The Assessment of Math Learning Difficulties in a Primary Grade-4 Child with High Support Needs: Mixed Methods Approach

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Received: June 2011 / Revised: January 2012 / Accepted: January 2012

Abstract
This mixed-methods study incorporated elements of survey, case study and action research approaches in investigating an at-risk child. Using an in-take interview, a diagnostic test, an error analysis, and a think-aloud clinical interview, the study identified the child’s major presenting difficulties. These included: inability to use the four arithmetic operations (addition, subtraction, multiplication, division) efficiently; not understanding the relationship between units, tens and hundreds; using any two of the four arithmetic processes (+, −, ×, ÷) in combination within one operation; treating each column as a separate problem; place value problems / wrong alignment of numbers; poor eye-hand coordination leading to dysgraphia; and memory lapses. The other problems that became apparent through this investigation and implied in the findings include possible causal factors such as dyscalculia, dyslexia, low self-esteem, low self-efficacy, and math anxiety. Further assessment, intervention and research are recommended to address problems of this vulnerable child.

Keywords: Mixed Methods Research; Survey; Case Study; Action Research; Authentic Assessments; Dyscalculia; Dyslexia; Dysgraphia

Introduction
Students’ academic and personal problems in institutions of learning can be identified and resolved in a number of ways that are familiar to educational psychologists, special educators, school counselors, and educational researchers. Normally, students’ problems tend to be numerous, multifaceted and complex in nature and require an interdisciplinary approach to understand them adequately. This then calls for a variety of procedures to be employed when investigating and addressing students’ problems in schools.

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ISSN:1307-9298
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Mixed method research designs

These are studies that combine quantitative and qualitative research paradigms in an attempt to compare or contrast the findings and understand the presenting problem(s) more fully. A researcher may, for example, investigate the same problem in a two-stepwise fashion or strategy starting as an exploratory quantitative survey and ending as an in-depth qualitative case study. Mixed methods research designs also use a mixture of data collection approaches (e.g. tests, questionnaires, observations, interviews, documents, and projectives) and adopt a wide range of data analysis techniques (both quantitative and qualitative). Investigators who use mixed method research designs also often report and interpret data and findings in different ways. In all these strategies, the data and findings are triangulated to confirm their validity. Mixed method research designs have several advantages and disadvantages but only three examples of each of these will be given here. The three main advantages of the strategy are that it: (1) incorporates the strengths of both qualitative and quantitative approaches; (2) provides a more comprehensive view of the problem or phenomena being studied; and (3) does not limit the data being collected. The major disadvantages or limitations are that it: (1) requires high-level expertise in both quantitative and qualitative methods to use it competently; (2) needs extensive data collection and resources; and (3) is prone to being used superficially such as claiming to have used several methods when in actual fact and reality only one was used. There are three main specific designs that are associated with the mixed method research approach and these are: (1) explanatory design - occurs when quantitative data are collected first followed by qualitative data collection; (2) exploratory design - whereby qualitative data are gathered first with quantitative data collection following later; and (3) the triangulation design – in which quantitative and qualitative data are collected simultaneously to provide a more comprehensive and complete set of data. As is the case with other research methods, the investigator’s decision and choice to use a mixed methods approach is often arrived at after a long and careful thought based on the consideration of a number of important factors such as the type/nature of problem to be researched, specific research questions or hypotheses to be probed, the feasibility of the research strategy, rationale or justification for using the method, and expertise in using appropriate data collection, analysis and interpretation techniques. It is pointless and redundant for eclectic investigators to use a research method when it is not warranted.

Authentic assessments for learning

In the past, student academic evaluations focused mainly on the assessment of learning (the quantity of knowledge and skills a student obtains as a result of attending school and receiving instruction from teachers – i.e. quantifying what one gets from undergoing a course of instruction). This quantity was usually reflected in test/examination scores and grades as manifested on the school report or transcript. Emphasis was placed on the so-called summative norm-referenced assessments that were used to mark the end of an educational cycle / level as well as rank and compare students for various purposes such as offering them admission, scholarship or employment. By doing so, examinations dominated the scene in schools and educational systems became examination-oriented (Mundia, 2010). Both teachers and students became obsessed with coaching and preparation for examinations respectively. In this way, examinations undermined good teaching which emphasizes understanding.

On the contrary, authentic assessments for learning stress that student evaluations should help learners to understand and master the knowledge and skills that they receive through teaching. These evaluations include both formal criterion-referenced assessments as well as
the informal formative assessments such as observations, experiments, interviews, portfolios, lesson studies, and assessments by the self, peers, and parents. There are no norms derived from the informal authentic assessments and the results cannot be used for comparison purposes. Despite this, these informal authentic assessments are seen or considered to be the key to meaningful learning based on understanding. They enable teachers and parents to identify the conditions and circumstances under which a student can maximize her/his potential to learn. Emphasis here is on assessing the learner holistically/globally for both academic and personal problems. The assessment results form the basis for improved teaching and learning. Though still relatively new and unknown, authentic assessments are already becoming well known in some developing countries (see Engelbrecht et al., 1999).

