

Perceptions and Performance of King Saud University Students' about Concept and Finding Limit of Functions Graphical and Symbolic

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Abstract

The purpose of this study was to investigate the misperceptions and performance of king Saud University students' about concept and finding the limit of some partial functions given in the form of both graphical and symbolic. In the collection of the data, the researcher employed the open-ended question test consisted of four items. Firstly, symbolic representation of the questions, later graphical representation of the same questions was given to the students. In the data analysis, descriptive analysis method was used and independent samples t-test was employed with $\alpha = 0.01$ in the analysis of the differences of students' algebraic and graphical performance. The result showed that some students had some misconceptions and misunderstanding related to the concept of limit. It was also observed that there were significant differences found between students' performances algebraic and graphical. The students had higher scores in graphical representation than symbolic.

Keywords: limit, misconception, mathematics education

1. Introduction and Theoretical Framework

1.1 Introduce the Problem

Mathematics defined as the study of measurement, calculations and geometry, In addition to modern concepts, including the Mathematical structure, variation and dimensions. In general, some may defined it as the study of abstract structures using logic and mathematical proofs, and more generally, also defined as the study of mathematics numbers and patterns. So, the main interest in mathematics which takes in consideration as an empirical science of creativity and innovation of the human mind is the sequence of ideas and methods of thinking (Abu zeineh, 2003).

Also, the mathematics feature as an integrated and coherent and sequential of mathematical knowledge by presenting abstract models which interpret some sensible and non-sensible of natural phenomena. Accordingly, the mathematics are considered the backbone of the development and the needs of different science branches to mathematics, it can hardly imagine a prosperous in various fields, but as far as we acquire it and comprehend in the branches of mathematics (Al Mejiadel & Al yafei, 2009).

Since the conceptions of modern mathematics were founded on the function such as limit, derivative and integral. Studying function became one of the most fundamental and important concepts in all branches of mathematical topics including calculus and abstract algebra and geometry (Froelich et al., 1991). Therefore, a lot of research emphasizes the importance of the function (Breidenbach et al., 1992; Leinhardt et al., 1990).

The functions can introduce by multiple representations such as equations consisting of algebraic notation, graphs consisting of a pictorial presentation, and tables consisting of ordered pairs of values (Brenner et al., 1997). The educational researches have indicated that students ingeneral prefer the symbolic on the algebraic representation of the function. However, graphs consider being the shortest way to obtain information, and the most useful and economical ways for summarizing large amounts of data about the function (Friel et al., 2001). Therefore, The National Council for Teachers of Mathematics focus on interpreting graphs, and considered it is an important for all the science and mathematics' curriculum (NCTM, 2000).

The concept of the Limit in mathematics consider to be one of the basic concepts in calculus, which written as, $\lim_{x \rightarrow a} f(x) = L$ and mean in simply, that f can be made as close as we want to (L) , by taking (x) close enough to (a) but not equal to (a) . In this process, f has to be defined near (a) , but not necessarily at (a) . It also

originated the concept of the limit in the context of the need to calculate the lengths and areas and volumes of shapes such as the circle and the spherical shapes, and the limit concept of the function can be summarized in that way to find a value that must be dependent variable take when the independent variable approaches to a certain value, so when the independent variable cannot be calculated by direct and algebra rules. Although the mathematics interesting material, tend to self-study and research where they often be a stumbling block to many of us. Due to the lack of our understanding of the origins and theory and laws, and is no doubt that this inability to understand not a defect in the same article, but stems from ourselves we are. However, the educational theories agree on the importance of error in mathematics and its role in the learning process, and the intention here of errors that can be detected through a learning path that interrupted the learner to acquire knowledge. He also believes that the learner does not acquire only what suits his abilities and in its interaction with the knowledge, so the error is generated has the while learning through misunderstanding or change the meaning or linking other concepts. From this perspective, mathematical errors become an essential based for use as a starting point to solve problems and the development of mathematical thinking, as well as to pay attention to the educational process of learning guide through the teachers analysis to the sources and causes of errors of learners, in order to address them and to help them overcome them, to build new concepts (Tsamir & Bazzinin, 2002).

