognitive autonomy support. They note that structure should not be confused with control. Even when a dimension is more structured than autonomous, the teacher should maintain respect for student thoughts, feelings and actions within the structure. Although the necessity of organizational and procedural autonomy support is not clearly defined in the literature, there is consensus with regard to the importance of cognitive autonomy support in relation to heightened student engagement and motivation.

Goals

Additionally, goal orientation can have a positive influence on performance and motivation in the face of a challenging task, such as that of learning mathematics (Grant & Dweck, 2003). The predominant view of goals that informs analysis in this study is that of Achievement Goal Theory. This theory is rooted in the belief of intelligence as being either fixed or malleable giving rise to either *performance* (self-enhancing) or *learning* (mastery) goal orientations, leading to various motivation driven behaviour patterns that depend on self-efficacy beliefs (Dweck, 1986; Pintrich, 2000).

In a *learning* goal orientation, “individuals seek to increase their competence, to understand or master something new” whereas in a performance goal orientation, “individuals seek to gain favorable judgements of their competence or avoid negative judgments of their competence” (Dweck, 1986, p. 1040). Grant and Dweck (2003) provide evidence that a learning goal orientation positively affects performance and motivation in the face of challenge while the performance goal orientation only positively affects performance and motivation if no challenge is present. In extension of Dweck’s (1986) theory, Dupeyrat and Mariné (2005) discover that for adults returning to school, “mastery [or learning] goals have a positive influence on academic achievement through the mediation of effort expenditure” (p. 43).

Further, Hannula (2006) shows evidence that “students may have multiple simultaneous goals and [that] choices between them are made” (p. 175). He claims that motivation is structured through the mediation of needs and goals with emotions and that a desired balance of goals can be promoted by offering students a safe learning environment that focuses “on mathematical processes rather than products” (Hannula, 2006, p. 176). Such an environment can be created through the provision of cognitive autonomy support and is possible within a flipped classroom context.

Background

In what follows, the context of the study and the methods used to collect and analyse data are overviewed.

Context

The context of this particular study is a full-term 60 hour adult mathematics upgrading course referred to as Math 084 offered through the Upgrading and University Preparation Department (UUP) at the University of the Fraser Valley (UFV). UFV is a fully accredited public multi-campus university primarily located in the Fraser Valley just east of Vancouver, British Columbia, Canada. The UUP department at UFV offers programmes in Adult Basic Education (ABE) for adults of all backgrounds and ages who want to meet their educational goals such as completing prerequisites for post-secondary programmes, earning the BC adult graduation diploma, or improving skills for personal benefit.

Math 084 serves as a requirement for the Dogwood Diploma (graduation diploma in British Colombia) and is the first out of two courses that together serve as a prerequisite for most undergraduate programmes that lead to career paths such as teaching, nursing, business diplomas, etc. The course covers a variety of topics including linear equations, linear inequalities, quadratic equations, radical equations, operations with polynomial, rational, and radical expressions, and function graphing. It is traditionally taught with 60 lecture hours and 30 individual or group work hours, which makes it a primarily teacher-centred learning atmosphere.

In contrast, the flipped classroom implementation of Math 084 fostered a student-centred learning atmosphere. Video lecture lessons[1], online quizzes, announcements, and practice problems were posted in an online learning management system (i.e., Blackboard Learn), and students were asked to preview this content prior to class as homework. More importantly, having the content available online afforded time during class for student-centred content-related discussions, group learning activities, practice time, and assessments. This means that the class design was aligned with the tenets of the Flipped Learning Network's (2014) description of flipped learning, which may emerge within a flipped classroom. Classes typically consisted of approximately 80 minutes of teacher facilitated discussions and/or group learning activities and 80 minutes of time for completing assignments. This means that classes were facilitated by the teacher, who decided on which activities to initiate based on their interpretation of student needs.

An example of an open ended group learning activity problem facilitated by the teacher during the course is the National Council of Teachers of Mathematics (2008) Barbie Bungee Activity. During this activity, students were asked to find the maximum number of rubber bands required to allow a Barbie doll to 'bungee jump' from a certain height without hitting her head. Students, in random groups, were given rubber bands and a doll and were asked to make the prediction for the number of rubber bands required. Eventually, through discussion, students noted the linear relationship between the number of rubber bands and the measure of the doll's descent. This led to further discussion on linear equations and slopes.

