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Evaluating Number Sense in Workforce Students

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Abstract

Earlier institution-sponsored research revealed that about 20% of students in community college basic math and pre-algebra programs lacked a sense of part-whole relationships with whole numbers. Using the same tool with a group of 86 workforce students, about 75% placed five whole numbers on an empty number line in a way that indicated lack of part-whole thinking. This concept, needed to understand fraction and percent relationships, carries over as a grasp of the relationship between details and the main idea in factual prose, in critical thinking in job situations, and on the current high school equivalency tests.

Introduction

The College and Career Readiness Standards (CCRS) (Pimental, 2013) upon which the new high school equivalency tests are built ask adults to perform at a level of critical thinking that they may not have been introduced to in their earlier education. This kind of thinking means that a person is able to look at the details and see the big picture, or look at the big picture and pick out the relevant details for a given situation. This kind of thinking requires keeping track of the WHOLE (the main idea) and the PARTS (the details) at the same time, while considering the relationships between them. While part-whole critical thinking is important in all academic areas and in problem-solving on the job, it is especially central to math success.

Based on research in the 1980s at the University of Georgia led by Dr. Leslie Steffe (Steffe & Cobb, 1988; Steffe et al., 1983; Steffe et al., 1982), the importance of part-whole thinking in math in the elementary grades began to be understood. This research led to the introduction of math curriculums and interventions such as Math Recovery (Wright et al., 2006) and "Singapore Math" (Ginsburg et al., 2005) that were more conceptually-based rather than skill/drill-based.

In the Math Recovery program, the starting point has been the identification of children's stage of number sense understanding. Determining the stage of number sense generally meant conducting one-on-one interviews and was time consuming, as I found when conducting a teaching experiment with 2nd and 3rd grade children that used Steffe & Cobb's interview model for pre- and post-evaluation (Steinke, 2001).

Using this interview method with adults, I showed that many adults had not developed part-whole thinking about numbers. This deficit was found in small groups of community college students, adult basic education students, and even teacher candidates at a university (Steinke, 1999). Understanding how students think about number relationships could be valuable to the classroom teacher when preparing math lessons. However, the time required for interviews made this method impractical in the day-to-day teaching/learning situation of adult education classrooms.

When teaching a basic math course at a community college, I noted that how students performed on an empty number line task, that is, where they positioned the numbers, seemed to show the same 3 Stages of number sense that Steffe & Cobb had outlined. That discovery led to three college-sponsored research projects to identify number relationship understanding using an empty number line. The studies were conducted in basic math [N=179], pre-algebra [N=167], Algebra 1 [N=319], and "math for pre-service teachers" [N=51] classes. Based on the number line assessment, 19%, 18%, 28%, and 17% respectively of students in the studies appeared to lack the concept (Steinke, 2010/2011).

The study reported here sought to determine the percentage of workforce students who may be lacking this part-whole concept by using the same number line assessment. This knowledge could inform teacher practice by determining the proper starting point of basic math instruction with lower-level math students. Acquiring grounding math concepts (described in the next section of this article) would allow such students to be more successful in later work with fractions, percents, and algebra.

Methodology

In a small-group meeting (3 to 8 students) during the first few days of class, students at a workforce education center were given an empty number line (0 to 20) and asked to place five whole numbers on it (Figure 1). The directions were read aloud to the group, especially pointing out the endpoints of the empty line. If students asked questions (such as, can we put on other numbers), the test administrator gave a noncommittal answer such as, "It's up to you." The intent was to give no further directions beyond what was printed on the page of the assessment. This is the same method that was used in the community college studies. The assessment tool is shown in Figure 1. The length of the line is 9.125 inches (23 cm).

GED Math - Preliminary Assessment NAME

The line below starts at zero and ends at twenty.
All the numbers in the box at the right belong somewhere on the line.
Make a mark on the line where you think a number goes.
Write the number by the mark you make for it.

17
12
2
5
1

0 20

Figure 1. Assessment tool with given numbers 17, 12, 2, 5, 1

The assessment was looking for a lack of understanding of two math concepts that are not usually taught in adult education classrooms because of an apparent assumption that adults have grasped these concepts.

The first concept is that there is a "same-sized 1" between all the counting numbers (the concept necessary to understand addition) (see Figure 2).

