FEATURE ARTICLE

Bifurcation and Hysteresis Effects in Student Performance: The Signature of Complexity and Chaos in Educational Research

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This paper addresses some methodological issues concerning traditional linear approaches and shows the need for a paradigm shift in education research towards the Complexity and Nonlinear Dynamical Systems (NDS) framework. It presents a quantitative piece of research aiming to test the nonlinear dynamical hypothesis in education. It applies catastrophe theory and demonstrates that students' achievements in science education could be described by a cusp model, where two cognitive variables are implemented as controls - the logical thinking as the asymmetry and the field dependence/independence as the bifurcation respectively. The results support the nonlinear hypothesis by providing the empirical evidence for bifurcation and hysteresis effects in students' performance. Interpretation of the model is provided and implications and fundamental epistemological issues are discussed.

Introduction

One of the main concerns in education is to get a better understanding of the factors affecting school performance and students' academic and social behavior, and to prepare them for their future citizenship. In this task, which is not an easy one, research methodology has a central and crucial role. In the mainstream, however, most researchers have been following the traditional methodologies for decades, while on the other hand, theoretical and philosophical discussions have shown that changes are needed in education research and specifically a paradigm shift towards the *Complexity* regime (e.g. Lemke & Sabelli, 2008). *Complexity theory* and *Nonlinear Dynamics* (NDS) have inspired researchers in all disciplines, since they provide a rich array of constructs, concept and tools, which can describe complex systems changing with time. Those concepts are *attractors*, *chaos*, *fractals*, *bifurcations*, *catastrophes* and the notion of *self-organization* (Cleick, 2008). The NDS language has been productively used in social sciences to describe systems and phenomena and it has enhanced our understanding about them. The same trend has also appeared in education literature, where complexity-informed analyses supported alternative approaches to research, teaching or curriculum development (e.g. Fleener, 2000; Hetherington, 2013; Ricca, 2012). However, descriptions and the use of NDS concepts have stayed at the metaphorical level. Complexity and NDS could undoubtedly serve as a meta-theory to embrace local and specific theories in educational sciences; but for achieving this, empirical research with the implementation of nonlinear methodologies and statistical tools are needed, compatible with the alternative to reductionist views and suitable to test appropriate hypotheses. It is important to point out that the NDS view is a choice to be fostered, but that the nonlinearity has to be confirmed by empirical research. In reality, at any level of complexity at which research might be focused on, things are not always definitely linear or definitely nonlinear. Nonlinearity and the signs of complexity and chaos are not always manifested or cannot be confirmed. The *nonlinear hypothesis*, then, is not an a priori assumption or guess, but it has to be tested empirically against its linear alternative. Moreover, even within the nonlinear regime, there are many alternatives; different dynamics might be present in a nonlinear system mixed with linear dependences and Gaussian noise (Gregson & Guastello, 2005). Thus, the development of the new paradigm is strongly dependent on a nonlinear research methodology, compatible with philosophical -ontological and epistemological - considerations.

Making a step towards the new agenda, within the *NDS regime,* the present work adds to the paradigm shift by presenting an empirical study from science education, where two cognitive variables are implemented to describe changes in students' performance by implementing a nonlinear model. These findings have important epistemological implications, which are discussed along with the relevant methodological issues.

A Need for Nonlinear Methods in Educational Research

In this part, an attempt is made to exemplify the fundamental limitations of the traditional linear (statistical) approaches and the need for a paradigm shift in methodology, coupled with world-view changes in ontological and epistemological considerations.