The teacher Knowledge and understanding to the common error of students help to develop strategies in teaching that address mathematical errors and misunderstandings, and on the other hand, the learner benefit from the error, and through verification of assumptions and perceptions formed has started, in other words we can say that mistakes can raise important issues to discover more in mathematics, because the teaching and learning of mathematics is built through a sense of the importance of those errors that cannot be ignored or only corrected (Borasi, 1987). Consequently, the common errors type of students, can be attributed to the nature of the material, and the student himself, and a teacher who is it the responsibility to minimize the effect of both the student and the material through the use of methods and learning strategies suit the level of students and have an effective impact on reducing errors (Al shareh & Abed, 2012; Tsamir & Bazzinin, 2002).

In recent years, concern has increased the identification of common mathematical errors in the cognitive structure of the students before learning concepts. Many studies have emphasized that existing knowledge in the cognitive structure of the students is one of the most important factors affecting the learning math correctly, the existence of the common misconceptions among students could lead to a negative impact on the effectiveness of learning, may be the result of ignoring the teachers' perceptions and alternative interpretations in learners before the start of the new learning (Elgandy & Shihap, 1999).

The purpose of Duru (2011) study was to investigate the pre-service teachers' conceptions about the limit of same partial functions given in the form of both graphical and symbolic, the misconception that the pre-service teachers have about the concept of limit, and whether there were any differences between pre-service teachers' algebraic performance and their graphical performance. The researcher used both qualitative and quantitative methods in collection the data (open-ended question test and interviews). The study tool consisted on five items. Firstly symbolic representation of the questions was administered to 95 pre-service teachers, later graphical representation of the same questions were given to pre-service teachers. Finally, semi-structured interviews were done with eight pre-service teachers. The result of study showed that some students had some misconceptions and misunderstanding related to the concept of limit. It was also observed that there were significant differences between pre-service teachers' algebraic and graphical performances. The Pre-service teachers had higher scores in graphical representation than symbolic. Also, a study conducted by Dreyfus and Eisenberg (1982) aimed to measure the performance of students in the concepts of mathematical functions provided through graphs and tables, figures and charts of functions. The results found that students with high ability to learn through graphs of functions, while the degree and the ability of students to study and learn the concepts of functions through the tables were low.

It has been observed through teaching limits topic in Math 150 course, which studied by all the preparatory year students of scientific disciplines at King Saud University, the occurrence of some students in some of the errors in calculating limit at a point graphically and algebraically. It was also noted that students had some difficulties and mistakes through calculating limit at a point when applying the properties and theories of limits. Therefore, it can be said that students have common errors and misconception; because they think that a partial function must be defined and continuous at a certain point to have a limit and that the limit is equal to the function value at that point. In addition to the errors in finding limit of the partial-function at a point whether it graphically or algebraically, can be classified into the lack of clarity of the limit concept, and procedural errors during of calculating the limit refer to the students weakness in the basic operations in the previous requirements in mathematical knowledge.

In light of the above, and through the researcher experience, it is clear that knowledge of the performance and perceptions of students about the limit concept, and methods of finding it is a matter of concern for the detection of the common errors that students located at the study of mathematics, especially in the first phase of a university education. After reviewing the literature and studies relevant to the limit and continuity, clearly seems few studies have examined the classification of students common errors on the limit topic in general at the Arab and local levels, while at the local level, no studies have addressed the issue of the limits, and investigated what errors happens by students on the level of school or college.

1.2 The Problem of the Study

Anyone who has followed the process of teaching and learning students noticed many of the behaviors that indicate the existence of weaknesses in learning mathematics. It also can be monitor a lot of practices that indicate the depth of the problem among students in learning mathematics. The student's behavior during math classes emphasizes the extreme weakness in learning basic mathematical concepts and skills. He also notes through their answers on tests not being able to the simple basics of mathematical operations they learned in earlier stages of education, that help them to learn new mathematical knowledge.

Mathematical knowledge is cumulative, meaning that new knowledge to the students build on previous knowledge learned through the various stages of education, some educators in general indicated that when students move from secondary education to higher education most of them fail during their education at the university, perhaps due to the weakness of the previous cognitive structure of the students in the basic concepts and requirements, which is pre request for studying some new mathematical topics (Jonatan & Peter, 2012).

