The Mathematics Educator
2015 Vol. 24, No. 1, 21–43

Preservice Secondary Teachers' Conceptions From a Mathematical Modeling Activity and Connections to the Common Core State Standards

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Mathematical modeling is an essential integrated piece of the Common Core State Standards. However, researchers have shown that mathematical modeling activities can be difficult for teachers to implement. Teachers are more likely to implement mathematical modeling activities if they have their own successful experiences with such activities. This paper describes one well-structured framework for implementing mathematical modeling with the Common Core State mathematics standards that incorporates the Standards for Mathematical Practice. One class of preservice secondary teachers engaged in a mathematical modeling activity, reflected on their experience, and discussed how they could implement similar modeling activities. This study describes the preservice secondary teachers' work on the modeling activity, their impressions of the activity, and how the mathematical modeling activity was effectively structured in connection to the Common Core State Standards.

The Common Core State Standards for Mathematics (Common Core State Standards Initiative [CCSSM], 2010)

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have the potential to foster teacher collaboration and improve the teaching and learning of mathematics. The standards do not dictate teaching methods, although the eight Standards for Mathematical Practice (SMPs) describe proficiencies that students should develop. One of the standards for mathematical practice, model with mathematics (SMP 4), places more emphasis on modeling in secondary classrooms. Additionally, 16 content standards are highlighted as modeling standards at the high school level. This focus is warranted due to the many applications of mathematical modeling in our global technology based society (Lesh, 2010). Engaging students in mathematical modeling activities can help them build the understandings and abilities that are needed for success not just in school but also in daily life. However, the successful implantation of mathematical modeling can be difficult for teachers (English, 2009; Galbraith & Stillman, 2006; Warwick, 2007). One reason for these difficulties is mathematical modeling often involves requiring students to make sense of complex systems within an interdisciplinary context (English, 2009). Additionally, teachers must redefine traditional roles to become more of a facilitator. To effectively implement modeling tasks teachers need to have robust knowledge of the content in those tasks, possible models that students may develop, the difficulties that students may encounter during the process, and knowledge of pedagogical practices that focus on student-centered classroom discourse (Shulman, 1986).

In order for preservice mathematics teachers to effectively develop students' ability to model with mathematics, it is important that they have opportunities to participate in research-based mathematical modeling curricula. If preservice teachers have successful experiences with mathematical modeling activities and are able to see how the activities connect to the CCSSM (both content and SMPs) then they may be likely to recognize the benefits of mathematical modeling and implement such activities in their future classrooms (Metz, 2010). The purpose of this paper is to describe an interaction with preservice secondary mathematics teachers (PSMTs) focused on mathematical modeling. The PSMTs completed a mathematical modeling activity and engaged in conversation regarding useful structures for implementing mathematical modeling in secondary classrooms that are integrated with the CCSSM. Specifically, the research questions that guided this study were:

(a) What are preservice secondary mathematics teachers' conceptions of modeling from a mathematical modeling activity that is connected with the Common Core State content Standards?

(b) In what ways was the preservice secondary mathematics teachers' work on the mathematical modeling activity connected to the Standards for Mathematical Practice?

Model-Eliciting Activities (MEAs)

A Model-Eliciting Activity (MEA) is a specific type of well-structured and researched modeling activity that is carefully designed based on six principles (Table 1). The principles ensure that groups of students are able to work through an iterative process of model development, providing teachers (and researchers) a lens to observe the thinking of students (Lesh & Doerr, 2003). MEAs were originally developed as a research tool to explore students' conceptual understanding while they solved real-life problems (Lesh & Lamon, 1992). Additionally, researchers have found that the use of MEAs helps students become more adept problem solvers and allows them to demonstrate understanding that might not be evident through traditional assessments or activities (Lesh & Doerr, 2003).

MEAs are structured so that students are not only asked to solve a problem for the immediate context of the problem but are also required to develop their solution so that it would be generalizable to other similar situations. This process helps students develop their problem solving strategies as well as their ability to extend what they have learned during the MEA to new problem solving situations (Lesh, 2010).