Sources of problems in learning mathematics

Many students at all levels of education in developing countries have problems in learning mathematics. The causes of these difficulties are many and wide ranging. Five of the numerous broad factors appear to be outstanding. First, some students seem to be negatively influenced by the stereotype beliefs held by many people that mathematics is a difficult subject (Heward, 1996). Second, for a number of learners their problems appear to stem from unsatisfactory teaching and the resultant lack of experience of success (Mundia, 1996; 1998). Third, still for other students their difficulties seem to be linked to the procedures used in evaluating mathematics learners (Somerset, 1987; Murray, 1996). Fourth, there are also students who unfortunately may have a genuine specific learning disability in mathematics (Thornton et al., 1983; Hall, 1994; Mercer, 1997; Bos & Vaughn, 2002). Fifth, poor performance in mathematics might also be attributed to inadequate funding of education which results in fewer teaching/learning resources and low quality of education (Kelly, 1986; 1991). The child described in the present triadic study (nick-named B) required the joint efforts of an educational psychologist / school counselor, one of the child’s parents (referrer) and a special educator, to solve. Dettmer, Thurston and Dyck (2002) discuss the viability and benefits of collaborative intervention strategy.

Development and persistence of math anxiety and phobia

Students who do not perform well in mathematics often develop math anxiety and phobia. Math anxiety and phobia in the context of the present study refer to the unreasonable worries about and fear of mathematics. This condition can be severe and persistent if not treated effectively through either educational interventions (e.g. provision of remedial instruction, learning support, and individualized educational plans), or via counseling. There are several counseling / therapeutic techniques that are used in treating anxiety and phobia. They include rational emotive therapy (RET), implosive therapy, systematic desensitization, operant conditioning, modeling, cognitive restructuring, and behavior therapy. Fogiel (1989) and Thompson (2003) discuss most of these procedures in detail.

Objectives of the study

The purpose of the present study was to identify the degree and nature of problems in math for a particular Year 4 (Grade 4) child. The study also sought to find and recommend ways the child’s math problems could be investigated further to gain additional insights. In short, the study sought to answer the following three research questions: (1) what exactly were the child’s problems in math?; (2) why were the problems recurring and persisting?; and (3) how could the problems be resolved and avoided in the future?
Research methods
The study used a mixed-methods design that incorporated the survey, case study, and action research elements. Initially, the study began as a field survey which was in form of a diagnostic math test administered to all the 29 students in a class that included a child referred to the researcher by parents for mathematics problems. The purpose of the survey phase was to determine the extent to which math problems were unique to the referred child and common to other students in his class. Upon confirmation of the problems, the referred child (pseudo-named B) was then included in the case study for further observation, interview assessments and analysis. The justification for adopting the case study procedure for the rest of the investigation was two-fold. First, the researcher wanted to make an in-depth functional analysis of the child with regard to mathematics. Second, only one child was referred to the psychologist by his parents for assessment. Throughout the present study, the investigation was done collaboratively in form of action research and as a triad involving the researcher, the class teacher, and one of the parents of the child pseudo-named B. This parent was the referrer. The rationale for incorporating an action research component was that the causes or sources of learning problems in math for a child can be many and multifaceted. Such numerous different factors might be best captured and understood from an interactionist view-point (Martin, 2010). A broad perspective on a student’s problems in math may include the child’s own weaknesses or difficulties, the unsatisfactory and harmful teaching at school, an inappropriate and disturbing home environment, and the overall quality of support given to the child in math at both school and home. Of course the causal factors are many more than indicated here and might include issues of teaching/learning resources, the curriculum, and assessment practices.

Data collection instruments
The data for this case study were collected through observations, school assessment reports and documents, an in-take interview with one of the parents, a researcher constructed 16-item diagnostic test covering contents (addition, subtraction, multiplication, division) already taught in the academic year which was administered to the whole class as a revision exercise; an error analysis of the child’s mistakes in mathematics from the diagnostic class wide test; and a think-aloud diagnostic interview based on the error analyses. The math test had an alpha reliability of .76 and high content validity. In addition, the test also had good ecological validity in that it was administered by the child’s math teacher as part of normal class activity and action research during a mathematics lesson. All the informal observations and interviews (in-take and diagnostic) were done inside the child’s usual classroom, a non-threatening habitat that the child was accustomed with. This helped the child not to behave or respond in defense and cautious manners. The in-take interview with one of the parents provided the child’s background information while the error analyses and think-aloud diagnostic interview provided valuable insights into the child’s strengths and weaknesses in mathematics.