A core issue in research is that of *causality*, which for science has been extensively analyzed (Bunge, 1979). While it might be considered that what holds for physics sciences holds also, *mutatis mutandis*, for social sciences, today no consensus has been reached on this issue in the literature. The diversity of traditional methodological choices, qualitative and quantitative, does not seem to solve the problems related to establishing causality because they usually are inspired by different theoretical orientations (Maxwell, 2004). The *experiment* as a scientific method has been the only suitable approach to establish causality; however the randomized control trials (RCT) and related designs such as quasi-experimental ones in educational settings and any social process are questionable from various points of view. Koopmans (2014a; 2014b) discusses the disadvantages of RCT in educational experiments, where the process under investigation is considered as a 'black box'. This approach focuses on measurements of input/output conditions ignoring the underlying processes or mechanisms which are actually reflect the nature of the phenomenon under investigation. Cognitive and social phenomena taking place in educational settings are more complex processes when considering the variety of individual differences, levels of motivation and self-regulation, or other cognitive and social skills. Causality through these processes cannot be considered with the linear, common sense cause-and-effect paradigm. These variables, intellectual and 'non intellectual' resources are expected to interact with environmental constraints (tasks) and with each other, in a fashion producing patterns of behavior evolving in time, which are totally ignored by the 'black box' approach. These patterns are dynamic or history dependent in Prigogine's notion, since there is no unique path to follow, and thus, the outcomes are not predetermined and cannot be reduced to certain initial states or variables. Thus, causality and reductionism are matters to be reconsidered.

Another issue is the magnitude and nature of anticipated changes in an educational setting. The above mentioned short-term interactions e.g. during the course of teaching interventions (inside the 'black box') lead to changes, which might not be always small and smooth, but they might be large and abrupt. For instance, quite often it is observed that a low-achiever exhibits, out of the blue, a burst of cognitive abilities when some crucial variables e.g. motivation or self-esteem exceeds a threshold value. Analogously, in developmental trajectories, nonlinear changes are the emergence of talent, and specifically the appearance of late bloomers (Simonton, 2000). These phenomena are nonlinear; in mathematical sense they are discontinuities, which cannot be captured and explained by the usual linear models. All the above identify fundamental ontological features of educational processes and deserve special attention.

Focusing on methodological issues, the dominant quantitative paradigm, in quasi-experimental designs or in cross-sectional studies, is based on linear statistics, which treats students’ achievement as ‘error’ around the mean and they are based on certain presuppositions e.g. the normal distribution. In fact, normal distributions are rarely obtained. Especially when considering very challenging tasks, students’ scores are not distributed normally around the expected value. Often in multiple regression models, it is suggested by residual analysis that the estimated coefficients may not be the best linear unbiased estimator, because the basic assumption of normality is violated and thus the inferential power of the statistical method is limited. The problem with distributional assumptions is ameliorated by implementing alternative distributions e.g. the generalized linear modeling (Hardin & Hilbe, 2012). In educational research the predictive power of linear models is typically low; however this is not an issue, since predictions are not made in the same sense as in other social sciences e.g. economics. A researcher merely attempts to provide empirical evidence that some independent variables have a significant contribution to the outcome, explaining a portion of the percent variance. What is more, the percent of the explained variance is usually small; this is attributed to the large errors and to other variables which are ignored; both are incorporated in the error term and the researcher seeks additional variables to include in the model in order to minimize the portion of unexplained variance. Interestingly, quite often, even with more variables in the model the increase of the explained variance remains unsatisfactory. Although methodological limitations of statistical origin exist, e.g. distributional assumptions and collineariry effects, the researchers usually persist in searching for the suitable set of predictors additively explain a large portion of the variance.

The crucial question is: "hypothesizing that all potential predictors become known, will they be able to explain a hundred percent of the variance?" In other words, is it possible to express the outcome of students' performance as a simple weighted linear sum of the contributing variables? Similarly, in a "black box" -education process, given that all important inputs are known and well measured, is it possible to express and predict the output as a linear function of the contributing components? This is not a merely a methodological question. The main thesis of this paper is that the limitations of linear models are not due the distributional presuppositions which do not hold or to the variables ignored by the theory, but that they are deeply epistemological (Stamovlasis, 2010). Linear models are compatible with ontological considerations that foster a mechanistic view, that is, the whole can be understood from its parts, by an additive manner. If this mechanical metaphor does not hold for educational processes, then the linear approach is epistemologically inadequate to access information or knowledge about the system under investigation.