In the existing literature, there are different variations on the mathematical modeling process (Perrenet & Zwaneveld, 2012). MEAs include common characteristics of the modeling process that are present in the CCSSM (2010) modeling cycle (p.72-73). For example, mathematical modeling is an iterative process that involves cycles of model construction, evaluation, and revision (English, 2009). The process starts with a problem

Table 1
Principles for Guiding MEA Development

Principle	Description
Model Construction	Ensures the activity requires the construction of an explicit description, explanation, or procedure for a mathematically significant situation
Generalizability	Also known as the Model Share-Ability and Re-Useability Principle. Requires students to produce solutions that are shareable with others and modifiable for other closely related situations
Model Documentation	Ensures that the students are required to create some form of documentation that will reveal explicitly how they are thinking about the problem situation
Reality	Requires the activity to be posed in a realistic context and to be designed so that the students can interpret the activity meaningfully from their different levels of mathematical ability and general knowledge
Self-Assessment	Ensures that the activity contains criteria the students can identify and use to test and revise their current ways of thinking
Effective Prototype	Ensures that the model produced will be as simple as possible, yet still mathematically significant for learning purposes (i.e., a learning prototype, or a "big idea" in mathematics)

(Lesh, Hoover, Hole, Kelly, & Post, 2000)

that needs to be translated into a mathematical model (Perrenet & Zwaneveld, 2012). Students must then develop their model within the problem context to ensure that their model is usable for the realistic situation (Warwick, 2007). After developing their model, students need time to reflect on the modeling process and the mathematics they used in their solution process (Perrenet & Zwaneveld, 2012; Warwick, 2007).

MEAs and the SMPs

In general, MEAs are team-based, client-driven, realistic problems that incorporate reflection and many of the CCSSM Standards for Mathematical Practice. Possible connections between MEAs and how they have the potential to address the SMPs are provided in table 2. The design of an MEA will likely enable the development of the first six SMPs based on the inherent structure of MEAs (Lesh et al., 2000). Depending on the specific MEA, and the solutions that are developed, the last two SMPs are likely to be developed in students as well.

Table 2
Possible MEA connections to SMPs

Standard for Mathematical Practice	How it can occur in MEAs
1. Make sense of problems and persevere in solving them.	As participants work through iterations of their models they continue to gain new insights into ways to use mathematics to develop their models. The structure of MEAs allows for participants to stay engaged and to have sustained problem solving experiences.
2. Reason abstractly and quantitatively.	MEAs allow participants to both contextualize, by focusing on the real world context of the situation, and decontextualize by representing a situation symbolically.
3. Construct viable arguments and critique the reasoning of others.	Throughout MEAs while groups are working and presenting their models.
4. Model with mathematics.	This is the essential focus of MEAs; for participants to apply the mathematics that they know to solve problems in everyday life, society, or the workplace. This is done through iterative cycles of model construction, evaluation, and revision.
5. Use appropriate tools strategically.	Materials are made available for groups as they work on MEAs including graph paper, graphing calculators, computers, applets, dynamic software, spreadsheets, and measuring devices.
6. Attend to precision.	Precise communication is essential in MEAs and participants develop the ability to communicate their mathematical understanding through different representations including written, verbal, symbolic, graphical, pictorial, concrete, and realistic.
7. Look for and make use of structure.	Participants in MEAs can use their knowledge of mathematical properties and algebraic expressions to develop their solutions.
8. Look for and express regularity in repeated reasoning.	As participants develop their models the patterns they notice can assist in their model development.

MEAs are meant to compliment the content of a course and address higher order thinking skills including the evaluation of others' ideas, synthesis of information, and analysis of relationships, patterns, and structures. Moreover, MEAs enable students to utilize the diversity of their knowledge bases and strengths of their classmates because MEAs can be solved using multiple strategies that incorporate varying levels of mathematics. The specific MEA used in this study, Historic

Hotel, was modified from an existing MEA (Aliprantis & Carmona, 2003) and focuses on the mathematics of quadratic functions.

Historic Hotel Model-Eliciting Activity

The Historic Hotel MEA is a modification of an economic problem typically used in undergraduate calculus classes that connects the economic concepts of profit, cost, and maximization (Aliprantis & Carmona, 2003). Given information on the number of rooms that will be rented for different room prices, students are asked to develop a method for determining the most profit that a hotel could earn from room rentals. The Historic Hotel MEA has the potential to address several Common Core high school modeling standards because it is based on the mathematical concept of optimization of quadratic functions. Specifically, three standards for functions that are particularly well connected to this MEA are:

- F-IF-4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
- F-IF-7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- F-BF-1: Write a function that describes a relationship between two quantities.