Due to the importance of behavior study and properties of functions through computing the limit of functions. Also, educational literature has pointed to the importance of studying the concept of limit through graphic representation to explain the behavior and characteristics of the Functions for all students through the continuation of their studies university according to various scientific specialties. In addition to discussions with the teachers of mathematics course (Math-150-Calculus) in the preparatory year, which includes the study of derivative of functions, including the study of the characteristics and behavior of functions through finding the limit, and continuity graphically and algebraically, where it seemed clear there are difficulties with the students, and the weakness in the previous cognitive structure.

Although the mathematical knowledge offered to all students at the same time, however, not everyone learns in the same way, and not everyone has the same ability to learn (Zaytun, 2007). So there is variation among students to learn one concept, and graded lack in the learning process, which negatively affects the cognitive and academic achievement of students.

Study Questions

The aim of any teacher in education is to help students to achieve the educational goals. But sometimes there are some factors that make it difficult to achieve these goals. The purpose of this study was to investigate the performance and the ability of the preparatory year students at King Saud University on the application and employing what has been learned within the Limits and Continuity and methods of calculation, in addition to stand on their performance and their perceptions about the limit concept of partial mathematical functions by recognizing the common error types in methods of finding limit for the piece-wise function at the point graphically and algebraically, and ratios of common error, through answering the items of the study tool. Also, the problem highlights by trying to answer the following questions:

- What is the performance of the preparatory year students at King Saud University in finding the limit of partial functions at the point graphically and algebraically?
- Are there any statistically significant differences in student performance due to the methods of finding limit of partial functions at the point graphically or algebraically?
- What types of common errors do by students of preparatory year at King Saud University about the concept and finding limit of partial functions at the point graphically or algebraically? And the ratio of common errors?

Importance of the Study

The importance of the current study played role from the importance of limit subject in calculus, and in the development of mathematical thinking among students, which in turn helps them to continue their university studies, also highlights the importance of the study through its attempt to analysis types of mistakes made by the students when they learn the limits, especially the subject of finding limit is considered very important because knowledge of calculus is based on the foundation of the subject of finding limits, where the limit describe the

behavior of the dependent variable when the independent variable takes outliers. For example, consider the function ($y = 1/x$). As we know, this type of function is undefined at ($x = 0$), because the division by zero is not possible in the real numbers. Therefore we cannot calculate the value of (y) when ($x = 0$), however, we can observe how the behavior of the function when (x) approaches zero from the right and left sides, and this is the concept of the limit. One of the most important uses of the limits in the science of Calculus is derivative, and integration, for example, the derivative of a function that is the limit of the change rate (the ratio between the value of the dependent and independent variable) when the value of the Independent variable approaching zero.

Objectives of the Study

This study aims to investigate erroneous perceptions of the Preparatory year students at King Saud University, through analysis student responses on a test of study, and to identify the varieties of the common errors and ratios of common errors about the concept and finding the limit of partial functions at a given point graphically or algebraically. The study also aims to identify the average of student's performance through items of study tests.

Limitations of the Study

The limitations of this research identified by the following:

- The instrument which was developed by the researcher, so the interpretation of the results depends on the instrument's validity and reliability.
- Sample size: the study sample consisted of 220 students distributes at 11 section, have been selected at random.
- Limited Sample of students male in the Deanship of Preparatory Year at King Saud University in the second semester (2012/2013) who have completed studying limit and continuity topics through calculus course (math 150-Calculus).

2. Method and Procedures

Descriptive approach and survey has been used to get the data and facts about the nature of the student's common mistakes and ratios, about the concept and finding limit of partial functions at the point graphically and algebraically.

2.1 Population and the Study Sample

The study population consisted of all male students 2280 of scientific disciplines enrolled in the second semester of the academic year (2012/2013), to study calculus course (math-150-Calculus) in the Deanship of the preparatory year at King Saud University. They are distributed on 105 sections. The sample consisted of 220 students, distributed in 11 sections, have been selected randomly.

2.2 The Study Tools

The study tool Consisted on two tests of limit and continuity topics, which was built in light of the expected appearance in student responses errors through the limit of partial function at a given point graphically or algebraically, which represented the subjects measured by tests items, error classes at the concept and computation of the limit at given point graphically or algebraically illustrated in (Appendix 1 and Appendix 2). The tests included 8 essay items, four items on the concept and computation of the limit graphically, and four items on algebraically representation.