An additional epistemological remark should be made at this point, rethinking unexplained variance. In classical psychometric theory, applied to educational measurements as well, a measurement y consists of a true score, t, and error e. Errors are assumed to be *identically* and *independently normally distributed* (*iid*). In fact, non-*iid* errors are often present in the residuals and these are indicative of nonlinear processes (Brock, Hseih, & Lebaron, 1990). Then it is epistemologically appropriate in statistically modeling to include nonlinear components that could convey information about the process under investigation. It is important however, that these components are theory laden and provide better interpretation of data, and not just contribute to the curve fitting procedure.

Conclusively, ignoring the dynamic nature of the processes involved in education, assumptions of linearity and the implementation of epistemologically incompatible methods might falsify our inference about outcomes and anticipated changes. If nonlinear changes occur, then an alternative approach for modeling discontinuous patterns of behavior is proposed next by the use of catastrophe theory.

Catastrophe theory

Catastrophe theory (CT) originates with pioneering work of Thom (1975) on morphogenesis and it is a branch of the NDS framework. CT is pertinent to natural sciences and to social and behavioural sciences as well (Zeeman, 1976; Poston & Stewart, 1978). In psychology and behavioral science many applications have been demonstrated within the emergent scientific paradigm of NDS. Some characteristic works are: the connection of CT to Piagetian stagewise development (Molenaar & Oppenheimer, 1985; Van der Maas & Molennar, 1992), to motivation and academic performance (Guastello, 1987), attitude change (van der Maas, Molenaar & van der Pligt, 2003), modeling cognitive overload phenomena (Stamovlasis, 2006, 2011), to mention a few related to educational research.

CT models describe discontinuous changes between stable states in a system observed under gradual increases in a number of independent or control variables. The most well-known and the most applicable model of catastrophe theory is the cusp model (Guastello, 2001; 2002) which describes the changes between two qualitatively different states by two controlling variables, the *asymmetry*, **a**, and *bifurcation*, **b**, factor respectively. Graphically the cusp model is depicted by a response surface (Figure 1), which mathematically is described by the function:

$$\delta V(y, \mathrm{a}, \mathrm{b}) / \delta y = y^3 - \mathrm{b}\, y - \mathrm{a} \qquad (1)$$

where ***y*** is the dependent variable.

When bifurcation variable, b, has low values, then the system is rather stable and a linear relationship best describes the link between the asymmetry variable and the dependent variable, that is, behavior changes gradually and smoothly. However, when the bifurcation variable takes on high values the behavior becomes bimodal. That is, beyond a critical point, a threshold value, the dependent variable (behavior) becomes unstable oscillating between the two modes of behavior. This is what introduces nonlinearity, turbulence and unpredictability in the system, pushing the behavior towards the chaotic regime.

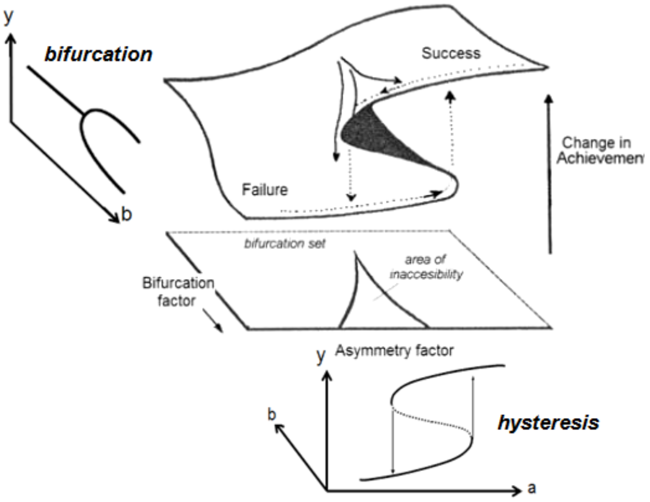

Figure 1. A three-dimensional representation of the cup catastrophe response surface. Bifurcation (upper left) and hysteresis (low center) effects are shown. For the present application logical thinking (a=*L*) and field dependence/independence (b=*F*) are the asymmetry and the bifurcation factors respectively.