Another instance of an activity facilitated during the course is that of student-generated examples[2]. This is not referring to Watson and Mason's (2005, 2002) concept development approach to learner-generated examples, but rather the opportunity for students to generate examples for purposes of involvement in the learning process. One instance of a student-generated example activity is when students were provided with a collection of 3-dimensional geometric objects and were asked to build a new object composed of two or more smaller objects. They were then asked to give their new composite object to another group to find the surface area and the volume of the given composite structure. This activity led to some interesting discussion and even a Google search regarding the surface area of a cone because it was not provided in the course textbook. Yet another case of a student-generated example activity is when students were asked to use whiteboards to develop exponential expressions

[1] Videos can be viewed by visiting Judy Larsen's YouTube Channel.

[2] Student-generated examples are used colloquially here in the sense that students were asked to generate examples for the purposes of assessment or engagement and not in the more defined sense that Watson & Mason (2005, 2002) indicate in respect to constructive concept development.

that needed simplification. They were then asked to pass their problems to another group for simplification. Interesting examples arose from such activities. One example in particular was that of a student who created a complicated exponential expression, but created it so that the entire expression was taken to the power of zero indicating that the student understood the implication of a power of zero (See Figure 1 below).

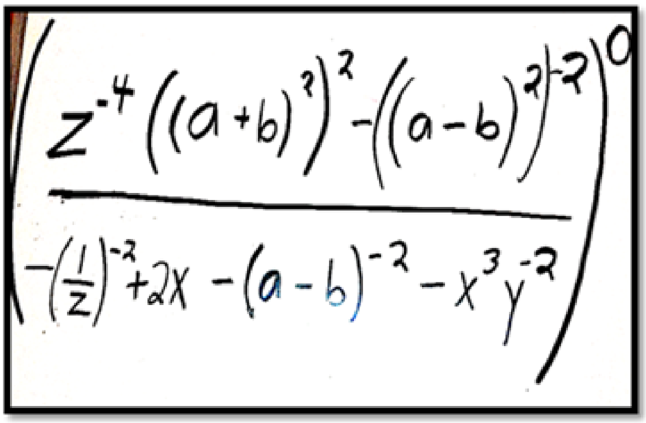

Figure 1. Student generated example 1.

Other group learning activities consisted of group concept review sessions. For example, students used whiteboards to develop reasoning for why certain properties exist, such as the rules for simplifying exponential expressions. Products from review sessions were often documented with a camera and posted on the course website to help provide study materials for students in preparing for tests.

Equally important to the choice of activities in the promotion of engagement and understanding was the method of grouping students so that they would productively collaborate. Liljedahl (2014) asserts that visibly random groups lead to positive observable changes such as “an elimination of social barriers, [an increase in] mobility of knowledge between students, [a decrease in] reliance on the teacher for answers, [and an increase in] engagement” (p. 130). During the first half of the term, students were always grouped together randomly to increase the likelihood of students working with as many other students as possible in alignment with Liljedahl’s (2014) suggestions for student grouping. Eventually, students found their favourite peers to work with and they settled into preferred groups.

Anything that contributed to a student’s final grade (assignments and tests), with a few exceptions, was completed and submitted during class time. In essence, the in-class workload and the out-of-class workloads were swapped or flipped as compared to a traditional class. Most importantly, class time provided students with opportunities to engage with collaborative problem-based learning tasks, a facilitative teacher, and a variety of learning tools.

Method

The Math 084 flipped classroom outlined above was implemented during the Winter 2013 term. The course started with 25 total students enrolled, 18 of whom completed the course. It should be noted that low completion rates are very common in these courses and many students often stop showing up due to life circumstances. Out of the 18 students who completed the course, two were registered, but were completing the course at a distance, and therefore were not part of the flipped classroom experience. This leaves 16 students who experienced the flipped classroom throughout the entire term, 14 of whom gave consent to participate in the research study. All 14 of these students appeared to be in their twenties and were completing the course either to satisfy prerequisites towards career-driven programmes or to complete their high school diploma.

Data collected consisted of observational data, interviews, and surveys (including in-class surveys and follow-up email surveys). As researcher and instructor of the course, I collected observational data throughout the term in relation to classroom interaction, goal statements, self-efficacy, anxiety, etc. and tabulated each observation into an Excel spreadsheet document for analysis. Interview and survey data was collected by an external co-investigator during the term while I was away from the room in compliance with local research ethics requirements. After final grades were posted, I was given access to all data collected by the external co-investigator.