Initial Sample and Case study participant
The survey component of the present study was based on an initial purposeful sample of 29 Grade 4 (Primary Year 4) children of whom 17 (59%) were females. The children ranged in age from 8 years 9 months to 10 years 3 months with a mean age of 9 years and 6 months. The t-test statistic for independent groups indicated no significant difference in age between the two genders (p > .05). The case study level of the investigation had only one male participant who was referred to the researcher (educational psychologist / school counselor) by one of his biological parents for problems in learning mathematics. The biographical information of this student reported in this section of the study was obtained from both the
school records and the in-take interview with one of his parents. At the time of collecting the
bio-data, the boy was aged 9 years and 7 months. He was the 5th and last born in an intact
nucleus family with five children (three boys and two girls). During the in-take interview, his
parent asserted that the family’s home environment was peaceful and not economically
disadvantaged.

Data analysis

Survey data were analyzed quantitatively while observational and interview data were
analyzed qualitatively. Qualitative data included error analyses.

Procedures

From the beginning to the end of this investigation, the researcher (psychologist and
counselor) was conscientious and mindful of the ethical and legal implications surrounding
the use or involvement of children (minors) in psychological research, assessments, and
therapy. In view of these sensitivities, the researcher consulted, for the sake of clarity, the
relevant legal experts and ethical codes of conduct for the psychological society and the
counseling association of which he was a member (regarding the contentious issues of
voluntary participation, privacy, anonymity, confidentiality, physical and psychological harm,
debriefing, and informed consent). No deception was used or involved in this study. As
indicated above, the child described in the present study was referred to the researcher
(psychologist / counselor) by his own parents. The parents of the child in the survey and case
study therefore consented to the child to be assessed. Legally and ethically, minors (children)
cannot, on their own, consent or volunteer to be in research or undergo assessment or be in
therapy without the agreement and permission of the parents or significant others. Prior to
carrying out the study, permission to conduct the survey and case study around the school
and classroom premises was also obtained from both the school authorities and the class
teacher as “loco parentis”. The parents, school authorities, and class teacher also allowed the
researcher to publish the results of the assessment in the present study as these might be of
value and useful to both the local and international communities of researchers, parents, and
teachers that are interested in improving young children’s learning and understanding of
mathematics. However, the parents, school authorities, and class teacher did not permit the
researcher to reveal any identifying information. Due to these ethical and legal constraints,
the researcher has kept anonymous and confidential the names, ethnicity and nationality of
the child in the present case study throughout the article. Instead, the child is given a
pseudonym B as his name in this study. The researcher and author had worked or taught in
five different countries (all of which cannot also be named) and the specific country from
which the data for the present study were obtained is kept anonymous and confidential.
Only general educational implications of the findings deemed by the researcher to be of
interest or value to the local and international community are discussed in the article.
Similarly and at their request, the names and affiliation addresses of the co-researchers
(cooperating teacher and collaborative parent) are also not revealed as part of the efforts
and process to conceal the identity of the child. The study met the ethical requirements for
using human participants (including children) in research stipulated by countries /

Results

Presented below are the results of performance for the child in the case study on addition,
subtraction, multiplication, and division problems. The findings from the error analysis and
the think-aloud clinical interview are also presented in this section. In addition to presenting
all the results, this part of the article will also attempt to address the first and second research
questions of the study, namely: (1) what exactly were the child’s problems in math?; and (2)
why were the problems recurring and persisting? Plausible explanations to answer these questions are offered.

School records, in-take interview and informal observations

Both the teacher’s observations that the child was struggling in math and the school assessment reports that indicated the child was two standard deviations below the mean concurred with the parents’ own observations. This repeated failure and lack of experience of success may somewhat have de-motivated the child.

During the in-take interview, the parent reported that the child’s birth, development, and health progressed normally throughout his childhood. The school records and parental input in the interview concurred that he attended school regularly with only a small and insignificant number of acceptable absences each year. Despite the good attendance recorded in the registers, school academic reports and observations (of both the class teacher and parents) suggested that the child had high support needs in mathematics. His poor performance could, however, not be attributed to negative impacts from the home environment or school atmosphere, both of which were considered to be conducive and supportive contexts (according to the in-take interview with the parent and the teacher).

Performance on math problems for the whole class (Appendix 1)

The performance of all the 29 students on math problems is presented in the table in Appendix 1. The items on this diagnostic test were arranged in an increasing order of difficulty starting with easy questions and ending with the most challenging items (with moderate or medium difficult items in the middle of the range). The four math topics covered were addition (Items 1–4), subtraction (5–8), multiplication (9–12), and division (13–16). The learner in the present case study is identified or labeled as student “A” in this table (Appendix 1). His total score (4) was the lowest but there were two other students (K and B) who trailed him and scored 5 and 6, respectively. These other two weak children were not included in the case study as they were not referred to the researcher (psychologist and counselor) by their parents. Both the weakest and strongest students in mathematics can be identified visually from this table. Apart from student A, who was referred for further assessment, none of the other two weak students was in therapy. Under the current and ongoing ethical and legal rules, children (minors) cannot be in therapy without the explicit consent of their biological parents or significant others. There was therefore no other way additional information could be obtained on these other weak children. Although the presenting problems for these other less able students are not known, it is quite possible they might have similar difficulties as the student in the present case study. In addition, it was also observed and noted that the school and class did not have any special needs support scheme.