An important issue when applying a catastrophe model is the appropriate choice for the variables implemented as controls. This is a theory-driven decision and should be based on a deeper understanding of the role of the candidate variables. In general, variables having strong linear association with the dependent variable are suitable for asymmetry factors, while for bifurcation factors, appropriate candidates are moderator variables, which belong to an opposite or are antagonistic to the asymmetry mechanism. The selected variables could be of any type, cognitive or affective in nature, individual differences or group characteristics, depending on the local theory implemented to interpret the process under investigation.

In the next section, the neo-Piagetian theory is briefly presented in relation to science education, with which the following application is concerned.

Neo-Piagetian Framework in Science Education

Science education research focuses on students' difficulties in understanding physical and chemical phenomena. One interesting and rather challenging area is teaching the material world in terms of atoms and molecules, which is of paramount importance for the contemporary person as a future scientist and citizen. The large amount of work carried out in order to investigate the teaching and learning difficulties on this topic has reveled a wide range of persisting student errors and misconceptions on the structure of matter, which hinder understanding phenomena related to the changes of states e.g. melting boiling, condensation and evaporation (Johnson & Papageorgiou, 2010). One trend in this research area is based on conceptual change theories (diSessa, 1988; Vosniadou & Brewer, 1992) and focused on difficulties arising from the subject matter itself, without however, providing explanations for their origin or correlating them with independent variables. On the other hand, psychological approaches to this domain, such as information processing models and neo-Piagetian theories, view cognitive processes as driven by mental resources explaining variation in performance on cognitive tasks. An example is Pascual-Leone's theory of constructive operators (TCO; Pascual-Leone 1969, 1970). According to TCO, cognitive performance is the responsibility of a variety of constructive operators, each of which performs a specific function: The *M*-operator deals primarily with mental capacity, the *C* operator with content knowledge, the *L*-operator with logical operations such as conservation and formal logic, the *F*-operator with field dependence/independence, and so on. Research has supported the construct validity of TCO, since these operators correspond to mental resources activated during cognitive tasks. Moreover they can be operationalized by psychometric variables measured at the behavioral level. Variables, such as, information processing capacity (M-capacity), logical thinking, field dependence/independence, or convergent/divergent thinking, have been proven to play a significant role in a wide range of tasks, and they affect students' performance in learning science (e.g. Lawson & Thompson, 1988; Tsaparlis & Angelopoulos, 2000; Tsaparlis, 2005).

Logical thinking and field dependence/independence were found to be strongly related to students' achievement score on the specific topic of the structure of matter and its changes of state (Tsitsipis, Stamovlasis & Papageorgiou, 2010). It was found that students' achievements in understanding and explaining phenomena, such as changes in physical states, could be modeled as a function of their level of understanding the structure of matter (the prerequisite knowledge) and the two above cognitive variables. The proposed linear model had low explanatory power and the disadvantages mentioned in the previous sections. In the next section, a brief description of the cognitive variables in question is provided.

Cognitive variables

The cognitive variables implemented in the present application were logical thinking and field dependence/independence:

Logical thinking: Logical thinking is a Piagetian concept and refers to the ability of an individual to use concrete- and formal-operational reasoning and is also referred as the developmental level (Lawson, 1978, 1985, 1993). Logical thinking was assessed by the Lawson test, a pencil-paper test of formal reasoning (Lawson, 1978), which includes the following reasoning modes: Proportional, combinational and probabilistic reasoning as well as reasoning related to the isolation and control of variables, conservation of weight and displaced volume. In science education, but not merely, logical thinking has been found to play a major role in students' performance.

Field dependence/independence: Field dependence/independence or disembedding ability is a cognitive style associated with one's ability to disembed relevant information from complex and potentially confusing contexts (Witkin, 1978). Those who efficiently separate an item from its context, without being confused by dominating field (context) are characterized as field-independent, otherwise they are classified as field-dependent. The two above cognitive styles are not distinct types, but a continuum of intermediate abilities exists, and thus a portion of individuals could be characterized as field-intermediate. Interdisciplinary research has shown that field dependence/independence is correlated with academic performance in various disciplines (Tinajero & Paramo, 1998), such as, language, mathematics, natural sciences, computer sciences, social sciences and art.