2.3 Validity Tests

To check the validity of the tests, was presented to a group of arbitrators four PhDs specialists in curriculum and methods of teaching mathematics. And two PhDs specializing in Educational Measurement and Evaluation. And 10 teachers of mathematics have master's degrees in mathematics, and who were teaching in the second semester. Each of them was given the test items and a list of common errors that have been prepared, and were asked to express an opinion on the occasion of the items of the target group of the preparatory year students at King Saud University. After reviewing the opinions of the arbitrators and suggestions have been modified, to achieve the purpose of the research and investigated the errors types, the application of tests procedures and instructions require that the student shows the steps resolved in detail, and in which is standing on the strengths and weaknesses in student performance, so the tests in this way can be considered that achieved a standard of validity.

2.4 Reliability

The reliability compute by using test and re-test, through the application the tests on an exploratory sample consisted 35 students, with an interval of 3 weeks, who completed studying the limit topics in Math-150-course. The Pearson correlation coefficient was 0.91 between the average performance of students in the first time and repetition (Oadeh, 2005; Gronlund, 1990).

2.5 The Study Procedures

The study included on the following actions:

- Review and analysis chapter limits and continuity, and methods of finding in the Calculus course (Math 150) studied by students in the preparatory year at King Saud University.
- Select 11 section from 105 section randomly, to represent the sample, with totally 220 student.
- The first test application in the period ranged from 30 to 35 minutes from the time of the lecture, where test consists of four items given in algebraic form to measure the performance of students about the concept and finding limit of partial function at a given point (Appendix 1).
- The second test applied on the sample study after two weeks from the first test application through a period ranged from 30 to 35 minutes. Where the second of four items given as graphically for the same partial function at a given point (Appendix 2).
- Identify the key answer before starting the process of correction.
- A random sample of the correct answer sheets, and monitors some errors and added to the initial list, which previously monitored through each item answers of the exploratory sample.
- Analyzed the errors appears in the students answers and monitored in accordance with the type of test graphically or algebraically. Also, has been taken into account adding any new error does not appear in the preliminary list to the list of items errors, and follow correct the same item to the end.

2.6 Statistical Treatment

Package statistical analysis of Social Sciences (SPSS) was used in the treatment of data. Frequencies and percentages were extracted to answer the first question of the study. Also, averages and standard deviations, and (t-test) for independent samples were used to answer the second question of the research. And to answer the third question of the research, it was extracting frequencies and percentages of errors.

3. Discuss the Results and Its Interpretation

To achieve the objectives of the study and stand on student performance and misperceptions and common errors about the concept and finding the limit of partial functions at the point graphically or algebraically. Student responses analyzed, to identify the misconception, common errors, through the ways of solution and methodology used by students. So results will be displayed according to the questions of the study.

-The first Question:

What is the performance of the preparatory year students at King Saud University in finding the limit of partial functions at the point graphically and algebraically?

To answer the first question, the percentages of the correct answer numbers on the items of two tests graphically and algebraically were calculated, as illustrated in Table 1.

Table 1. Number of the correct respondents on the items of graphically and algebraically tests, to the students who tried to answer and their percentages

Items	Second test: graphically					First test: algebraically				
	(1)	(2)	(3)	(4)	Total	(1)	(2)	(3)	(4)	Total
Number (wrong answer)	50	58	62	55	(225)	99	95	75	79	(348)
Number (correct answer)	170	162	158	165	(655)	121	125	145	141	(532)
Percentage(correct to wrong)	77.3%	73.6%	71.8%	75%	74.4%	55%	56.8%	65.9%	64.1%	60.5%

Table 1 shows that the number of students who correctly answered the items of the first test (algebraically) and their percentages. It is clear that two students from among three students 60.5% answered correctly, which represent finding the limit of partial function at point given in algebraic formula also 65.9% the highest percentage of correct answers obtained by the students respondents at the first test were on the third item, which represents finding the limit of partial function at point given in algebraic formula. This means that 34.1% of students did not answer the item correctly. This percentage indicate that the concept of limit not clear for some students according to the purpose of the item, which measures the ability of students in finding the limit through partial function not defined at the point, and the limit does not exist at same point.

It also notice that the percentage 55% came to represent the lowest percentage of correct answers obtained by the students on the first item, which means that 45% of the students did not answer the item correctly. This percentage refers to the low of student performance. Since the item measures students' ability to find the limit of the given function algebraically, were the function defined at that point and the limit does not exist.