Test and item statistics and alpha reliability analysis (Appendix 2)

Item 6 in the table in Appendix 2 was deleted (omitted or excluded) from analysis because it was too easy and every child got it correct (see table in Appendix 1). The facility value, standard deviation, and discrimination index for this item were all 1.00 (easiest item). Theoretically, the range of the total scores on this diagnostic test was 0–16 but the actual obtained score spread was 4–15. Student B’s total score of 4 (25%) was 6 points below the class average of 10.069 (SD = 2.776). B’s score was therefore more than 2 standard deviations below the mean (Z = -2.186). Given that the test as a whole had good alpha reliability, good content validity, good ecological validity, and was not biased in favor of either gender, B’s poor performance could therefore not be accounted by (or attributed to) all these factors. A close scrutiny and analysis of his performance provided additional insights. His sub-scores by
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Topic were: addition (2 out of 4); subtraction (1); multiplication (0); and division (1). Of the four questions he got correct answers (Items 1, 3, 6, 13), Item 6 was the easiest question (p = 100%). In the table in Appendix 2, item mean scores are synonymous with item difficulty indices (facility values or p-values) because the data were dichotomous/binary (see Appendix 1). From the table in Appendix 2 it can be noted and observed that Items 1 and 3 were the second easiest questions with similar p-values (Facility = .93). Furthermore, Items 1 and 6 are too elementary and perhaps suitable for Grades 1-2 (Year 1-2) students. Moreover, even the only relatively challenging item that he got right (Item 13), is also more appropriate for Grade 3 (Year 3) students. Overall, the test diagnosed B as a very weak student in mathematics. His knowledge and skills in math were very shaky and fragile. In four years of primary schooling, this student only mastered the addition and subtraction of single digit numbers (units) that do not involve or include zeros. This evidence suggests that the child might be about two Grades (two academic Years) behind in mathematics as he seems to be operating at the level of a Grade 1 or Grade 2 pupil. He is weakest in division but this may be due to the fact that this was the last topic the class learnt before the diagnostic test was administered. He might, however, benefit from further assessment, early identification of causal factors, and early intervention, both educational and psychological (to build a strong and firm foundation).

**Error analysis of math problems for the case study participant (Appendix 3)**

The observations presented below are based on the results of the diagnostic test displayed in the table in Appendix 3, the error analyses of these results, and the subsequent think-aloud diagnostic interview with the child that allowed him to verbalize his strengths and weaknesses. Although the psychologist encouraged the child in a variety of ways (e.g. through probes) to talk during the diagnostic and think-aloud interview, the student said little other than uttering short sentences such as “I added”; “I subtracted”; “I multiplied”; and “I divided”. His few verbalizations during the diagnostic interview demonstrated that he was perhaps deficient in language facility and possibly lacked both understanding and meaningful learning in mathematics. Despite this problem, the error analysis and think-aloud clinical interview generated significant and valuable insights (hints and clues) into the math problems of this child from observing and analyzing the way he practically solved math questions on the diagnostic test. These problems might be attributed partly to the child and superficial teaching. Listed and briefly discussed below are the main results from the error analysis and think-aloud diagnostic/clinical interview.

**Addition errors (Items 2 and 4)**

**Item 2:**
- Treats each column as a separate problem (Thornton et al., 1983)
- Fails to regroup when adding (Thornton et al., 1983)
- Number fact error- fails to master and do simple addition (Rivera & Bryant, 1992)

**Item 4:**
- Handles each column as a separate problem (Thornton et al., 1983)
- Inability to regroup when adding
- Place value problems e.g. aligns numbers incorrectly (Trafton, 1987)
- After probing he adds left-right (tens and hundreds) across the row e.g. 3+8= 11
- Cannot add large and small numbers
- Fails to carry simple addition manipulations
- Incomplete work

Subtraction errors (Items 5, 7, 8)

Item 5:
- Poor visual functioning - dysgraphia e.g. cannot write numerals such as 9 correctly
- Memory impairment e.g. he forgot that he borrowed 1 from 6
- Commits number fact errors e.g. has not mastered simple subtraction (Rivera & Bryant, 1992)