Analysis of data was performed according to the tenets of analytic induction, a qualitative method of analysis rooted in grounded theory. Much like in grounded theory, the inductive analyst recursively codes the data looking for themes to emerge; however, analytic induction allows for an *a priori* proposition or theory driven hypothesis to be used as a lens to deductively analyse the data in contrast to grounded theory, which begins inductively through open coding (Glaser & Strauss, 1967, cited in Patton, 2002). In this research study, the key *a priori* theory used in the deductive phase of the analysis was that of Stefanou et al.'s (2004) distinction between types of autonomy support. Other theories used in the analysis pertained to goals, self-efficacy, and anxiety in the context of mathematics education (Ashcraft, 2002; Bandura, 1997; Biggs, 1985; Dweck, 1986; Hannula, 2006; Jang et al., 2010; McLeod, 1992; Zimmerman, 2000).

Prior to the theory driven deductive phase of analysis, a preliminary analysis of data was performed to draw out data relevant to the goal of this research, which is to characterize student experiences in a flipped classroom. During this preliminary investigation, it quickly became evident that there were three levels of student interaction in the class. The class design provided students with a diversity of learning tools, and although most students utilized all learning tools during the first part of the term, they eventually gravitated towards certain learning tools as they pursued completion of the course. In particular, some students chose to utilize class time completely in order to gain better understanding of topics. These students willingly participated in all classroom activities. Others chose to focus more on out-of-class learning materials such as the online videos and the textbook. These students tended to attend less regularly or chose to opt out of activities offered during class time. There were also those who shifted between these types of interaction throughout the term.

For each of these three types of interaction, two participants whose actions were reflective of each of these types of interaction were carefully selected. This means that the six participants selected as cases consisted of two students who participated completely in both in-class and out of class components of the flipped classroom (Group 1), two students who at first participated completely with the flipped classroom model but later fell behind and chose only to participate in out of class components (Group 2), and two students who tried participating in the flipped classroom model completely, but quickly participated only in what was absolutely required in the course (Group 3). These cases are summarized in Table 1 below.

Table 1.
Grouping of Cases

Group 1	Students who completely engaged in both in-class and out-of-class components.	Alexa (A) Kristy (A+)
Group 2	Students who at first engaged in both in-class and out-of-class components, but chose to opt out of class time activities near the end of the term.	Mark (A+) Ryan (A-)
Group 3	Students who tried engaging in both in-class and out-of-class components, but as soon as they could opt out of the activities, they did.	Lindsay (B+) Vanessa (A-)

Note. All names are pseudonyms to maintain anonymity.

These cases were reflective of the three types of interaction in the course because out of the 14 participants, five were categorized as Group 1, five were categorized as Group 2, and four were categorized as Group 3. Further, grades obtained by these cases were within the grade range obtained by the majority of the students in the class (11 out of 14 students attained a B+ or higher and no students completed the course with a grade lower than a B-).

The data related to these six participants was aggregated to form cases that reflected various student experiences in the course. Each case was then analysed and coded according to the key a priori theories of autonomy, goals, self-efficacy, and anxiety in the context of mathematics education. This case analysis was followed by a cross-case analysis that inductively derived common themes across the data. As previously noted, the scope of this paper has been limited to only factors of autonomy and goals in order to provide adequate depth and detail.

Results

Results are presented by providing sample case data from the study as well as a cross-case analysis of key factors that are of focus in this paper: autonomy and goals.

Cases

In what follows, three cases are briefly outlined to provide samples[3] from each of the three groupings described in the previous section: Kristy (Group 1), Mark (Group 2), and Lindsay (Group 3). These are chosen for their strength in presenting key issues resulting from the study pertaining to autonomy and goals in a flipped classroom environment.

Kristy

Kristy was selected as a case in Group 1 because of her complete engagement in both in-class and out-of-class activities. She attained an A+ in the course and serves as an example of someone who experienced the flipped classroom to the fullest extent. Kristy attributed her success in the course to several factors including the ability to progress through lectures at her own pace, the time available to discuss concepts that were troubling during class, and the opportunity to teach others in the class. Although she was initially shy and nervous about being in the course due to her past negative experiences with mathematics, she soon found the learning environment comfortable and conducive to learning. She claimed in an initial survey that “up until this term, [she had] never liked mathematics and never grasped the concept.” She noted that in high school, she kept falling behind with notes, didn’t receive enough individual attention and was not shown things in a kinaesthetic manner, which resulted in poor achievement. Although she initially expressed concern about doing things the “right” way during classroom activities, she soon discovered that seeing multiple approaches is beneficial to understanding the concept. She summarized her engagement in classroom activities on the follow-up survey:

> Although I want to say that the at home lectures were the most valuable part of the class, the group activities played an equal role in how well I learned the mathematical concepts. Being forced (I use the term lightly) into group activities during class allowed me to get to know my classmates, which made me feel a lot more comfortable asking any questions I had. Secondly, the other ideas and approaches that students had towards problems allowed me to see different ways of understanding the questions and different techniques to use when finding an answer … The way I see it is, solving a problem is great, but being able to

[3] For complete case data, see Larsen (2013).

