Exploring The Connections Among Number Sense, Mental Computation Performance, And The Written Computation Performance Of Elementary Preservice School Teachers

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ABSTRACT

The purpose of this study is to a) explore connections among number sense, mental computation performance and the written computation performance of elementary preservice school teachers; and b) explore the correlation among mental computation skills, computation skills, effect issues and number sense. The sample was composed of students in six intact entry-level mathematics sections of a course populated by preservice elementary school teachers. One hundred fifty-five participants from these six classes completed data collection tasks during the Spring 2002 semester for the study. Regression analyses were used to investigate the correlation of written computation skills, mental computation skills, and affective domain with regard to number sense. Three of these subscales of Conference Learning Mathematics, Mathematics Anxiety, Effectance Motivation of Mathematics, Mental Computation Test score, and Written Computation Test score were found to positive significantly correlate with Number Sense Test score success at the α= 0.001 level. Overall, the six independent variables considered in this study accounted for 57.1% of the variation in Number Sense Test score, with Mental Computation Test, and Written Computation Test having the strongest effects.

INTRODUCTION

Number sense refers to a person's general understanding of numbers and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful and efficient strategies for managing numerical situations. (Reys & Yang, 1998; McIntosh al., 1999). It also includes the ability and inclination to use this understanding in flexible ways to make mathematical judgments and develop useful strategies for handling numbers and operations. Number sense is highlighted in current mathematics education reform documents because it typifies the theme of learning mathematics as a sense-making activity (National Council of Teachers of Mathematics, NCTM, 2000). The development of number sense is important in mathematics education. The National Council of Teachers of Mathematics, in their *Principles and Standards for School Mathematics*, note that number sense is one of the foundational ideas in mathematics in that students “(1)

Understand number, ways of representing numbers, relationships among numbers, and number system; (2) Understand meanings of operations and how they related to one another; (3) Compute fluently and make reasonable estimates. (NCTM, 2000, p.32).

Considerable research has dealt with the mathematical performance of elementary school students, but far less research has dealt with what their teachers understand. The few studies that have investigated the mathematical understanding of elementary teachers and preservice elementary teachers indicate that many exhibit weakness in mathematics, may misapply mathematical rules, do not understand true meanings of mathematical concepts, and that they are, generally, not prepared to teach the mathematical subject matter entrusted to them (Cuff, 1993; Hungerford, 1994).

In many cases, however, much of the attention to developing number sense is a reaction to an over emphasis on computational procedures that are often algorithmic and devoid of number sense. For instance, the reaction of a student when asked if a calculation seems reasonable is often to recalculate rather than to reflect on the result in the light of context and numbers involved (Wyatt,1986). Mclnotosh et al. (1992) claimed that high skill in written computation is not necessarily accompanied by number sense. This finding confirms that the content emphasized in mathematics is what is learned and is consistent with the statements (Sowder,1988, p. 227) that "correct answers are not a safe indicator of good thinking" and "teachers must examine more than answers and must demand from students more than answers." Yang (1997) conducted research based on 234 students in four schools in one city in Taiwan, and provided strong evidence that Chinese students perform at very different levels on written computation when compared to their performance on number sense. Johnson (1998) found that preservice elementary teachers have a gap in their rational number understanding and that they rely on the use of algorithms when approaching non-standard problems. The misconceptions they exhibit tend to be similar across different representations of rational numbers. The findings of Rasch (1992) and Hungerford (1994) suggest that preservice elementary school teachers exhibit difficulties with rational numbers that may be indicative of a lack of intuitive conceptual understanding of the meaning and properties of the number system. Thus, the scope of number sense was restricted to the understandings that could be derived mentally, without resorting to computation, rules, or algorithms.

Although considerable attention to number sense is occurring in the United States, the term " number sense " is rarely heard in preservice teacher research. Believing that understanding the level of number sense should play an important role in preservice teaching programs, the motive for conducting this study rises from a deep concern for the development of number sense for preservice teachers. Of the many variables affecting the development of number sense, the four variables that will be considered for this study are estimation ability, computation ability, mental computation, and affective issues.

THE PURPOSE OF THE STUDY

The purpose of this study was to a) explore connections between number sense, mental computation performance and the written computation performance of elementary preservice elementary school teachers; and b) explore the correlation among mental computation skills, computation skills, affect issues and number sense.

A quantitative approach will be used to address the question, with the following null hypotheses:

H_{o1}: There is no significant correlation among mental computation performance, written computational performance or affective issues and preservice elementary school teachers number sense, at the $\alpha= 0.01$ level.

REVIEW OF LITERATURE

According to the history of formal mathematics education, the aim has been to help students understand what they are doing and why (Von Glaserfeld, 1987). Elementary school teachers need, in the midst of learning general pedagogy and all the subject matter required of a generalist, to develop a comprehensive understanding of mathematics that goes beyond citing rules, definitions, and procedures. It is important that they have positive attitudes about mathematics, have a comprehensive understanding of mathematics, know that mathematics is more than a set of skills and procedures, have acquired number sense, and have models for teaching mathematical notions and conceptions at a given level of instruction (NCTM, 2000). Studies of the mathematical subject matter knowledge possessed by prospective elementary teachers have revealed that many gaps exist. Fewer studies have focused on the mathematical understanding that provides the foundation for computation ability. Some mathematics educators (Bums, 1994; Kamii &Lewis, 1991) believe that teaching standard paper-and pencil algorithms interferes with the development of number sense.

The literature reviewed for this study will begin with the number sense and then discuss researches related to number sense. The second section focuses on describing mental computation, and computational estimation. The Final section includes a discussion of the attitudes and beliefs about mathematics exhibited by elementary teachers.