Item 7:
- Inability to regroup when subtracting a large number from a small number; he then rearranges the problem e.g.
  \[ 0 - 6 = \_\_\_\_\_ \text{ becomes } 6 - 0 = 6 \text{ (in Q7)} \]
- Zero difficulties e.g. 0 - X = X in the above statement
- Subtracts the smaller from the larger digit in each column (Thornton et al., 1983)
- Takes each column as a separate entity/problem
- Disturbance in quantitative thinking e.g. is unable to understand the relationship between units, tens, and hundreds
- Number fact errors e.g. is incapable of doing subtraction involving large numbers (numbers with two or three digits)

Item 8:
- Zero difficulties e.g. subtracts the smaller from the larger digit in each column (Thornton et al., 1983)
- Handles each column as a separate problem
- Aligns numbers incorrectly (place value problem)
- Finds it difficult to subtract a two digit number from a three digit number.
- Relationship between units; tens, and hundreds is not clear

Multiplication errors (Items 9, 10, 11, 12)
- Confusing multiplication with addition e.g. in item 9 he first multiplied 2 x 7 and got 14. He wrote 4 and carried 1 which he then added to the multiplier 2 to get 3 thereby obtaining 34 as the answer (item 9)
- Poor visual- motor functioning – dysgraphia e.g. inability to write 4 properly (Item 9)
- Poor visual – motor functioning – dysgraphia e.g. aligning numbers incorrectly (Items 9, 10, 12)
- Zero difficulties e.g. P x 0 = P (Item 11)
- Number fact errors e.g. lacks mastery of basic multiplication skills
- Confusing addition with multiplication e.g. he treated 4 like the number 1. He then added 8 + 1 = 9. Finally he dropped down to get 49 as the answer (Item 12).
- Lack of information errors
Division problems (Items 13, 14, 15, 16)
- Wrong placement of the quotient digits (Items 13-16)
- Zero difficulties e.g. \(0 \div P = P\) (Item 14)
- Failure to recognize when the quotient is larger than both the numerator and denominator (Item 16)
- Know this division sign/symbol \(\div\) (Items 13-14) but does not know the other division sign/symbol in the last two items (Items 15-16)
- Number fact errors e.g. failure to comprehend simple division problems (Items 14-16)
- Confusing division with either addition or multiplication (Item 16)
- Poor visual – motor functioning- dysgraphia e.g. unable to write 4 correctly (Item 13)
- Inability to change and state a simple division problem in words (Items 14-16)

Persistent and recurring errors (Across the four processes)
- Number fact errors e.g. failing to do simple addition, subtraction, multiplication, and division.
- Using or mixing or confusing any two of the four processes \((+, -, \times, \div)\) within one operation
- Bugs - the following systematic procedural errors occurred repeatedly and consistently:
  - Wrong alignment of numbers
  - Hand-eye coordination
  - Treating each column as a separate problem
  - Zero difficulties
  - Place value problems
  - Misunderstanding the relationship between units, tests and hundreds
  - Memory lapses; short-term memory
  - Dysgraphia (observed from the writing of responses/solutions to math problems)
- Potential problems that from observations seem to perpetuate the child’s difficulties in mathematics and that may need further assessment:
  - Dyscalculia (not measured in the present study but implicated as a possible cause)
  - Dyslexia (not assessed in the present study but suspected to be a likely causal factor)
  - Mild to moderate learning disability (LD) in math (not evaluated in the present study but amply observable and evident from the child’s interactions with the researcher during the diagnostic / clinical interview)
  - Low self-esteem in mathematics due to the negative effect of repeated failure
  - Low self-efficacy in mathematics because of lacking experience of success
• Math anxiety / phobia (no clues or hints on these from the present study but worth probing and preventing)

Discussion

Instead of discussing the results of the investigation presented above already with detailed explanations, this section of the article will address the third research question of the study, namely: how could the problem be resolved and avoided in the future? At this stage of the development of math problems in student B, primary prevention of the difficulties encountered would not be feasible because the problems have already occurred. More efforts should therefore be directed at the long-term (rather than short-term) secondary or perhaps tertiary preventative strategies. Failure to resolve the problems satisfactorily at this stage would perpetuate the difficulties and disable the child from mastering BODMAS (brackets, operations, division, multiplication, addition, and subtraction) or PEDMAS (parenthesis, equations, division, multiplication, addition, and subtraction) later at the upper primary and lower secondary school levels, respectively.

There are four broad ways the math difficulties of the student in the present case study may be resolved and in which problems of this nature might be avoided in the future. Based on the outcomes of the present study, the following long-term intervention strategies might be beneficial: (a) use of informal authentic assessments to evaluate math skills; (b) teaching interventions to address math anxiety and phobia; (c) administrative decisions to adjust math assessments and make them friendly; and (d) improving teacher education through initial and continuous training programs. Each of these is, in turn, explained below. The suggestions discussed in this section of the article (regarding authentic assessments, teaching interventions, administrative supports, and teacher education) are intended to assist a client / learner like student B to minimize his difficulties during math lessons and tests. They are to be implemented by regular teachers, the school administrators, and teacher educators respectively.