> explain how the problem works means you truly understand it. I was almost testing myself by teaching others. It became another way of studying for me … I was beginning to "know" math. I was starting to truly understand the concepts because I was able to study as much as I needed to since the lectures were always available to me.
>
> (Follow-up survey)

In addition to her in-class engagement, her out-of-class engagement was also notable. On a survey taken at the middle of the term, she responded that outside of class time, she watched all of the videos (sometimes more than once) and took detailed notes from them. She also re-watched the videos if she was stuck on a concept, and if she couldn't figure something out, she made note of it and moved on knowing that she could ask about it during the next class. She made decisions about how much practice she needed to complete on each section claiming on a survey during the middle of the course, "If I feel strong on a concept I don't do all the examples and if I feel weak I do more than the given" (Week 8 survey). She showed appreciation for the videos on the follow-up survey:

> Having the lectures in video form allowed me to study them at my own pace and take notes a lot more accurately. The option of being able to pause or rewind the video instead of asking the instructor to stop or repeat was great as well since it does not stop anyone else's learning process.
>
> (Follow-up survey)

During an interview, Kristy noted, "I feel like I'm walking out of the classroom knowing something, I'm not just wasting my time trying to get a letter grade, I'm actually taking something away from the class too" (Week 3 interview). By attending almost every class, participating completely in all in-class tasks, leading group discussions, and working on material at home frequently, she was able to develop a new interest in mathematics.

In summary, Kristy participated completely in all components of the flipped classroom throughout the entire term. However, she may not have been as successful in the flipped classroom environment had there not been initial organizational structure and continued cognitive autonomy support provided. She noted that she greatly appreciated being placed in random groups at the beginning of the course (an organizational structure) because otherwise she would have been too shy to communicate with others. At first, she was also uncomfortable with not knowing how her assignment was supposed to look, indicating a resistance towards procedural autonomy support. She also reacted negatively when I probed her to think on her own, indicating an aversion towards cognitive autonomy support. However, this quickly diffused as she participated in the course consistently and completely. Eventually, she came up with her own ways of solving problems and taught others comfortably, indicating that she found use in the cognitive autonomy support that was being provided as part of the classroom culture. Even though she entered the class wanting to satisfy prerequisites for a programme path, indicating a performance goal orientation, she managed to cultivate a learning goal orientation within the flipped classroom environment. Given that Kristy decided to continue in the course after finding out that she no longer needed the course as a prerequisite, the flipped classroom was a truly empowering experience for her.

Mark

Mark was selected as a case in Group 2 because he first engaged in both in-class and out-of-class components, but chose to opt out of class time activities near the end of the term when he wanted to get farther ahead with the material more efficiently. Mark attained an A+ in the course and showed complete interaction with the flipped classroom during the beginning of the term, but became more motivated to work individually after missing a few classes in the second part of the term due to a bad case of the flu. His favourite part about the flipped classroom model as stated on the follow-up survey was that he could "come to class with questions and actually get the questions answered instead of being stuck out of class time." He also noted on this survey that he appreciated the freedom he had to learn content at his own pace and out of class time. From the beginning of the term, Mark showed inquisitiveness and engagement. He noted on his initial survey that he chose to take Math 084 "to get a better understanding of Math" because he is "just fascinated by how it works." Even though he completed Math 11 and 12 in high school eight years ago, he noted that he did not find it enjoyable at the time and he found that he had forgotten too much of it when he recently attempted to complete a first year calculus course. This informed his choice to take Math 084.

As mentioned, after a series of absences due to being sick in the latter part of the term, it was observed that Mark began to opt out of activities and worked on his own in the back of the classroom. During these times, he took the liberty to choose when to engage in the entire class and when to engage in his own work. He did this by looking up when something interesting was happening and looking down at his work when he felt he didn't need to be paying attention. In a survey completed near the end of the term, he noted, "I used class time more for doing homework [as the term progressed] so that I could ask questions." He clarified this later claiming, "Near the end of the term, the topics we were doing I was very familiar with and I wanted to get ahead on my homework so that I could go back and check and think of any questions I could ask before the final" (Follow-up survey). He was actively engaged in course content out of class time throughout the term and was able to develop his own method of studying for a test by taking questions from each section and making mock tests for himself.