Research Questions and Hypotheses

This paper is part of a series of investigation aiming to build bridges between educational research and nonlinear dynamics. It tests the *nonlinear hypothesis* in educational context. Specifically, the working hypothesis behind this design is to predict students' achievements in understanding and explaining physical phenomena (Task 2, challenging task), given their previous achievement on the structure of matter. (Task 1, prerequisite knowledge). Two independent predictors were implemented, which are operationalized as mental resources involved in the task execution.

There are three interdependent research hypotheses: 1) There are bifurcations and hysteresis[1] effects in students' achievement in challenging tasks. 2) Students' achievement scores in science education, particularly in understanding and explaining physical phenomena, could be understood in relation to two cognitive variables within a cusp catastrophe model, where logical thinking acts as asymmetry and field dependence/independence acts as bifurcation factor. 3) Cognitive processes involved in this educational setting such as conceptual change in learning science, can be nonlinear dynamical process.

Method

Participants and measures

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[1] The term hysteresis is a Greek word (ὑστέρησις) meaning 'lag behind'. It might be more appropriate for time series terminology; however, it fits to the present context as well, emphasizing the 'hysteresis' as far as the performance level (or achievement score) is concerned.

The subjects (N=205, 48% male) were students in the 10th grade high school, aged 16, who were taking a compulsory course in physics. The research took place almost one year after the time they had been taught the relevant matter. For the depended measures two tests were implemented, which have been developed and used in the related literature (Tsitsipis, et al., 2010; 2012). Test 1 (Time 1) was an instrument that assessed the students' understanding of the particulate nature of matter. Test 2 (Time 2) was an instrument targeting in assessing students' understanding of physical changes such as boiling, melting and evaporation. The mean score of all items was used as a final score. Both Test 1 and Test 2 were parts of the same examination paper; thus, time is implicit here. Data were collected during one school year through paper-and-pencil tests.

In addition, all students were assessed for the two psychometric variables:
Field dependence/independence (F): F ability of the subjects was assessed by a version of the Witkin, Oltman, Raskin, & Karp (1971) Group Embedded Figures Test (GEFT). This is a timed test (20 min) in which the subject's task was to locate and outline simple figures concealed in complex ones. In this study a Cronbach's alpha reliability coefficient of 0.84 was obtained.
Logical thinking (L): Pupils' logical thinking abilities were measured using the Lawson paper-and-pencil test of formal reasoning (Lawson, 1978). The test consists of 15 items involving, conservation of weight, displaced volume, control of variables, proportional reasoning, combinational reasoning and probabilistic reasoning. The students had also to justify their answers. A Cronbach's alpha reliability coefficient of 0.79 was obtained for the present sample.

Statistical Analysis and Results

Cusp catastrophe analysis with empirical data can be performed by four different statistical approaches proposed by Oliva et al (1987), Guastello (1987), Cobb (1998) and Grasman, van der Maas and Wagenmakers (2009) respectively. They use different modeling and optimizations techniques e.g. least squares or maximum likelihood method, and can be performed with ordinary software or more sophisticated ones, e.g. modeling in cusp-package available in R.

In the present analysis the least squares regression method by Guastello (2002; 2011) was implemented. To this method, bootstrap estimates were added (Stamovlasis, 2012). The dependent measure in this study was the standardized students' change in achievement from Time 1 (Test1) to Time 2 (Test 2). Students' raw scores were transformed to z scores corrected for location and scale s:

$$z = (y - y_{\min})/s \tag{2}$$

Location correction is made by setting the zero at the minimum value of y. The scale is the ordinary standard deviation.

The specific equation to be tested for a cusp catastrophe model is:

$$\delta z = z_2 - z_1 = b_1 z_1^3 + b_2 z_1^2 + b_3 F z_1 + b_4 L + b_5 \tag{3}$$

z is the normalized behavioral variable, while L and F are the normalized asymmetry (Logical thinking) and the bifurcation (Field dependence/independence) respectively.