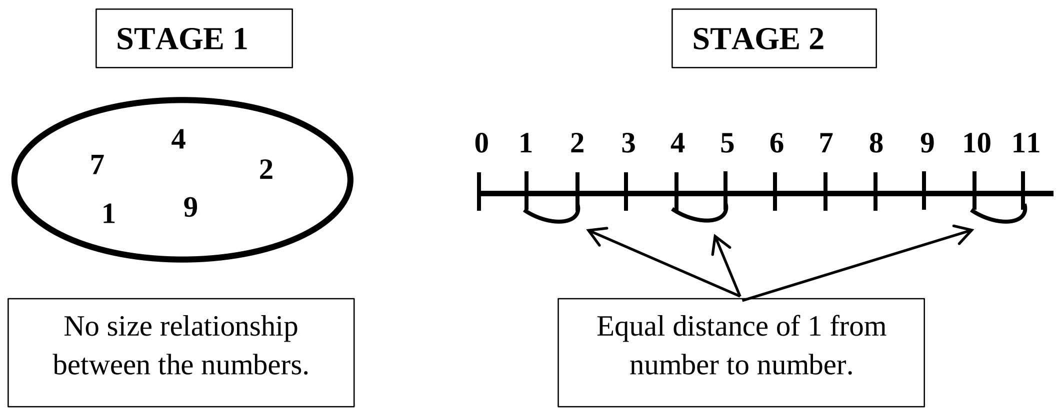

Figure 2. Stage 1 vs. Stage 2 sense of number relationships

People lacking the concept think of each number as a discrete quantity, separate from and unrelated to any other quantity. When counting items, such people count the items that they can see and only the items that they can see. In other words, the items must be physically present to exist for such people. These are the people, children and adults, who must count all the items, starting from 1, in order to add two groups of items, even if they have just counted the separate groups of items. People who understand numbers this way are said to be Stage 1 in their thinking about the physical relationships of the numbers, per the Steffe & Cobb model of number sense development in children.The second concept is part-whole coexistence. This is the understanding that a number exists as a whole and at the same time contains within it all the combinations of addends (the parts) that can be summed to create that whole. For example, 11 contains within it 4 + 7 or 3 + 3 + 3 + 2 and many other combinations while it continues to exist at the same time as the whole 11. The important point here is the understanding of the parts and whole existing at the same time as opposed to understanding that either the parts exist or the whole exists (see Figure 3). Students who have an "either – or" understanding of parts and whole (that is, who lack the "coexistence" concept) are said to be Stage 2 in their thinking per the Steffe & Cobb model.

People who think of numbers in a part-whole coexistence relationship are said to be Stage 3 in their number sense.

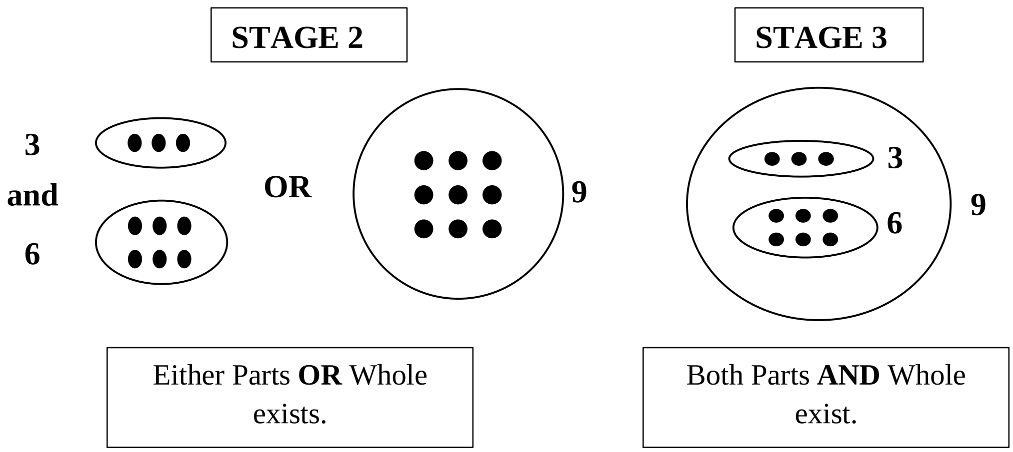

Figure 3. Stage 2 vs. Stage 3 sense of number relationships

Results

The results were analyzed by this researcher and reviewed with the teachers at the workforce instruction site. As with the co-reviewers in the community college projects, the workforce-site teachers had been introduced to the Steffe & Cobb 3 Stages model of number sense. They had the additional advantage of examples of number lines from the community college projects with which to compare the number lines (and stage of number sense) of their student. Stage 1 number lines show that people are not aware of the size relationship between the whole numbers. On the assessment, this shows up as neglecting items that are not physically present in front of them. People at Stage 1 put the same distance between 1, 2, 5, 12, and 17, spacing the given numbers evenly across the empty line. They ignore the numbers that are not given as though they did not exist. The order of the numerals is correct, but there is no sense of the size relationship of the numbers, as shown in Figure 4.