Use of informal authentic assessments to evaluate math skills

Experimental assessments

Students (young and old and at all levels of education) with special needs in mathematics and other subjects might benefit from the use of experimental assessments. Experimental assessments are informal assessments conducted by teachers, educational psychologists, school counsellors and parents to discover the effective learning conditions under which a child with difficulties in learning math works best. For example, a series of parallel tasks could be devised for a student to perform: (1) alone at home; (2) alone in class; (3) in a group in class; (4) in a group outside class; and (5) other structured conditions. Careful observations of the student will indicate what a learner can and will do in various test situations. This type of assessment will not tell us anything about how a student performs in relation to others in class but will be insightful in suggesting how she or he can be assisted. At present, experimental assessments are not used in developing countries' schools and homes and therefore unfamiliar to teachers and parents. Teachers and parents in these countries may need in-service training workshops to equip them with the necessary skills for using experimental assessments.

Observational assessments

For students with mild and moderate difficulties in learning math, alternative methods of assessment are essential. One way to do this is to observe the pupil's everyday activities (cognitive, psychomotor, and affective) and use some kind of a developmental scale as a
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standard by which to reach a measure of a pupil’s achievement. The kind of activities observed can then be related to the age that the nondisabled children achieve each one and an approximate developmental level determined. Observations of this nature can also be used in the assessment of the student’s everyday behaviour. To reduce cautious or defensive reactions and improve the quality of the data obtained, both informal and formal observational schedules should be well constructed and administered by trained adults with whom children are familiar in the usual home or school environment (Van Tassel-Baska, Quek, & Feng, 2007). With the implementation of many educational reforms in developing countries regarding the curriculum, teaching strategies, examinations, inclusive education, and gifted education, the use of observations will be helpful to teachers and parents in identifying how regular and exceptional students are functioning in the reformed school system. However, both regular teachers and parents may need in-service training workshops to acquire observational skills. Special needs teachers, educational psychologists and school counsellors may serve as facilitators in these workshops.

Learning assessments

A learning assessment could be designed and implemented to inform the teacher what a child is capable of learning. In an ordinary achievement test, a particular task is given only once (occasionally two times) and the examinee is recorded as passing or failing. In a learning assessment, the pupil is first taught something in the assessment situation. She or he is then given the assessment task as many times as she needs to succeed or until the instructor gets a useful measure of the rate at which she/he learns. There are no norms for this kind of lesson study assessment and we cannot say how the child’s learning compares with her/his age group or with that of a large group of non-disabled peers. However, three benefits are derived from a learning assessment of this nature. First, we would know more about the learning pace of the pupil. Second, we would get to know the type of reinforcements that are effective to the child. Third, the learner would eventually experience success, a desired motivating event. Teachers (both pre-service and in-service) in most developing countries would require additional training to be able to use learning assessments effectively. Current serving teachers could obtain this training through in-service workshops conducted by special education teachers, school counsellors, and educational psychologists.

Parental assessments

Children are raised in two main environments: home and school. At home children are always and constantly under the critical eyes of the parents. Some of the educated parents help their children with school work regularly or many times. Many of these literate parents even make detailed developmental notes in diaries or journals regarding their children. It would therefore be reasonable to expect parents to be partners in the assessment of school children. With reasonably high literacy rates in many developing countries these days, most parents may be able to assess their school children at least at the primary school level. At the secondary school level, many well educated parents would be able to assess their school children if they were given appropriate training in diagnostic observations, clinical interviews, and educational measurement to enhance and improve their assessment skills. When necessary, parents could be part of an assessment team (dyadic or triadic) with special education teachers, school counsellors, educational psychologists, and other specialists such as speech therapists. Parental assessment is especially critical when dealing with students with high support needs in challenging behaviours (e.g. defiant, oppositional, violent, school refusal, learning difficulties, autism, dyslexia, dyscalculia, emotional and behavioural disorders or EBD, and attention deficit hyperactive disorder/ADHD). At the moment it is
absurd to note that parents in most developing counties are only often told, rather than asked to assist in assessing their children's school work (Engelbrecht et al., 1999). More parental involvement in assessment is needed as advocated by many parent-teacher associations.

**Peer assessments**

Learners can help teach each other in studying any subject and this is called peer tutoring. Similarly, learners can also help to assess each other to some extent, a system known as peer assessment. Many practising teachers have for example seen students form informal study groups, discussing taught topics and asking each other spelling, arithmetic, science and social studies questions. In answering academic questions from peers, students express themselves freely without fear or intimidation and without anxiety, tension and stress. Peer assessment has potential to help students to master concepts and skills learned in various lessons. To be more useful and valuable, students involved in peer assessment need to be trained in social interaction skills, critical observation, and constructive appraisal. This is where special education teachers, school counsellors, and educational psychologists can be used as resource persons to train students in the effective use of peer assessment. Peer assessment may be challenging and difficult to use with young children in lower primary school or preschool level who might not be able to master its required skills. In addition, this form of assessment works better when a student has a buddy and if students work in small groups. When properly used, however, peer assessment can be a valuable learning tool. Peer assessment does not replace teacher assessment but rather enriches the process of student evaluation (Engelbrecht et al., 1999).