To showcase Mark's search for understanding, it is worthy to mention his classroom interactions during the first half of the term. First of all, Mark consistently asked questions that demonstrated his desire to test his own conjectures and search for generalisations. One example of such a question, noted in the observational data, was when he inquired about whether there existed a general method for finding the domain and range of any function after he determined the domain and range for a few rudimentary functions. Secondly, Mark was often observed attempting to complete activity problems in several different ways and working collaboratively with others, encouraging them to think in various ways. In his follow-up survey, he noted that his favourite type of activity was "one that allows you to come to the same solution but with multiple paths." Based on my observations, he thrived within activity problems that were open-ended because he worked towards creating difficult scenarios in order to challenge himself. One example of this was when he created a very complicated three dimensional shape consisting of a cone nested within a cylinder with a half-sphere on top (See Figure 2 below). He then encouraged his group to figure out the volume and surface area of the shape. We hadn't learned how to find the surface area of a cone, so it led the class to learn more than was expected. Combining several shapes also gave students the opportunity to learn how to alter formulas they had learned.

Figure 2. Student generated example 2.

Mark was also interested in developing reasoning. In a survey during the early part of the term, Mark reflected on an activity that asked students to justify reasoning for various exponent rules on the whiteboards in groups. He claimed that the activity was "very helpful in understanding the way rules for exponents work instead of just memorizing them” and that that is his “favourite way to learn things” (Week 3 survey).

In summary, Mark participated in all components of the flipped classroom until about two thirds of the way through the course, when he began to opt out of class time activities. Interestingly, Mark exhibited a learning goal orientation right from the beginning with his original intent for taking the course being to get a better understanding of mathematics. During the beginning of the course, he readily communicated with others and was intrigued by the activities, using all tools that were available to him. He participated in the classroom culture by proposing interesting ideas to others and helping them with their work. Based on the examples provided of his interactions in the problem solving activities, it is evident that Mark embraced cognitive autonomy support during the first part of the term. Additionally, his increasingly independent thinking and learning throughout the term contributed to his ability to make good use of the organizational and procedural autonomy support that became increasingly available. After being sick for a while and being away from class, he began to come to class without engaging in classroom activities. Due to his absences and low classroom involvement, I perceived his actions as that of someone who had fallen behind in his work and needed to catch up. However, Mark was actually moving ahead. He wanted to learn further material, engaging autonomously with it, so that he would know what to ask questions about. However, as the term neared completion, time constraints seemed to alter his goals. He began to participate less and less in the classroom activities, and although his goals were still predominantly of learning, he showed goals of performance in his expressions of concern around completing course requirements. Overall, Mark’s experiences with the course were very positive because even when he was sick and had to miss class, he was not greatly inconvenienced by it because of the accessibility of learning materials.

Lindsay

Lindsay was selected as a case in Group 3 because although she initially tried engaging in both in-class and out-of-class components, she soon opted out of class activities after falling behind with the material and realizing that the activities were not required towards course completion. Remarkably, even though it was noted that she was absent a lot during the last third of the term, she was able to complete the course with a B+ by watching the videos, completing examples from the videos, and completing assigned graded textbook problems. Although Lindsay engaged in the course in an individual manner, it proved to be more beneficial for her than another completely individually paced course she had previously taken because she had a greater variety of resources available. Lindsay also noted on the follow-up survey that although her primary goal with Math 084 was to get a good grade and complete her prerequisite requirements towards an animal health technician program, the flipped classroom environment was beneficial for her because it helped her learn how to ask questions and provided her with enough material out of class time to work through and catch up with when she fell behind.

Lindsay particularly enjoyed learning from the videos out of class time because she was able to “go through [each video] slowly and do the example questions one step at a time” (Week 8 survey). She also noted that she really appreciated the opportunity to “pause and rewind the video whenever” she needed to (Week 3 survey). At the end of the term, Lindsay wrote, “The ability to watch lessons at home and at [my] own pace was probably the thing I liked the most about the class” (Week 14 survey). On the Week 8 survey, she noted that she watched every video in great detail, took notes from the videos, and paused the videos in order to try the example questions on her own before proceeding with the video. She also claimed on this survey that she referred to textbook examples often and tested her understanding by completing the online quizzes. It was observed that when she didn’t understand a concept well, she gravitated towards re-watching the videos before asking any questions.