Although students may use other mathematical concepts in their solutions, it is important for teachers to have main concepts in mind that they will explore after the MEA. By making such considerations, teachers can use MEAs at the beginning of a unit or chapter as a formative assessment. Other concepts that are used by students will only enrich the main mathematical concepts that students learn by engaging in the MEA. These concepts will be further explored throughout the unit that follows the MEA (Stohlmann, 2012).

The Historic Hotels MEA has been used with middle school and high school students. Aliprantis and Carmona (2003) used the MEA with three classes of seventh grade students. Their goal was for students to develop a mathematical and economic model that would incorporate the concepts of maximizations, linear and quadratics relations, variable recognition, profit, cost, and price. The researchers investigated the process that students used to obtain their answer, and how that process demonstrated the students' mathematical knowledge. Students used different representational systems in their work including symbolic algebra, tables, graphs, operations without variables, and written descriptions. The most frequent representations used were tables. The researchers found that not all students were able to produce a mathematically correct answer, but they did demonstrate better mathematical models than expected. The fact that students were able to demonstrate mathematical understanding, though not fully developed, illustrates the benefit of using MEAs as a formative assessment tool to understand and build on students' prior knowledge.

Backman (2008) used the Historic Hotel MEA as an introduction to a unit on quadratic equations with ninth and tenth grade students in an honors-level mathematics class. During the study, the researcher collected student work, field notes, and surveys. The different strategies that groups used to solve the MEA included tables, equations and the table feature of a graphing calculator, and operations with the list feature of a graphing calculator. Backman found that the MEA could be used effectively as a formative assessment of student knowledge in order to provide a reference for further instruction and engage students in the study of quadratic models. The MEA was also a powerful instructional tool in that students made connections to the MEA as they progressed through the unit on quadratics. At the conclusion of the unit all students were able to develop a solution for a similar problem in a new context of planning a concert.

For our study, we shared these prior studies with the PSMTs after they completed the MEA in order to illustrate strategies students might try. This information highlights the benefit of using MEAs as a formative assessment tool to engage students in realistic mathematical problem solving. One of the main components of pedagogical content knowledge is teachers' ability to respond appropriately to student work through knowledge of possible student strategies (Hill, Ball, & Schilling, 2008).

Our study is valuable in that it shows how MEAs connect well to the CCSSM. Curriculum that is research-based and linked with standards will likely have more teacher buy-in. Current commercial textbooks often do not adequately allow students to fully engage in mathematical modeling (Meyer, 2015). This creates a need for teachers to use supplementary activities, we suggest MEAs, to give students the most effective mathematical modeling experience. Also, there is currently little research about preservice teachers' conceptions about mathematical modeling.

Methods

The participants of this study were 17 preservice secondary mathematics teachers enrolled in a secondary mathematics methods course at a large southwestern research university. The PSMTs developed solutions to the MEA and responded to reflection questions after completing their work on the MEA. Two researchers took field notes throughout the implementation of the MEA and the class discussion that followed. Although the context of the MEA was modified, the general structure of the MEA remained similar to its usage in previous studies.

From prior experience of working with teachers to implement MEAs, we knew that the time needed in order to effectively implement MEAs is often a concern. To model pedagogical strategies, we asked the PSMTs to complete part of the work on the MEA outside of class to demonstrate time-effective implementation of MEAs. Before the class period, the PSMTs were asked to individually read an article outlining historical information about the Golden Gate Casino, which is currently the oldest operating casino in Las Vegas. The authors of this article also provided background information on the casino's renovation that provided the real-life context for the activity. The MEA was implemented in one class period and followed a format of opening reading, readiness questions, problem statement, group work, group presentations, and time for revision. The PSMTs responded to several questions addressing the responsibilities of a hotel manager, the benefits and drawbacks of room renovations, and factors that could influence the price of a room (To see the complete MEA visit http://wordpress.unlvcoe.net/wordpress).

At the beginning of class, the opening reading and readiness questions were discussed. Following this discussion, the problem statement (Table 3) for the MEA was discussed as a whole class. The realistic client for this MEA is Mark Brandenburg, the president of the Golden Gate Casino. The PSMTs were asked to develop a model for the specific situation given in the problem statement as well as develop a generalized model that could be effectively used for similar situations.