**Self-assessments**

In this mode of assessment a learner tests himself or herself and then reflects on his/her own capabilities (strengths and weaknesses). Self-assessment and self-reflection are key ingredients in the process of active learning. By doing this repeatedly, students learn to take control and responsibility for their own learning and eventually gain self-discipline and self-direction. Self-assessment helps students to master the learned contents and skills in several ways. Through repeated self-testing, a student eventually experiences success which raises her/his intrinsic motivation. Under self-assessment, a student competes against himself/herself and self-competition has fewer negative effects compared to competition with others. In self-competition there is no damaging effect of anxiety, tension, and stress. The success achieved through self-assessment and self-evaluation helps to boost a student’s self-confidence and self-esteem. In view of all these advantages, self-evaluation is a valuable process for all learners (disabled, gifted, nondisabled, young, and old). It ought to be taught to and practised by students. At present, students in most developing countries may not know how to use self-assessment effectively. There is therefore a felt need to mount awareness or sensitization programs in schools to alert students about the potential benefits of using this study strategy to improve achievement particularly in challenging subjects such as mathematics. Students who develop and maintain a habit of self-evaluation often perform better in school work (Engelbrecht et al., 1999). Like peer assessment, the application of self-assessment among learners in lower primary and preschool levels may prove difficult because of the young children’s inability to master its salient skills.

**Portfolio assessments**

A portfolio could be a file or container holding work examples or samples of tasks performed by a student (Engelbrecht et al., 1999). The examples of work samples could include pieces of writing, drawing, crafts, diary/journal entries, or anything that the student and teacher think is of value in assessing a learner. The pieces of work can then be evaluated by teachers,
parents, members of the parents-teachers association (PTA) or governing body as well as special support teams such as special educators, school counsellors, and educational psychologists. It is important to ensure that different pieces of work examples or samples are included in the portfolio. Diagnostic observations of the work samples, clinical interviews of the student on the work samples, and counselling discussions based on the portfolio contents can reveal quite a lot about a learner’s weaknesses and strengths and provide insights about the best possible and available interventions. Portfolio assessment is important and valuable to all learners but more so for those with special needs and the gifted students. Teachers and students need training in assembling and critically evaluating a portfolio. Portfolio assessment is a relatively new form of evaluating students in developing countries. This kind of assessment has been proposed as an alternative to standardized and norm-referenced tests (see Taylor, 2000; Venn, 2000; Salvia & Ysseldyke, 2004). Recent studies show that portfolio assessments and other informal evaluations would be good non-traditional ways of accommodating the needs of students with disabilities as well as a simpler procedure for identifying gifted / talented students instead of using complicated traditional strategies such as tests of intelligence which may have linguistic and cultural bias (Lohman & Lakin, 2008; Olszewski-Kubilius & Kulieke, 2008). Portfolio assessment may also be used in conjunction with problem-based assessment (PBA).

**Teaching interventions to address math anxiety and phobia**

1. An error analysis of the client’s performance in mathematics tasks needs to be made to determine where problems occur and the nature or extent of the problems. Both the weaknesses and strengths can be used in the intervention program.

2. Based on the error analysis, a diagnostic interview should be conducted to determine how and why the client makes these errors in mathematics. The findings can be used to improve strategies to teach the learner and assess him effectively.

3. Instruction should emphasize direct teaching (expository rather than discovery learning) of mathematics concepts and skills. Use of simple examples and demonstrations is recommended.

4. Mathematics lessons should be broken down into small steps and tasks that clearly show the client the relationships and inter-relationships between concepts within one lesson and concepts between different lessons.

5. The use of concrete materials e.g. visual aids, audio-aids; and audiovisual aids is highly recommended to facilitate explanation, demonstration, interpretation, and application of mathematical concepts and coefficients. This is important because the client is functioning at the concrete level.

6. Peer tutoring and cooperative learning are pro-social learning strategies that are highly recommended for the client. These techniques will eventually enable the child to have experience of success which will help increase his interest and motivation in mathematics thereby reducing anxiety and phobia.

7. Use positive reinforcements (including token economies) to encourage the client to maintain interest and motivation in mathematics.

8. Increase the funding of education so that the quantity and quality of teaching and learning resources can be improved.
Administrative decisions to adjust math assessments and make them friendly

1. The client should be given time extensions to complete math tests and examinations. Additional time will help accommodate his slow speed to read questions, conceptualize contents and process the responses.