Lindsay tended to work individually and as a quiet observer during class time. She tried engaging in the group activities during the first third of the course, but always seemed overwhelmed in the group setting. When in her proximity, she would often ask me probing questions seeking confirmation of the work her group was doing. During the latter part of the course, it was observed that Lindsay began to use class time even more individually. As the material became more difficult, Lindsay began to be absent more often. She soon fell behind with the material and began to treat the flipped classroom as a place to learn individually. During one set of consecutive absences, she emailed me explaining that she needed to stay home because she wanted more time to go over the videos and complete missing graded problems. It was evident that she was avoiding group work and desired to complete course requirements as efficiently as possible. This is evidenced in the following survey response:

> One thing I didn't really like was the amount of group work we had to do. Sometimes it was helpful but sometimes it seemed to complicate things … [As the term progressed], I used class time to hand in work, work on graded problems and do tests. I [made] sure when I [got] stuck on something to ask for help.
>
> (Week 14 survey)

However, on a survey taken during the middle of the term she wrote, "This class has helped me realize that asking for help more when I need it is OK" (Week 8 survey). During the latter part of the term, she watched the videos in great detail and then came to class to clarify concepts that she struggled with. I observed that most of her clarifications pertained to implementation strategies of the various procedures outlined in the videos and used in the textbook. These clarifications were very important for her.

In summary, although Lindsay tried to engage in all components of the flipped classroom, she quickly began to avoid components that expected her to adopt cognitive autonomy support. Upon entering the class, Lindsay held a strong performance goal orientation with her main reason for engaging in the class being to satisfy a career prerequisite. The organizational structure of requiring students to work in random groups at the beginning of the term allowed her to experience a classroom culture of learning. However, during the times when she was asked to work with others, she tended to observe the others in the group rather than initiate discussion. She seemed to resist cognitive autonomy support within problem solving opportunities and often became confused by other students' approaches to solving problems. This was at times frustrating for her and it may have interfered with her performance goal orientation because it compromised the efficiency of learning the material. As soon as more organizational and procedural autonomy support was available, she chose to focus on the videos as her main learning tool and was grateful for their accessibility. When she was behind with the material, she did not feel adequately prepared to participate in group activities, causing her to avoid class time. Although she missed a lot of class time in the second half of the course, she was able to complete the course successfully due to the availability of the online videos. The flipped classroom seemed to be beneficial for her because as she noted, it helped her learn how to ask questions.

Cross-case analysis

The aggregated case data and case by case analyses of all six cases in the study evidenced a bifurcation in how participants experienced the flipped classroom during the second part of the term once students became accustomed to the class structure. A cross-case analysis clarifies that the bifurcation was made possible, in part, by the autonomy support provided in the structure of the Math 084 flipped classroom's learning environment. Student goals were interrelated with the bifurcation, and attendance surfaced as an emergent theme. These results are overviewed in the subsequent paragraphs.

Autonomy

The flipped classroom, as implemented in this study, offered students an opportunity for autonomy by allowing them to engage in a variety of components: learning activities, classroom community, and accessible learning materials. Most importantly, cognitive autonomy support was provided during class. This can be seen in Alexa’s survey response:

> If I was confused about anything, we would explain everything in great detail and have debates about it … I learned different ways to solve problems during the activities and others learned from me. This was fantastic.
>
> (Follow-up survey)

At first, all students participated in all components of the class when procedural and organizational structure was provided in an autonomously supportive way, through the use of random groups, due dates, specified assignment submission procedures, etc. As the term progressed, more autonomy support was provided over procedural and organizational dimensions in the class. Simultaneously, a bifurcation of student experiences occurred. In particular, election of cognitive autonomy support began to change. Figure 3 below serves as a subjective visual representation of students’ expressed desires for either high cognitive autonomy, occasional cognitive autonomy, or no cognitive autonomy as coded from the case data in relation to the time in the term.

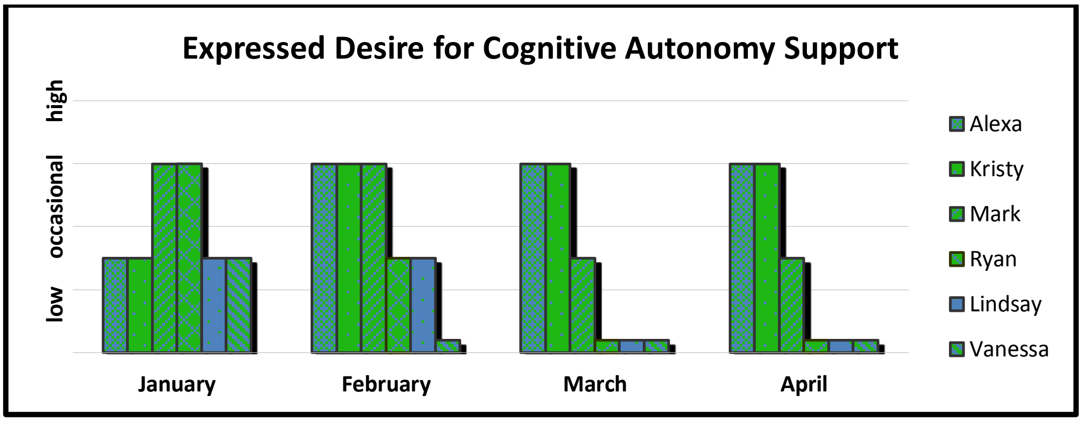

Figure 3. Cognitive autonomy over term.