The PSMTs worked in groups of three or four and were given approximately 35 minutes to develop a solution to the problem. After the allotted time, each group shared their solution with the class. The groups then were given time to revise their original solutions. Following the time provided for revisions, the instructor led a discussion on the general purpose, format, implementation, and motivation of using MEAs. The connections between the MEAs and the CCSSM SMPs and content standards provided earlier in this paper were also discussed. Lastly, the PSMTs were asked to answer the following reflection questions:

(a) What mathematical or scientific concepts and skills did you use to solve this problem?
(b) How well did you understand the concepts and skills you used?

(c) How well did your team work together? How could you improve your teamwork?

(d) Did this activity change how you might think about mathematics or how you might teach mathematics?

Table 3
Historic Hotels MEA Problem Statement

Mark Brandenburg, president of the Golden Gate Casino, has offered your team the chance to be interns at his hotel. There are a wide variety of tasks that must be coordinated to have a well-functioning hotel. A hotel in Las Vegas needs employees with a wide variety of skills and knowledge backgrounds. However, Brandenburg recognizes that hotel management is different for casino hotels versus non-casino hotels so he would like you to learn about both types. He has a task for you that focuses on the price of the hotel rooms. The Golden Gate Casino currently has 102 rooms. Mr. Brandenburg would like you to figure out how to make the most profit on just the hotel rooms. On a typical weekend all of the rooms are usually occupied when the daily rate is \$60 per room. He has found that for every dollar increase in the daily \$60 rate, one less room is rented. So, for example, if he charged \$61 dollars per room, only 101 rooms would be occupied. If he charged \$62, only 100 rooms would be occupied. Each occupied room has a \$5 cost for service and maintenance per day. Mr. Brandenburg would like to know how much he should charge per room in order to maximize his profit and what his profit would be at that rate. Also, he would like to have a procedure for finding the daily rate that would maximize his profit in the future even if the hotel prices and the maintenance costs change. Write a letter to Mr. Brandenburg explaining how he can calculate his profit and how much he should charge so that his profit is maximized. Be sure that your method works even if hotel prices and costs rise in the future. Include a good reason for each step of your procedure. Your team will also be responsible for presenting your solution, procedure, and letter to our class.

The PSMTs' work, researcher field notes, and responses to the reflection questions were analyzed to identify and describe the models developed by the PSMTs, as well as general themes emerging with regard to their impressions of this MEA type. Memos (Corbin & Strauss, 2008) were written based on the field notes to describe the PSMTs' work during the MEA and the solution strategies that they developed. The two researchers who took field notes wrote individual cases for each group. Comparisons were made between the individual cases and any discrepancies were reconciled or removed from the narratives. Cross-case analysis was then conducted through coding the data for evidence of the Standards for Mathematical Practice (Patton, 2002). Again, the two researchers who took field notes individually coded the data for connections to the SMPs.

Finally, comparisons were made and discrepancies were reconciled or removed.

Results

The PSMTs worked in five groups on the Historic Hotels MEA. The instructor for the class limited his questions to asking groups what they had come up with so far and does their model meet the needs of the client in order to allow groups to develop their own models. When each group presented their solution, they also shared the progression of ideas that led to their solution. To present as robust illustration of the process as possible, we provide narratives of each groups' solution strategy, comments from the other groups, and the revisions that each group made after every group presented.

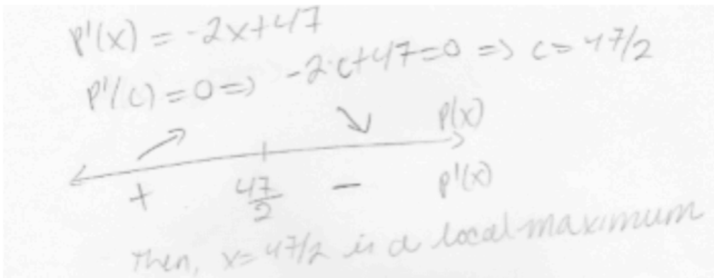
Group 1

At the outset, Group 1 attempted to develop an equation that related profit, revenue, and cost. They ended up with a function for profit, P(x), that took the revenue and subtracted the cost where x was the amount of increase on the $60 room rate (Figure 1). They expanded the initial function equation so that the equivalent quadratic expression appeared in standard form. Next they took the derivative of P(x), tested the critical value, and stated there was a local maximum. Based on this information they stated that the hotel manager should charge $83.50 and would then have a profit of $6,162.25. They also developed a generalized function that could be used for similar situations. In explaining their generalized function, they noted that, "a new situation might not have the same relationship that a one dollar increase in room rate leads to one less room being occupied."