The alternative and the most antagonistic linear model is the pre/post model:

$$z_2 = b_1 L + b_2 F + b_3 z_1 + b_4 \tag{4}$$

while two additional models :

$$z_2 = b_1 L + b_2 F + b_3 \tag{5}$$

and

$$\delta z = b_1 L + b_2 F + b_3 L F + b_4 \tag{6}$$

are also examined.

In order to support the nonlinear hypothesis, that a cusp catastrophe is appropriate model to describe students' achievements, the regression equation (3) should account for a larger percent of the variance in the dependent variable than the linear models. In addition, in order for the cusp to be the appropriate model for the data both the cubic and the product terms in equation (3) must have significant weights and the confidence intervals (95% CI) should not span the zero point. Table 1 shows the regression slopes, standard errors, *t*-tests, confidence intervals and model fit for cusp catastrophe model and the controls linear models.

Model	**Adj R^2**	**b**	**seb**	**t**	**95% CI**		**Model F**
Pre/Post	0.51						76.2****
z_1		0,372	0,057	6.49****	0,484	0,259	
L		0,379	0,060	6.21****	0,499	0,258	
F		-0,137	0,055	-2.42*	-0,028	-0,246	
Cusp	0.69						105.2****
z_1^3		0,077	0,020	3.82***	0,116	0,038	
z_1^2		-0,554	0,091	-6.09****	-0,376	-0,732	
F X z_1		-0,077	0,035	-2. 20*	-0,008	-0,146	
L		0,375	0,061	6.17****	0,495	0,255	
			Bootstrap estimates				
Cusp	0.67						107.1****
z_1^3		0,081	0,022	3.91***	0,124	0,038	
z_1^2		-0,568	0,096	-6.11****	-0,380	-0,756	
F X z_1		-0,078	0,036	-2. 10*	-0,007	-0,149	
L		0,383	0,065	6.15****	0,510	0,256	

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$, **** $p<0.0001$

Table 1. Regression Slopes, Standard Errors, *t*-tests, Confidence intervals and Model Fit for Cusp Catastrophe Model and the Controls Linear Models. Bootstrap estimates are included.

The pre/post linear control model based on equations (4), was able to predict a significant portion of the variance in the dependent variable [Adjusted R^2 = 0.51, F(3.201) = 76.2,

$p < 0.0001$]. The weights of predictors are significant and the 95% CI do not span the zero point, thus the model could be considered as significant, explaining 51% of the variance. The linear control models based on equations (5) and (6) were insignificant explaining a negligible portion of the variance in the dependent variable

The cusp model, based on equation (3) was able to predict a significant portion of the variance in the dependent variable, change in achievement, from Time 1 to Time 2 [Adjusted $R^2 = 0.69$, $F(4.201) = 105.2$, $p < 0.0001$]. Each of the terms in the cusp model significantly predicted change in the behavioral variable: the cubic term [$t(200) = 3.82$, $p < 0.001$], the quadratic term [$t(200) = -6.09$, $p < 0.0001$], the bifurcation parameter, field dependence [$t(200) = -2.20$, $p < 0.05$] and the asymmetry or normal parameter F [$t(200) = 6.17$, $p < 0.0001$].

In addition, bootstrap estimates of the cusp were performed. The results are shown in the lower part of Table 1 and they support the statistical significance of the cusp catastrophe model.

Model Interpretation and Epistemological Remarks

The cusp model in the present application reveals that both linear and nonlinear changes in behavioral variable might be expected and the pattern of these changes can be described by the two cognitive variables L and F.

At low values of F, changes are smooth and at high values of F they are discontinuous. At low values of L, changes occur over the lower mode and are relatively small. At high values of L, changes occur around the upper mode and are small too. At middle values of L, and depending on the value of F, changes can occur between modes and are relatively large. At the control surface we can observe the bifurcation set mapping in the unfolding of the surface in two dimensions (Figure 1). The cusp bifurcation set induces two diverging response gradients, which are joined at a point, the *cusp point*. At the cusp point the behaviour is ambiguous, while the two diverging gradients represent varying degree of probability that a student might succeed of fail (Figure 1).