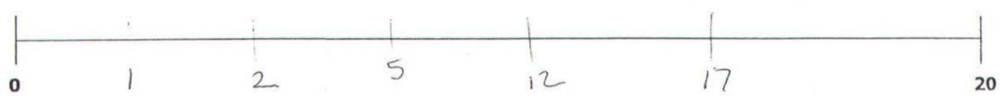

Figure 4. Stage 1 number line

In the current workforce study, 12 of 86 number lines (14%) were evaluated as being Stage 1.

On the number line assessment, students at Stage 2 (those who seem to lack the part-whole coexistence concept) proportion the distances between 1, 2, 5, 12, and 17 somewhat correctly. However, they do not take into account the length of the entire line when placing the numbers. This is in spite of the assessment administrator specifically pointing out that the empty line begins at zero and ends at 20. This results in two main types of errors: 1) an obvious leftward skewing of the entire set of numerals, often to the left of the center of the line (see Figure 5); or 2) a proportional spacing of the digits in-and-of-themselves that is too far to the left for 1, 2, and 5, and too far to the right for 12 and 17, leaving too great a gap between 5 and 12 (see Figure 6). In both cases, the size of "1" is internal and individual. Furthermore, since Steffe & Cobb identified giving a correct answer to an interview question on the first try as a hallmark of Stage 3 students (those with part-whole understanding), number lines with erasures and corrections are deemed Stage 2 (lacking the part-whole concept) (see Figure 7). There are other one-of-a-kind errors not shown in this article that could cause an assessment to be classified as Stage 2.

Figure 5. Stage 2 numbers on line skewed left

Figure 6. Stage 2 numbers on line with middle gap

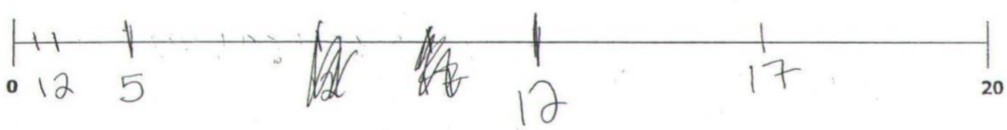

Figure 7. Stage 2 numbers on line with errors corrected

People at Stage 2 in their sense of number relationships recognize that each counting number is the same-sized “1” larger than the number before it. However, the increment of “1” that they use is not the size of “1” on the existing line. Stage 2 thinkers focus on either the size of parts (the size of their internal, individual “1”) or the size of the entire line, but not the spatial relationship of both at the same time.

In the workforce group, 52 of 86 assessments (60%) were evaluated as being Stage 2.

Many of the assessments in the study revealed a correct sense of number relationships on a number line (Stage 3), as shown in Figure 8. In some of these “correct” number lines, the 12 appears to be positioned slightly farther to the left than it should be. This is likely due to a documented effect that shows that humans judge the distance between two larger neighboring numbers to be less than the distance between two smaller neighboring numbers (DeHavia & Spelke, 2009; Longo & Lourenco, 2010; Mundy & Gilmore, 2009). That is, the distance between 12 and 13 somehow feels smaller than the distance between 2 and 3, even though both pairs of numbers are the same-sized “1” apart.

Figure 8. Stage 3 numbers on line

An additional reason for classifying a number line as Stage 3 in number sense is the small marks at the approximate locations of 5, 10, and/or 15 on the line. As shown in Figure 9, the person appears to have considered the size of the whole line first and then put the given numbers (the parts) on the line in relation to the whole.

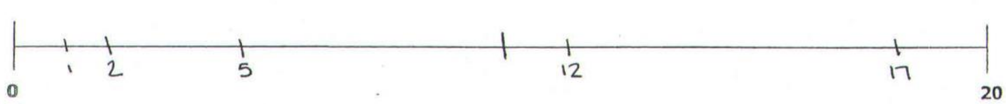

Figure 9. Stage 3 with location of 10 marked

In the current study, 19 of 86 assessments (26%) were evaluated as being Stage 3. In other words, 74% of these workforce students (Stage 1 and Stage 2 combined) were not thinking of number relationships in the manner that is required to understand fraction and percent relationships in basic math.