Based on another group's comments, Group 1 ended up changing part of their solution. Group 2 commented that based on the equation, Group 1 "needed to have a room rate that was a whole number to match the situation given in the problem statement that a dollar increase leads to one less room being rented." Group 1 ended up deciding on charging $83 for the room rate.

$$P(x) = \overbrace{(60 + x)(102 - x)}^{\textbf{Revenue}} - \overbrace{5(102 - x)}^{\textbf{cost}}$$

$$P(x) = -x^2 + 47x + 5610$$



In general
P(x) = (daily rate + increase)(number of hotel rooms – increase) – maintenance (number of hotel rooms – increase)

- Ratio of dollar increase to room decrease may not always be 1

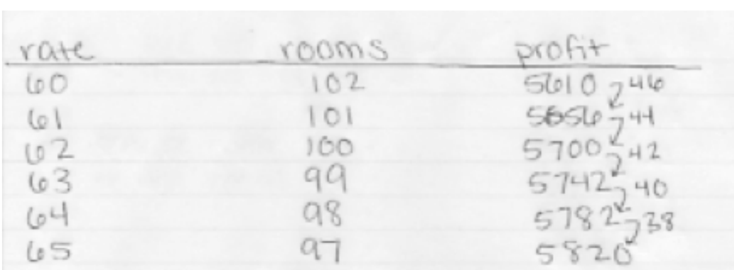
Figure 1. Group 1's Solution.

Group 2

Initially the group tried to use an equation, table, and graph to help them develop their solution. Each group member individually worked with one of the representations to begin. One group member was not successful in developing an equation and knew something was wrong because he was getting negative numbers for the profits. Another group member started to make a table with the different combinations of room rates and occupied rooms to see if a pattern could be found. The third group member began to graph points with room price and profit. The group members then started to share ideas and were able to generate an equation based on how one group member calculated the profit for her table. They ended up getting the same equation as group one, took the derivative and then set it equal to zero to get the maximum value. They checked the profit using their equation for a room rate of $83 and for $84 and found that the profit was the same. Group 2 shared that "since many hotels have restaurants attached that a hotel manager would want more rooms occupied to make more money." For this reason they stated that the price should be set at $83. This group did not have a generalized model determined and did not include one in their final solution. The group mentioned that if a person made a presentation to a CEO of a hotel they would want to show their information in many different ways including graphs, tables, and equations.

Group 3

Group 3 developed an equation that already accounted for the $5 dollar maintenance fee, profit = (55 + n)(102 – n), but had trouble remembering how to use a derivative so they decided to use a table (Figure 2). They adjusted their profit in the table by subtracting the maintenance cost for each room. In explaining their strategy, they noted that "the difference between the profits started at 46 and then each successive difference decreased by 2." Using this pattern they realized that if they increased the room rate by 19 more dollars then the difference would become zero and give them the maximum profit. Based on this they shared that the profit would be $6,162 with a room rate of $83.



rate	rooms	profit
60	102	5610 46
61	101	5656 44
62	100	5700 42
63	99	5742 40
64	98	5782 38
65	97	5820

Figure 2. Group 3's table.

Group 3 did not notice that two room rates would give the same profit. In their final solution they added this information to show that either room rate would give the same profit. In their final solution, they also included a generalized model (Figure 3). The generalized model was based on their original equation.

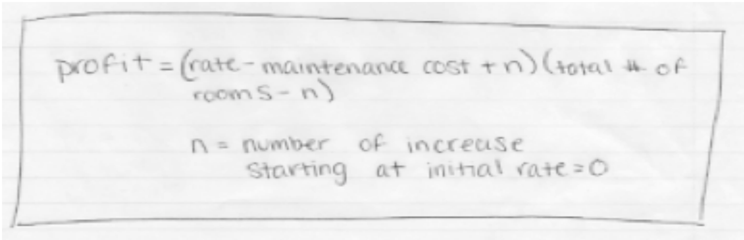


Figure 3. Group 3's generalized model.