When procedural and organizational autonomy support was provided during the latter half of the term, students split into those engaging in the flipped classroom completely and those interacting with it in a more or less self-paced manner by either opting out of classroom activities or choosing to not attend class.

Falling behind seemed highly associated with absence. Some students fell behind because of external factors which influenced absences (Mark and Ryan), while others chose to be absent because of internal factors such as falling behind (Lindsay). Falling behind can be extremely frustrating and can lead adults to withdraw from a course (McAlister, 1998). In the flipped classroom outlined in this study, procedural and organizational autonomy allowed self-pacing to be a management skill, a sort of coping mechanism for falling behind. Remarkably, students who fell behind were able to catch up through the use of the video resources that were provided as part of the flipped classroom. Had these students not been able to access content delivery materials out of class time, they may have not been able to complete the course with so many absences, which could have led to withdrawal or failure.

Attendance

Some students downgraded to the self-paced mode of interaction after a series of absences because they had to catch up with the material that they fell behind with. Mark and Ryan encountered uncontrollable challenges in their lives that caused them to be absent due to external factors, altering their interactions in the class, while Lindsay chose to be absent due to internal factors related to her choice of interaction with the class. In particular, Mark missed several classes due to illness. After this period of absence, he began to work individually. At first, he used the self-paced mode of study to catch up with material he missed, but then he continued to use it in order to move ahead of schedule. Similarly, Ryan missed several classes due to a funeral, and then an English paper that took more precedence for him. After these periods of absence, Ryan used his class time in a self-paced manner in order to catch up with the material. He did not return to engaging in the complete class experience. Lindsay was also absent a lot. However, unlike the others, there was no significant external reason for her absence. She noted that she was absent when she was too far behind to participate in class activities. Absence seemed more like a coping mechanism for her.

Goals

Finally, the bifurcation into two key types of learning experience may be more prominently attributed to a variety in student goal orientations. Students who engaged in all components of the class (Alexa, Kristy, and Mark) tended to exhibit learning goal orientations. Whereas students who treated the class in a self-paced manner (Ryan, Lindsay, and Vanessa) by opting out of the more collaborative class components, tended to portray performance goal orientations. Those with strong performance goal orientations evidenced a focus on efficiency in completing the course requirements. For example, Ryan agreed on the follow-up survey that he tended to avoid coming to class when he was behind because he “felt [he] could use [his] time more effectively outside of class, rather than covering more material [that he] would not understand.” In contrast, Alexa and Kristy pursued learning activities regardless of whether they contributed to their grade of not.

Summary of analyses

Essentially, the students who engaged in the complete flipped classroom as presented in this paper were taking advantage of the collaborative elements of the class that provided them with cognitive autonomy support, primarily within the problem solving learning activities. These students held strong learning goal orientations. Meanwhile, the students who experienced the classroom in a self-paced manner focused on less collaborative components of the classroom where they could work individually and efficiently in an effort to satisfy their performance goal orientations. These students also tended towards embracing cognitive structure rather than cognitive autonomy support. The bifurcation of student experiences in the class occurred half way through the term when procedural and organizational autonomy support was more prominent. Students tended to opt-out of attending class or participating in group activities during this time if they were so inclined. It is interesting to note that once students downgraded to using the course in a self-paced manner, they did not return to using the elements of the course completely. However, what is most important is that all of the cases were able to successfully complete the course with a final grade of B+ or higher regardless of the manner in which they chose to experience the course. A visual summary of these analyses is provided in Table 2 below.