2. General instructions on tests / examinations and specific instructions on how to respond to individual items should be written clearly in easy language. Both types of test instructions (general and specific) should also be verbally given to the student to ensure he understands them.

3. Ensure that the assessment instruments administered to the client contain a careful balance of both objective and short guided essay items. These types of items require many different cognitive strategies to answer. They also require different academic skills to answer satisfactorily.

4. Items on tests / examinations should not differ much in difficult so that the student's performance is easy to monitor and compare across items within one assessment instrument and between evaluation instruments.

5. The assessments should be criterion-referenced (emphasizing mastery / competence) rather than norm-referenced (stressing relative rank or position when compared to peers). What a student can be able to do is more important than how he competes with other students.

6. Continuous assessment, CA (formative evaluation) should be weighted more than final examinations (summative evaluation). CA gives a better picture of a person’s abilities because it is multiple and multifaceted and permits an observation of the individual’s non-cognitive skills over an extended period of time.

7. Using the results from frequent assessments under CA, it may be possible to monitor and chart the client’s progress and difficulties. Both strengths and weaknesses would be useful in the student’s intervention program. 

Improving teacher education through initial and continuous training programs

1. Imparting special needs education instructional skills to regular early childhood education teachers to enable them to effectively help students with high support needs in learning mathematics.

2. Mount continuous professional development (CPD) workshops in mathematics to upgrade the knowledge and skills of early childhood math teachers.

3. Educational psychologists, school counselors, and special educators could help mount sensitization campaign messages (e.g. motivational talks, study skills, etc) to help prevent the development of math learning difficulties in young children. Such campaigns could use mediums popular to young children such as internet, movies/films, psychodrama, and role play.

Conclusion

This mixed-methods study incorporated elements of the survey, case study and action research approaches in investigating the research problem. Using a range of data collection strategies, the study assessed the academic problems of a young Grade 4 (Year 4) student who was referred for having difficulties in learning mathematics. From the diagnostic test, an error analysis, and a think-aloud clinical interview, the study identified some of the child's
difficulties. The major presenting problems included: inability to use the four arithmetic operations (addition, subtraction, multiplication, and division) efficiently; not understanding the relationship between units, tens and hundreds; using any two of the four arithmetic processes (+, -, x, ÷) in combination within one operation; treating each column as a separate problem; place value problems or wrong alignment of numbers; poor eye-hand coordination leading to dysgraphia; and short-term memory / memory lapses. The other problems that became apparent through this investigation and are implied in the findings include possible causal factors such as dyscalculia, dyslexia, low self-esteem, low self-efficacy, and math anxiety / phobia. Further assessment and research is recommended to probe and confirm the role of these variables in young learners with math difficulties such as student B, to gain additional insights. Future research should also be directed at examining the learning styles and study strategies in mathematics of young children with high support needs in this subject.

**Limitations of the study**

The present study was informed by three main limitations. First, it only reports on the outcomes of the assessment phase of a child who was brought into counseling by parents for inability to do math in Grade 4 (Year4). The consultation with the psychologist was limited to assessing the child only. The resulting copies of the technical report on assessment (that included recommendations for intervention) were given to the child’s parent, math teacher, and school authorities. There was therefore no other data on this case and the results of the entire educational intervention that followed the assessment phase were ethically and legally outside the objectives and beyond the scope of the present study. Second, as a case study, the results cannot show cause-and-effect relationships among the variables investigated because of internal validity problems. While the results of a case study such as the present inquiry are only exploratory rather than confirmatory, the findings may, however, provide useful research questions and hypotheses for further follow-up research studies. Third, because of the small and non-probability sample in the case study (n = 1), the results or findings cannot be generalized to other students with similar problems as the case study design is deemed to have low external validity. Despite these shortcomings, the present clinical case study has high practical significance and might be of value to both the local community in the country in which it was conducted as well as the international community elsewhere in the world.

Lawrence Mundia holds a PhD in educational psychology and measurement from Cornell University (USA), an MEd special education and MEd guidance and counselling both from the University of Southern Queensland (Australia). He is a chartered psychologist (CPsychol), an associate fellow of the British Psychological Society (AFBPsS), and an HPC(UK)-registered practitioner psychologist and counsellor. Currently he teaches educational psychology, psychological assessment, educational testing, special education, and school guidance and counselling in the Sultan Hassanal Bolkiah Institute of Education at the University of Brunei Darussalam. Recently, students with mental health problems have received more research consideration, attention and priority. He is keen on researching child and adolescent mental health problems in educational institutions.
References


**Appendix 1:** Performance on math problems by the whole class (N = 29)

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*Decimal points are omitted and all values are to three decimal places

\(^a\)Item 6 is deleted/excluded from analysis (Facility value = 1.00; see Appendix 1)

\(^b\)Items student B got right/correct
Appendix 3: Performance on math problems by the case study participant ($n = 1$)