Group 4

Group 4 developed the same equation as group 1 and 2, and set the derivative equal to zero. They evaluated their equation for x =23 and x =24 and found that the profit was the same. They reasoned that, "based on psychology, people look at dollar amounts and not cents so that a price of $83.50 would be seen as a room rate of $83." They decided to charge this amount because they could make $39.50 more profit with the same number of rooms filled for a rate of $83. This group was able to develop a generalized model similar to Group 1 but kept 102 in the equation for the number of rooms.

Group 5

Group 5 started with the idea that the situation would be able to be modeled with a parabola because they were trying to "maximize profit." They noted that the values for the variables they defined would be integers greater than or equal to zero (Figure 4). Through discussion they realized that in an equation they could adjust the money earned from the room rate by initially subtracting the maintenance fee as indicated on the right side of the table. They then used trial and error to find different profits for different values of n. Group 5 identified the greatest profit as $6,162 with a room rate of $83.

Group 1 noted that the Group 5 equation, P =(102-n)(55 - n), would not work for the generalized model. Because of this group 5 wrote a new equation for their generalized model that was similar to Group 3 except they kept 102 in for the number of rooms. That is, in general P = (102- n)(c - m + n).

r = number of rooms rented c = cost of room m= cost of maintenance integers ≥ 0 wrote equation c*r - m*r initially c= \$60 \$5 = m 102 = r plugged these values into the equation 60*102 – 5*102 = 102*55 = \$5610	wrote equation for profit P = (102 – n)(55+n) n is integer $n \geq 0$ n = 10, P = (92)*(65) = \$5980 n = 20, P = (82)*(75) = 6150 n = 23, P = (79) * (78) = 6162

Figure 4. Group 5's solution.

Across the five groups there ended up being some commonalities in the groups' models though the process was not the same for how the groups ended up with their models. The models that were developed showed how MEAs can be used as a formative assessment. All five groups developed an equation to model the situation with group 1,2, and 4 having the same equation and groups 3 and 5 having the same equation. Four groups had the idea to use a derivative, with one group not remembering how to use this concept. Two groups ended up charging 83 dollars for the price of the room, two groups found that 83 or 84 dollars would give the same profit but ended up selecting 83 dollars, and one group ended up charging \$83.50 to increase their profit.

MEA Discussion

After completing the MEA, the instructor gave the PSMTs an overview of how MEAs could be incorporated into a curriculum, practical advice for implementation of MEAs, the benefits of MEAs, how MEAs are integrated with the CCSSM, and prior research on the Historic Hotels MEA. The preservice teachers posed the following questions about the mechanics of implementing MEAs: (a) How many students were in each group in the studies mentioned? (b) Would these activities work with larger classes of 30 to 40 students? (c) Would students work together or just use the time to talk? (d) Would low-level students be able to do these type of problems? (e) What about students that get distracted easily? (f) If there is not much space in a classroom and groups are right next to each other, how do you ensure that groups will not bother each other?

There was a class discussion on general ideas related to classroom management, building classroom community, and also on the benefits of using well-structured modeling activities. The instructor identified the aspects of MEAs that allow students to collaboratively work on ideas, and allow all students to contribute their knowledge and experiences. The self-assessment principle for developing MEAs ensures that students are able to iteratively express, test, and revise their models. The information provided in the problem, other group members, and other groups allow for this iterative process to take place so that groups can better meet the needs of their client.

Reflection Questions

After the discussion on MEAs, the PSMTs responded to reflection questions. The PSMTs were asked if the Historic Hotel MEA changed how they think about mathematics or how they might teach mathematics. Fourteen of the preservice teachers stated that they would like to use this type of problem in their classroom because of the perceived benefits of using "real-life problems" and increasing student engagement. Two of the PSMTs still expressed doubts that students who "lack average skills" would be able to develop a solution for a MEA. Additionally, one PSMT indicated that he would like to see "more examples of MEAs that correspond to the topics I would teach" before deciding if it would be useful for students.

Connection to SMPs

Upon reviewing the observations, the eight SMPs were evident to varying degrees in the PSMTs' solution development processes (Table 4). The first six SMPs were evident in our analysis of the data for all of the groups and occurred throughout the MEA. The last two SMPs were only evident once in our analysis of the data for two of the groups.