It is noticeable that the percentages of correct answers on the items of the first test located between the two percentages 55% and 65.9%. Therefore this is an indication of not having some students in general to the concept and finding the limit at the point of partial function through the algebraic formula.

Table 1 also shows that the number of students who answered correctly on the items of the second test (graphically) and their percentages. It is clear that three students from among four students 74.4% answered correctly on the second test items, which represent finding the limit of the same partial function at point but through graphical representations. The highest percentage of correct answers were 77.3%, obtained by the students in the second test, were on the first item, which represents finding the limit of function at point through graphical representation. This means that 22.7% of students did not answer them correctly. This percentage indicate that the concept of limit not clear for some students according to the purpose of the item, were to measure the ability of students in finding the limit, when the limit does not exist and the function defined at same point , but through the graphical representation. Also, the percentage 71.8% represented the lowest of correct answers obtained by student's respondent on the third item, which means that 28.2% of the students did not answer this item correctly. This percentage refers to the low in the student performance, where the item measures students' ability in finding the limit of the partial function at point, but through the graphs of the function, which not defined and the limit does not exist at that point

We notice that the percentages of correct answers to the items contained in the second test lied between the two percentages 71.8% and 77.3%. Therefore this is an indicator that some students in general not having the concept and finding the limit at point through the algebraic formula.

In light of the importance of studying the behavior and properties of mathematical functions and their application, weakness of performance of some students can be attributed to the lack of mathematical knowledge which is previous requirements for studying mathematical functions and applications properties, Such as: finding the solution set for various kinds of inequality, and determine the domain of mathematical functions. It can also be attributed this to the weakness of the students in the principles of logic and mathematical reasoning, or to the previous experience gained by the students in the different stages of the studying.

-The second Question:

Are there any statistically significant differences in student performance due to the methods of finding limit of partial functions at the point graphically or algebraically?

To investigate if there is differences in the average performance of students on the items of the first test given in algebraic formula and the second test given by graphical representation to find the limit of partial functions at point. It has been the averages illustrated for the performance of students on all the items of the two tests, as shown in Table 2.

Table 2. Results of (T-test). The differences between the means of performance according to the test type (graphically or algebraically)

items	test	means	St.dev	sig
1	Graphically	2.4955	0.98620	**0.0002640
	Algebraically	2.1409	1.09509	
2	Graphically	2.4409	1.00281	**0.0010170
	Algebraically	2.1227	1.15408	
3	Graphically	2.4182	1.01928	0.4593430
	Algebraically	2.3545	1.03444	
4	Graphically	2.5955	0.79667	**0.0004050
	Algebraically	2.2818	1.06528	
Total average	Graphically	2.4875	0.55013	**0.0000040
	Algebraically	2.2250	0.79986	

** : Statistically significant differences at the level of significance ($\alpha = 0.01$) between the means performance of students on the items of the first test and the second.

The results of (t-test) indicate that there is significant differences between the mean performance of students on the first test 2.225 given in algebraic formula, and the mean of student performance on the second test 2.4875 given in graphical representation, and favor the mean of student performance on the second test, which includes Finding the limit of functions given in graphical representation.

As seen through comparisons between the means of students performance on the items of the two tests, exist a statistically significant differences in performance on the test items of second test (graphically), excepting the third item, the result indicated that the mean of student's performance on item given through algebraic formula 2.3545 and mean of student's performance on the same item through the graphical representation 2.4182 is not statistically significant. Meaning that the mean of students performance in finding the limit of mathematical function "which is not defined and the limit does not exist at the same point" does not differ whether the function was given as algebraic formula or graphical representation. And can be attributed with statistical significance between student performance differences that graphical representation of functions be effective tools in giving a visual perception about the characteristics and behavior of mathematical function more than the use of formulas and algebraic symbols. Similar findings with the results of the study of each of (Duru, 2011) and (Keller & Hirsch, 1998) which found differences in student performance on the test given by graphic representation on the test given by algebraic formula, and the preference of students to the study of mathematical functions through multiple graphical representations on the content for the study of mathematical functions given by algebraic.