These evaluations involve a degree of subjectivity: When is a number line good enough be called Stage 3?

As a check on this subjectivity, a group of adult educators who were being introduced to the assessment tool and its evaluation parameters were asked to review all these workforce results. At least four people reviewed each assessment, though each person reviewed one-fourth of the total assessments. Tests were scored as 1, 2, or 3 (for the stages) or sometimes 2.5 if there was unresolvable uncertainty between Stage 2 and Stage 3.

The average of the adult educators' scoring put 32 of the 86 assessments at 2.5 or higher; that is, half or more of the scores were at 3. At this conservative scoring, 63% of the tests were judged NOT Stage 3, that is, lacking part-whole coexistence. When the cut-off for the average was raised to 2.75 (three of four reviewers scored the line as a 3), the adult educators had 20 of the 86 assessments at Stage 3. With this higher gateway, 77% of the total were judged NOT Stage 3.

With the adult educators' results so close to the original determinations made by me, it seems safe to say that, based on the number line assessment, two-thirds to three-quarters of the adult students in this sample lacked the concept of part-whole coexistence. My experience teaching basic math and pre-algebra to community college students tends to confirm that the students in that sample with 2.5 scores (not clearly Stage 3) lacked the part-whole concept. This observation would tilt the results toward affirming the higher percentage of not Stage 3 in the workforce group.

Summary/Discussion

Math is the area of the high-school-equivalency test in which adults have struggled the most over the years. These assessment results go a long way to explaining why. Without the correct understanding of the physical relationship of numbers, math becomes a skill in rote symbol manipulation.

These results beg the question: Why do these adults not have these concepts? After all, they are expected to be developed in the early elementary grades. In normal, middle-class children, this should happen by about age 8.

The answer may be two-fold: 1) these adults have the concept, using it in other areas of life such as driving a car, and do not realize they need to think of numbers as "coexisting" parts and whole; 2) these adults lack the concept in all areas of life, which shows up as weakness in life-planning and decision-making.

In the first category are adults who may have been the youngest in their elementary–school classes. These adults are victims of the "Matthew effect" (Bedard & Dhuey, 2006**;** Musch & Grondin, 2001). They have late spring or summer birthdays. They were expected to learn math concepts for which their brain had not yet developed the connections. Children's brains develop the ability to keep two things in mind at the same time (required for part-whole thinking in subtraction) after age 7. In other words, if you have to learn subtraction in second grade and you turn 8 in the summer after second grade, you may not have the brain development (specifically, the proper connections between the forebrain and anterior cingulate(Houdé et al., 2011; Rueda et al., 2004)) to understand how subtraction works. You fall behind in math and, all too often, never catch up nor feel good about math.

In the second category are adults who should have normal brain development but who have been affected by what is being called "toxic stress" in childhood. This stress can be caused by poverty (lack of a safe neighborhood; food and shelter concerns) or emotional or physical abuse. There are many recent publications on childhood stress and its long-term effects (Child Welfare Information Gateway, 2015). Not only are children unable to learn at the point-in-time of the stress, but their brains and emotional systems can remain affected by the stress into adulthood. One brain-imaging study (Hanson et al., 2012) found that older teens who were raised in living conditions of constant stress had fewer connections between the forebrain and the amygdala than did youth of the same age who had been raised in more normal circumstances. This is the exact connection that is required to keep more than one thing in mind at the same time, in other words, to understand part-whole coexistence.

A third group – adults with developmental disabilities or traumatic brain injury – may also lack these two concepts. Whether such individuals have a capacity to grasp the concepts may be a question for study by other researchers.

In sum, adult math textbooks from major publishers assume adults have these two concepts: 1) the "equal distance" of 1 between neighboring whole numbers; and 2) part-whole coexistence. Those textbooks seem unaware that these concepts need to be taught, or at least reviewed, in adult education classes. Lack of the first concept is the reason students plot coordinate grid points incorrectly (always off by 1) or read line graphs and bar graphs incorrectly (misreading the scale). Lack of the second concept is why so many students struggle with fractions. Fractions are the essence of the part-whole relationship: the PART of the whole amount that I care about compared to (in relationship to) the WHOLE amount.

Assessing for the concepts allows teachers to tell students one of two things. For those without the concept: "I know what you need." For those with the concept: "You are OK in your thinking; let's work on skills."

After all, knowing what the student needs to learn next is the essence of good teaching.

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