Table 4
Standards for Mathematical Practice integration with MEAs

Mathematical Practice	How it occurs in MEAs	How it occurred in the Historic Hotel MEA
1. Make sense of problems and persevere in solving them.	As participants work through iterations of their models they continue to gain new insights into ways to use mathematics to develop their models. The structure of MEAs allows for participants to stay engaged and to have sustained problem solving experiences.	The preservice teachers in this study were engaged throughout the activity and iteratively developed their solutions that meet the needs of their client.
2. Reason abstractly and quantitatively	MEAs allow participants to both contextualize, by focusing on the real world context of the situation, and decontextualize by representing a situation symbolically.	The preservice teachers developed equations to model the situation, but also interpreted their findings in terms of the original situation to determine their profit and room price.
3. Construct viable arguments and critique the reasoning of others.	Throughout MEAs while groups are working and presenting their models.	The preservice teachers provided a rationale for the development of their models. They also provided critiques of other groups' models including the generalizability of their models and their hotel room price.
4. Model with mathematics.	This is the essential focus of MEAs; for participants to apply the mathematics that they know to solve problems in everyday life, society, or the workplace. This is done through iterative cycles of model construction, evaluation, and revision.	The preservice teachers developed a model to determine the room price that would maximize profit for a hotel. Also, four out of the five groups developed a generalized model that would work for similar situations.
5. Use appropriate tools strategically.	Materials are made available for groups as they work on MEAs including graph paper, graphing calculators, computers, applets, dynamic software, spreadsheets, and measuring devices.	The preservice teachers in this study chose to use pencil, paper, and calculators for calculations.
6. Attend to precision.	Precise communication is essential in MEAs and participants develop the ability to communicate their mathematical understanding through different representations including written, verbal, symbolic, graphical, pictorial, concrete, and realistic.	The groups were able to describe the development of their model and to show this symbolically as well. The groups could have improved their written communication by defining their variables more clearly, but overall described their solutions well with correct units.
7. Look for and make use of structure.	Participants in MEAs can use their knowledge of mathematical properties and algebraic expressions to develop their solutions.	Group 3 and group 5 developed a simplified form of the equations developed by the other groups by taking into account the maintenance fee $(102 – x)(60 + x) – 5(102 – x) = (102- x)(55 + x)$.
8. Look for and express regularity in repeated reasoning.	As participants develop their models the patterns they notice can assist in their model development.	Based on recognizing patterns in tables, group 2 and group 3 were able to develop equations. Group 2 used the process they used in making their table to develop their equation to find the profit. Group 3 used the differences of the profit in their table to determine when the maximum profit would occur.

Discussion

We conducted this study to determine PSMTs' conceptions of modeling from engaging them in a mathematical modeling activity. Furthermore, we endeavored to identify and describe ways in which their work was connected to the Standards for Mathematical Practice. Researchers indicate that implementing mathematical modeling activities can be difficult for teachers (English, 2009; Galbraith & Stillman, 2006; Warwick, 2007).

Due to the increased emphasis on modeling in the high school CCSSM, it is important to provide structures for teachers to effectively implement modeling activities. Model-Eliciting Activities (MEAs) are an example of well-structured activities that can be used in such ways, as well as provide opportunities for formative assessments for teachers. Above, we have demonstrated some of the important benefits of implementing mathematical modeling through MEAs including how they can be used in a time efficient manner as well as how they are connected to the CCSSM and the SMPs.

In this study the PSMTs worked with mathematical ideas that include: developing a function that models a relationship between two quantities, interpreting key features of graphs and tables, derivatives, critical values, solving equations, making generalizations, defining variables, and interpreting solutions in a realistic context. In previous research with the Historic Hotels MEA, seventh grade students used mostly tables in their solutions but also graphs, equations, and written descriptions (Aliprantis & Carmona, 2003). Beckman (2008) found that ninth and tenth graders mainly used the graphing calculator list and table features as well as tables made by hand. However, later in the unit all students were able to develop a quadratic equation to a similar problem and work with maximization ideas to develop a solution.

Through experiencing the Historic Hotels MEA and learning about how secondary students had worked with this activity, the PSMTs may be well prepared to implement mathematical modeling activities. For teachers to successfully implement modeling activities they need all three types of content knowledge that have been described by Shulman

(1986): curricular, content, and pedagogical. The PSMTs understood the information presented in the MEA, what was being asked in the problem statement, and were able to experience and discuss how to implement an MEA. Teachers also need to know the mathematical content that is covered by the MEAs in order to facilitate groups' ideas and model development. Through their work on the MEA the PSMTs were able to develop their content knowledge. Pedagogical content knowledge is also essential so that teachers will have ideas about what solutions students may develop and appropriate ways to respond to students' thinking. The PSMTs were able to see what solutions students may develop through their own work and hearing about what models students had developed in prior studies. Throughout the class period, these ideas were incorporated in order to help PSMTs form positive views about MEAs.