Also can be attributed to the decline in the mean of some student's performance to misunderstanding and misconceptions about the limit concept and the way of finding it, where some students believe that the limit of the function and the value of the function at the point is the same. Also some students believed that if the function defined at the point then the limit has the same value at that point. Also, some students believed that if the function does not defined at the point then the limit does not exist at that point, and finally, some students believed that if the limit of a function exists at point, then the function continuous at that point.

-The third Question:

What types of common errors do by students of preparatory year at King Saud University about the concept and finding limit of partial functions at the point graphically or algebraically? And the ratio of common errors?

To answer the third question, the frequencies and percentages calculated for the number of times the error in the students' solutions appearance, Has also been monitoring the repeat error class when it appears and follow correct answer student on the same item to monitor other errors. As well as monitoring the number of times the error is repeated when the student himself or others, Then chosen errors that more than 20% as common error according to the standard that has been relied upon to classification errors. In order to facilitate the discussion of the results, it will be display according to the type of the first test (algebraically) and second (graphically) as clear in Table 3.

Table 3. Errors Classes Frequencies and Common percentage by subject and item number in the two tests, and the ratio of common class error by subject within the same class

Errors class	Subject and Item No.		Limits exist and function defined at the point		Limits does not exist and function defined at the point		Limits doesnot exist and function defined at the point		Limits exist and function defined at the point		The total number of frequencies of the class and the error percent	
	(1)	(2)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)
Errors:finding limit at the point from right	.Alg	.Grap	.Alg	.Grap	.Alg	.Grap	.Alg	.Grap	.Alg	.Grap	.Alg	.Grap
	17	12	20	11	23	12	12	14	72	49	24%	20%
Errors : finding limit at the point from left												
	20	8	18	15	16	13	9	14	63	50	32%	20%
Errors in judgment the existence or not of the limitat the point												
	14	10	16	13	14	11	10	12	54	46	26%	18%
Errors: adoption of the function value on existence the limit, or not												
	24	9	16	10	14	15	12	18	66	52	36%	21%
Errors:Confusion between the right and left the point when finding limit												
	21	11	18	12	12	17	6	14	57	54	37%	21%
Errors : the basic arithmetic operations in substitute and simplify at limit computation												
	13	-	15	-	13	-	16	-	57	-	23%	-
Total												
	96	50	88	61	79	68	49	72	312	251		

* : First Percentage means $(\text{error class frequency} \div \text{total frequency of class}) = \text{the percentage of common error for each subject by class.}$

** : Second Percentage means $(\text{error class frequency} \div \text{total error frequency by subject}) = \text{the percentage of common error by subject.}$

First test: common errors among students about the concept and finding limit of partial functions algebraically.

The results of the analysis in the Table 3 indicate that 24% represent higher percentages misconceptions among students about the concept and finding the limit through the test items given in algebraic formula, represented by finding limit at the point from the left, followed by the percentage 21% represented this misperceptions in the adoption of the value of the function on the existence or not of the limit.

For the detection of misconceptions among students about the concept and finding the limit by class of error and the percentage of common as illustrated in Table 3. The first item included the lowest common errors percentage by the students reached 14% and Class errors in basic arithmetic operations in computation and simplification to find the limit for “function defined at the point and the limit exist”, while the highest percentage 25% of the perceptions of students misconceptions about the concept and finding limit of the functions algebraically, where in the class of errors “adoption of the function value at the point on the existence of the limit, or non-existent value”. The second item included calculating the limit of function defined at the point and the limit does not exist, where the percentage of less percentage common errors was 17% and the error in basic arithmetic operations in substitution and simplification class at the calculation of limit the function, and were 23% the higher percentage of Class errors in finding limit of a function at point from right side. And the third item included the lowest percentage common errors of the students were 15% and from Class “confused between the right and left point at the limit computation”, and also, the percentage of error class “computing the limit at the point from the right” were the highest ratios of common errors which reached 29%. The lowest percentage of errors on the fourth item was 12% and from class “confuse left and right of the point through computing the limit of function were “not defined at the point and limit does not exist”, and was 32% the highest percentage of type errors in basic arithmetic operations in substitution and simplification through finding the limit of a function were “not defined at point and the limit exist”.