From the above geometry of behaviour one can conclude that for certain mental resources and cognitive tasks, a point, the *bifurcation point,* there exists, beyond which the system enters the *bifurcation set,* the area where discontinuous changes occur (Figures 1). Any subject within the area of *inaccessibility* could be pulled towards either attractor (success or failure). The phenomenon of *hysteresis* is observed on the bimodality (Figure 1, lower part), that is, subjects with the same parameter values (L, F) oscillate between the two states/modes. This is a dynamic effect and indicates sensitivity of the parameters, that is, small differences in L and/or F, may lead to sudden shifts between the two modes representing success and failure.

Based on the initial research hypotheses, it can be concluded that: 1) There are bifurcations and hysteresis effects in students' achievement in challenging tasks, such as understanding and explaining state changes phenomena. 2) Students' achievement scores in this subject matter education can be understood in relation to two cognitive variables within a cusp catastrophe model, where logical thinking acts as asymmetry and field dependence/independence acts as bifurcation factor. 3) Execution of a cognitive task in learning science could be a nonlinear dynamical process. The third conclusion, that a nonlinear dynamical process is being investigated, is based on the first and second hypotheses, but also on the following epistemological discussion.

Statistical analysis supported the existence and the superiority of the cusp structure in the data comparing to the linear alternatives. Though, detecting a cusp model is more than a curve fitting procedure; it has serious epistemological implications. The cusp is a phenomenological model which describes rather than explains the behavior. Explanation of why that particular geometry of behavior is being followed cannot be given within a linear framework, but can be understood merely by considering nonlinear dynamics. The revealed bifurcations and hysteresis effects are the signatures of chaos and complexity; they imply a dynamical system, where abrupt changes are seen as transitions between two operating *attractors* as a result of an underlying *self-organization process* (Nicolis & Nicolis, 2007). This is just the point where advances in methodology have challenged fundamental philosophical assumptions and especially systems' ontology (Jörg, 2011). Bifurcations cannot occur in a linear and mechanical system. This comes to support also the initial argument that the linear methodology is incompatible with the system under investigation. The ontology that the present findings suggest is that of a *Complex Adaptive System* (CAS), where *self-organization mechanisms* and the dynamics of the system are the causal interpretation of the nonlinear phenomenology (Molenaar & Raijmakers, 2000; Stamovlasis, 2011).

A general faith, (it is written in any introductory text book of educational research methodology) is that ontological assumptions about reality determine the epistemological assumptions and finally the methodological choices. This traditional thought does not hold any longer; advances in methodology have changed the ontological views and induced a new epistemology. The above support the claim stated in previous sections, that further growth of the new paradigm needs empirical research evidences for the nonlinear hypothesis by means of research methodology compatible with the new epistemology.

Educational research can focus on various levels of complexity: Students at the individual level can be considered as dynamical systems and their development, academic and social behavior can be modeled accordingly (Lewis, 2000; van Geert, 2003). Classes or groups of interacting learners could be viewed as dynamical systems, where the nature and patterns of interaction could determine outcomes in unexpected ways (e.g. Stamovlasis, Dimos & Tsapalis, 2006). Studies could also be focused at a 'meso-scopic'- the school level, where schools as evolving systems could be traced by time series analysis demonstrating the dynamic of their functioning (e.g. Koopmans, 2011, 2013). Finally, viewing an educational system as a whole, its dynamics at the macro-level could be analyzed with NDS tools, predicting its efficiency and its deviations from the expected outcomes (e.g. Guevara, López, Posch, & Zúñiga, 2014).

Summarizing, the present research applied a catastrophe theory model in empirical data from students' conceptual understanding of physical phenomena. Nonlinearity in academic behavior explained by individual differences has certain implications for teaching and learning, related to anticipated performance and to adoption of suitable pedagogical approaches. The emphasis here, however, is on the philosophical - ontological and epistemological- implications with which the methodology of educational research is directly concerned. Hence, co-evolution of methodology, epistemology and ontology has been demonstrated and a contribution has been made to the emergent perspective of the complexity and *NDS*.

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