Researchers have suggested that being in a supportive community that values an idea can lead to change in views (Ambrose, 2004). The PSMTs' success in completing an MEA was a positive experience that they reported made them more likely to try a modeling activity in their own teaching. According to Rogers' (1995) diffusion of innovation theory there are five stages to adapt to a new idea (in this case MEAs): knowledge, persuasion, decision, implementation, and confirmation. At the beginning the individual does not have enough knowledge about the idea. In the second stage, "persuasion," the individual actively seeks for information. After accepting the idea, the individual starts implementing the new practices and evaluates the results to determine if the practices are effective. In this study, through a positive experience discussing and working through an MEA, most of the preservice teachers went through the knowledge, persuasion, and decision steps to valuing mathematical modeling. It is yet to be seen if the PSMTs will implement and see the benefit of MEAs in their teaching.

In order for teachers to implement new activities, time concerns and connections to standards must also be addressed. The PSMTs in this study were able to see how the SMPs and content standards were integrated with the MEA and also experience an implementation model for MEAs that uses class time effectively. This information can be used to change teachers' previous perceptions of MEAs. For example, Yoon, Dreyfus, and Thomas (2010) found that, in practice, teachers often see MEAs as separate "rainy day" activities instead of being integrated into an instructional unit.

Two additional important aspects of MEAs include participants reflecting on the activity and generalizing their solutions. Perrenet and Zwaneveld (2012) found that preservice secondary teachers did not see reflection as a part of the modeling cycle. They suggested that reflection should occur by having participants discuss the generalizability of their models to other situations. Four of the five groups of PSMTs in this study developed generalized models. They also reflected on the mathematics that they used in the MEA, how well they understood the mathematics, how well they worked as a team, and if this activity changed how they might teach mathematics.

The PSMTs in our study were encouraged and challenged to give each other and other groups feedback on their solutions. Through this process all five groups developed a solution that either completely met the needs of the client or partially met the needs of the client (group 2 did not develop a generalizable model). We maintain that through these discussions, the PSMTs became more aware of their own mathematical thinking, thereby informing their overall understanding of potential strategies their own students may employ.

Limitations and Future Research

There are important limitations to this study that could be used to help shape future research. The sample size in this study was relatively small, and as such it is difficult to generalize at this point to state that future preservice secondary mathematics teachers would have a similar experience with this MEA and implementation format. However, the well-structured nature of MEAs should enable groups to develop solutions that meet the needs of their client. The PSMTs' prior mathematics achievement was not looked at in this study, which could have provided more information about the sample.

Although two researchers took copious field notes, audio recordings were not used in this study and so some of the discussions during the group work time could have been missed. The researchers anticipated this and in the groups' presentations made sure to emphasize that groups explain their model development process.

There are several areas for further research connected to this study. The use of audio and video recordings in future research could enable further analysis of preservice teachers' conceptions of mathematical modeling. The connections between MEAs and the SMPs could be tested further in secondary and preservice methods classrooms to determine if other MEAs have similar results to the Historic Hotels MEA. Future research could also focus on developing more models of students' thinking on MEAs that could be shared with preservice and in-service teachers in order to develop their pedagogical content knowledge to effectively implement mathematical modeling. Future research could also look at the PSMTs subsequent implementation of mathematical modeling activities in their practicum or student teaching classrooms.

Conclusions

One of the most effective ways to help teachers improve their teaching is to help them become more familiar with their students evolving ways of thinking about mathematical ideas (Carpenter & Fennema, 1992). The use of MEAs enables teachers to document students' ways of thinking. These activities also engage students in realistic modeling experiences that can develop students' skills needed in daily life and future careers. In order for preservice teachers to appreciate the importance and benefits of mathematical modeling, they need positive experiences in their teacher preparation program (Beswick, 2011). The CCSSM state that, "modeling is best interpreted not as a collection of isolated topics but in relation to other standards" (p.57). The preservice teachers in this study experienced and discussed how mathematical modeling can be integrated with mathematics standards. MEAs are one useful structure to accomplish this purpose.

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