When comparing common percentages errors within the error classes, we find that the confusion between the right and left of the point through finding the limit of function not defined at the point and the limit exist, formed the highest percentage 37%, followed by the common percentage of the adoption of the function value on the existence of the limit, or non-existent. Came percentage 11% less common ratios in the error class “confused between the right and left of the point through finding the limit “for function is not defined at the point and the limit exist”.

Second test: common errors among students about the concept and finding limit of partial functions graphically.

The results of the analysis in the Table 3 indicate that the percentage 21% represent highest percentages of misconceptions among students about the concept of the limit and finding it through the test items given in graphical representation, marked by the adoption the value of the function on the existence of the limit, or non-existent, in addition to confuse between the left and right of the point through finding the limit.

To detect the perceptions of students misperceptions about the limit concept and finding for partial functions at point graphically through ratios of common mistakes classes among students as shown in the Table 3. It is clear that the first item gets the lowest ratio of common errors by students were 16% and in errors Class “finding the limit of the function at the point from the left” the function were “defined and the limit exist at the point”, while the highest percentage 24% were from errors Class “finding the limit from the right side”.

The second item included finding the limit of the function defined at point and the limit does not exist, where the percentage of less percentage in common errors was 14% and through the Class errors “finding the limit of the function at point from the right side”, and were 28% represented the highest percentage of errors Class Errors in judgment on the existence or not of the limit. And the third item included, the lowest percentage in common errors of the students were 15% and through the Class errors “Errors in finding the limit at point from right”, and the percentage of error class “confusion between the left and right of the point” through finding the limit of function were “not defined and the limit does not exist at the point”, the highest ratios of common errors which reached 25%.

The lowest percentage of common errors on the fourth item was 17% and class error “finding limit value from the right side” for the function “not defined at the point and the limit exist”, and the 25%, was the highest percentage of class error “errors in judgment on the existence of the limit, or non-existent” for finding the limit of function were not defined and the limit exist at the point

When comparing the percentages of common errors classes within the same error class, we find the highest percentage 35% in the class error which included “errors in the adoption the value of the function on existence the limit, or non-existent” through finding the limit of function “not defined at the point and the limit exist”, followed by 31% which represents the percentage of error class ratio “Confusion between the right and left of the point when finding the limit”, and the percentage 14% Came less common ratios in the common error class of errors in judgment on the existence of the limit, or non-existent through finding the limit of the function not defined and the limit does not exist at the point.

Since the mathematical knowledge cannot be transferred is ready from one person to another according to the theory of constructivism, because it should be built by each learner, because the idea of the limit is complex, and it is difficult to understand by the students of this idea, possibly due to a link is understood in terms of the language of more than mental image of the students, such as “approaching to or implies to... and other”. In addition to the corresponding words refer to the concept of the limit in the English language as the students for the first time studying math in English language. Educational literature also indicates that there is no effective methods to help students overcome the difficulties they face during their studies of the concept of limit (Cottrill et al., 1996). So in order to understand the limits and other concepts in calculus, students must be their perception and the ability of the scientific explanation for the behavior of mathematical functions and charts its own, and this includes the idea of continuity, and the students also need to have knowledge and skill to represent mathematical functions graphically especially if it is given as algebraic and interpretation.

4. Conclusion and Recommendations

Results of the study showed that most students aware that if the value of limit from right-side is equal to the value of limit from the left side and equal the value of the function at the same point, then the limit is exist. Also the results showed that there is a misunderstanding of some students about the concept and finding the limit consisted judging that the limit exist or not through the value of the function at the point. If the function defined at the point then the limit will be exist, and if the function not defined at point then the limit does not exist. Some difficulties as observed among students in identifying the right and left to find the limit at point through the algebraic formula. Some misconceptions in judgment as noted on the existence of the limit, or lack thereof through graphic representation, particularly if the limit exist and the function discontinuous at the same point, also noted the existence of procedural errors in finding the limit at a given point through the algebraic formula, represented with an Errors in the basic arithmetic operations in computation and simplification at limit computing.

In light of the study results can be recommended as follows:

- Perform a diagnostic test for the preparatory year students admitted to the university, and identify the actual errors that appear in the performance of students in mathematics and then address these errors as soon as they occur.
- Need to focus faculty members to master the basic concepts and skills associated with the concept of limits and ways of finding, because its importance in understanding the other topics in mathematics, such as the rate of change, and the derivative, and integration.
- Further studies in the field of the common errors analysis of the students about the concept of the limit and finding, and compare them with the common errors among students at various universities.