Table 2.
Summary of Analyses of Components by Case

	Alexa	Kristy	Mark	Ryan	Lindsay	Vanessa
Cognitive Autonomy Support	●	●	●	☾	☾	☾
Attendance	●	●	☾	☾	☾	●
Learning Goal Orientation	●	●	●	○	☾	☾
Performance Goal Orientation	☾	☾	○	●	●	●
● high	**Complete**			**Incomplete**		
○ occasional	**Flipped Classroom**			**Self-Paced Classroom**		
☾ low						

Conclusions

The main intent of this research is to describe how students can experience a flipped classroom that is designed to promote flipped learning (The Flipped Network, 2014) and a transactional student-centred learning environment (Elen et al., 2007). Although the flipped classroom in this study afforded the capacity for a collaborative student-centred learning environment where the teacher was guided by student learning needs, it also provided students with an autonomous opportunity to choose ways in which they could interact in the class. In summary, students in the adult mathematics upgrading course Math 084 bifurcated into experiencing the flipped classroom in one of two ways: the complete flipped classroom and the self-paced option that the flipped classroom afforded.

Students who experienced the complete flipped classroom tended to exhibit strong learning goal orientations and engaged themselves autonomously in the collaborative learning tasks provided, the facilitative role of the teacher, and the social culture of learning in the classroom community. These students had more consistent class attendance and were less swayed by changes in organizational and procedural structure than their self-paced counterparts. On the contrary, students who experienced the flipped classroom as more of a self-paced classroom tended to exhibit strong performance goal orientations, often resisting cognitive autonomy support in an effort to maintain efficiency in completing tasks, and did not embrace engagement in collaborative learning opportunities.

Interestingly, this bifurcation of student interaction coincided with the increase in provision of procedural and organizational autonomy support in the latter half of the term during which class times were less structured procedurally and organizationally. As more organizational and procedural autonomy support was provided, self-paced performance oriented students tended to focus on completing the minimum requirements of the course. The bifurcation of student experiences also coincided with increasing student absences. Once students fell behind in the material or experienced a series of absences, they typically resorted to treating the course in a self-paced manner, an interaction that they continued until the completion of the term. It should be noted, however, that these students may have easily dropped out of the course had they not been provided with an extensive set of resources to help them complete the course as many adult students do when they fall behind in course material (McAllister, 1998).

Both the complete flipped classroom and the self-paced option that the flipped classroom afforded were highly student-centred and allowed students to pursue their goals orientations in the context of the course, regardless of their nature. Some students evidenced a shift in goal orientation from performance oriented to learning oriented, likely due to the contagious nature of engagement during collaborative problem based activities, but others did not exhibit this shift. Hence, it is important to note that although it is desirable for students to pursue goals of learning, it is not always what they desire. This speaks to the ever-present tension between student and teacher goals. It is also a good reminder of the fact that a goal cannot be forced onto anyone. Instead, the goal can be encouraged and nurtured through providing opportunities for developing deeper understanding if a student so desires. This is the essence of a student-centred learning environment.

The flipped classroom in this study provided students with an invitation to pursue goals of learning without forcing it to be the only option. Students could still complete the course and satisfy the prerequisites they needed by interacting in a self-paced manner, but more importantly, those who became interested in developing deeper meaning in mathematics were given the opportunity to do so through the collaborative nature of the classroom learning environment. Cognitive autonomy support in particular served as a determining factor in classroom interaction. This research supports the premise of Jang et al.'s (2010) theory that classrooms conducive to engagement give both structure and autonomy. In particular, organizational and procedural dimensions should be structured, while the cognitive dimension should be provided with autonomy support in order to promote student engagement in opportunities for collaboration with peers during meaningful classroom activities. Therefore, the main result of this research is that it affirms that cognitive autonomy support is an essential ingredient in promoting student engagement in learning opportunities.

Implications

The most important implication of this research for the adult mathematics education community is that it is an illustration of a learning environment that is conducive to providing adult students opportunities for pursuing goals of learning while maintaining accessibility of prerequisite completion through self-paced options. Although this study was conducted as a small scale exploration of six case studies in one particular implementation of the flipped classroom, it provides a basis for future research opportunities.

Future studies may want to look at exactly how each of the two ways of interaction in a flipped classroom (complete and self-paced) affect student understanding of the material in comparison to each other and in comparison to a control group that is not taught according to a flipped classroom model. Student achievement in a flipped classroom could also be studied further. All participants in this study who completed the course did so with a B- or higher. This leaves room for investigation of whether the flipped classroom in general pushes students into either succeeding in the course or dropping out of the course, or if it was just an instance that occurred within this small scale study.

Finally, a flipped classroom is merely a mindset with no clear method of implementation. Further implementation approaches could be explored. For example, content delivery videos could be used as content review rather than content preview. Class time could be treated in a more structured manner. Assessment strategies such as standards based grading could be also be explored. There are many opportunities for exploration of various approaches to flipped classroom implementation. That is the beauty of the flipped classroom model: it provides a mode by which teachers can accomplish their goals of evolving a student-centred learning environment without compromising the delivery of the curriculum.

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