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Appendix A

First Test: Items in the Algebraic formula

Errors Classes	Item Subject (algebraically)
<p>The following symbols identified the common errors as:</p> <p>E1: computing limit of function at point algebraically from right.</p> <p>E2: : computing limit of function at point algebraically from left.</p> <p>E3: judgement on the limit existence or not at point algebraically.</p> <p>E4: adaptation the value of function on existence of limit or non-existence.</p> <p>E5: Confusion between the right and left of the point when finding limit.</p> <p>E6: Errors in the basic arithmetic operations in substitute and simplify at limit computation.</p>	<p>First item: finding limit of partial function at point Algebraically “limit exists and the function defined at the point”.</p> <p><i>Q1:</i> Discuss the limit of</p> $x = -2 \text{ at } f(x) = \begin{cases} -2, & x < -2 \\ 1, & x = -2 \\ x, & -2 < x \leq 2 \end{cases} \quad ? \text{ 判断}$
	<p>Second item: finding limit of partial function at point Algebraically “limit does not exist and the function defined at the point”.</p> <p><i>Q2:</i> Discuss the limit of</p> $f(x) = \begin{cases} 3, & x > 2 \\ -2, & x < 2 \\ x, & -2 \leq x \leq 2 \end{cases} \quad ? \text{ 判断 at } x = 2$
	<p>Third item: finding limit of partial function at point Algebraically “limit does not exists and the function not defined at the point”.</p> <p><i>Q3:</i> Discuss the limit of</p> $f(x) = \begin{cases} 1, & x < -2 \\ x^2, & -2 < x \leq 2 \end{cases} \quad ? \text{ 判断 at } x = -2$
	<p>fourth item: finding limit of partial function at point Algebraically “limit exists and the function not defined at the point”.</p> <p><i>Q4:</i> Discuss the limit of</p> $f(x) = \begin{cases} x^2 + 4x, & x < 1 \\ 5, & x > 1 \end{cases} \quad ? \text{ 判断 at } x = 1$

Appendix B

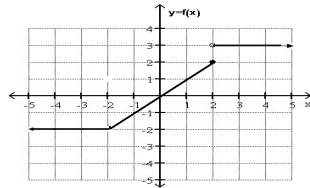
Second Test: Items in the graphical representation

Item Subject (graphically)

Second item: finding limit of partial function at point Algebraically “limit does not exist and the function defined at the point”.

Q2: Use the graph below of $y = f(x)$, to discuss

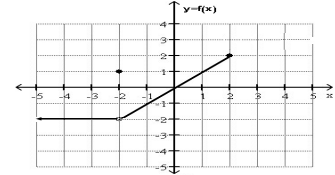
$$\lim_{x \rightarrow 2} f(x)$$



First item: finding limit of partial function at point graphically “limit exists and the function defined at the point”.

Q1: Use the graph below of $y = f(x)$, to

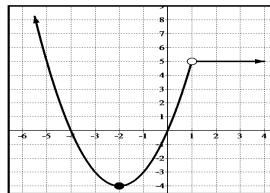
$$dsscuss \lim_{x \rightarrow -2} f(x)$$



Fourth item: finding limit of partial function at point Algebraically “limit exists and the function not defined at the point”.

Q4: Use the graph below of $f(x)$, to dsscuss

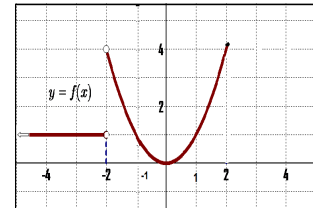
$$\lim_{x \rightarrow 1} f(x)$$



third item: finding limit of partial function at point Algebraically “limit does not exists and the function not defined at the point”.

Q3: Use the graph below of $y = f(x)$, to

$$dsscuss \lim_{x \rightarrow -2} f(x)$$



Errors Classes

The following symbols identified the common errors as:

- E1:** computing limit of function at point algebraically from right.
- E2:** : computing limit of function at point algebraically from left.
- E3:** judgment on the limit existence or not at point algebraically.
- E4:** adaptation the value of function on existence of limit or non-existence.
- E5:** Confusion between the right and left of the point when finding limit.

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