NUMBER SENSE

By referring to how number sense was exhibited, Greeno (1991) characterized number sense in terms of flexible mental computation, numerical estimations and qualitative judgments. His perspective on number sense encompassed recognition of the role of equivalence in the decomposition/recomposition of numbers, the use of approximate numeric values in computational contexts and the making of inferences and judgments about quantities with numerical values. Teaching basic facts has always been a part of any successful mathematics program and is very important in developing mental math skills and flexibly applying estimation skills. (Leutzinger,1999). Leutzinger (1999) addressed that too much time is spent on repetitive practice instead of exploratory experiences, which give students the opportunity to develop thinking strategies. Calvert (1999) agrees that practicing procedures without thought or understanding enforces a dependence on standard paper and pencil algorithms. Mental math strategies and flexibly applying the knowledge of number senses and operations to computational situations, need to be developed from conceptual understanding rather than memorized rules (NCTM,1995).

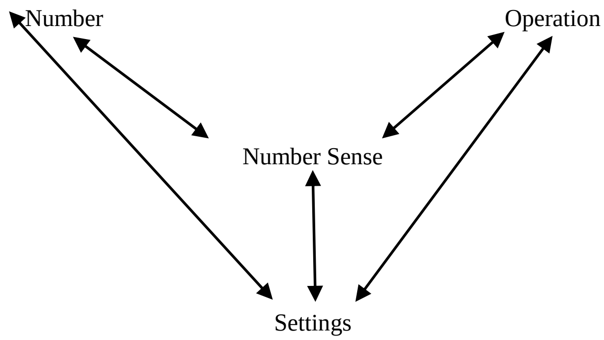
McIntosh, Reys, & Reys (1992) state that "fully conceptualizing (the concept of) operation implies understanding the effect of the operation on using numbers including whole and rational numbers" (p. 7). *The NCTM Standards* (1989) highlight that "operation sense also involves acquiring insight and intuition about the effects of operations on two numbers" (p. 43). For example, children should sense that the sum of two numbers, each of which is greater than 50, must be greater than 100.Kaminski (1997) found that the use of number sense can assist individuals in their understanding of, and calculation in, mathematics. He reported on these aspects of number sense by studying six primary preservice teacher education students. Kaminski chose those who experienced difficulties with whole and rational number numeration and computation in addition to those who exhibited a desire to understand more about students' use of number sense. The researcher found the preservice teacher students in this study displayed underdeveloped sense of number, exhibited a preference for using exact written calculations and seldom utilized approaches involving estimation, and desired to follow a set line of reasoning without reviewing the appropriateness of the strategies employed or reasonableness of results obtained.

McIntosh, B. Reys, & R. Reys (1992) proposed that the connection between the components shown in Figure 1 imply monitoring similar to that used in metacognition and present in problem solving activities. "A person with good number sense is thinking about and reflecting on the numbers, operations, and results being produced (p.5)."

Carroll (1996) asserted that good mental computation and estimation ability is evidence of number sense and also enhances the development of number sense in addition to improving metacognitive skills. If mental and estimation computation are taught using a problem-solving approach rather than as a sequence of strategies or skills, students tend to invent their own strategies and then mental and estimation computation will involve higher-order thinking. The most important single element of number sense is an understanding of numbers (Sowder and Schappelle, 1994). The NCTM Standards (2000) states that children must understand number meanings if they are to make sense of the ways that numbers are used in real-life situations. Hiebert (1989) has noted that children are often surprisingly proficient at solving mathematical problems outside of school because in these settings the context provides a means for understanding what they are doing. These same children when given more formal school-related tasks are often unable to complete the task because they cannot relate the formal or written symbolism to their own informal conceptualizations (Hiebert, 1989).

Mack (1990) examined the development of students' understanding of fractions by looking at ways students are able to use their informal knowledge to give meaning to symbols and procedures associated with fractions. Her study "suggests that knowledge of rote procedures interferes with students' attempts to construct meaning algorithms " (p. 30). The degree of a student's understanding is determined by the number, accuracy, and strength of connections. A concept is well understood if it has many links to other aspects of knowledge that are accurate and strong (Hiebert and Carpenter, 1992).

Figure 1.: Interconnections Of Major Components Of Number Sense.
Adapted From " A Proposed Framework For Examining Basic Number Sense "
(P5) By Mcintosh, B. Reys, & R.Reys, 1992, White Rock: British Columbia, Canada.



Understanding a number as a quantity of specific magnitude and being able to judge how it compares to another number is basic to number sense (Sowder,1988). A sense of the size of a number including the ability to compare and order numbers is a corner stone of number sense. For example, students know that place value of 5 is more than place value of 3, or 1359 is less than 1500. This characteristic also includes a general sense of large numbers, often using benchmarks to reveal their size. Markovits and Sowder (1994) state that "understanding number magnitude within a number domain encompasses the abilities to compare numbers, to identify which of two numbers is closer to a third number, to order numbers, and to find or identify numbers between two given numbers" (p. 6).

Sowder and Markovits (1989) believe that a mathematics curriculum devoted to more work on number magnitude can increase the understanding of numbers and the number system. For example, research has indicated that " percent " is a difficult topic in the middle grades' mathematics curriculum (McGivney & Nitschke,1988). Sowder and Markovits believe that meaningful understanding of the size of fraction and decimal numbers can help students in developing number sense in general. Sowder and Markovits (1989) found that instruction focusing on the concept of fraction number size could improve students' understanding of fractions. Behr, Wachsmuch, Post, & Lesh (1984) and Behr, Wachsmuth, & Post (1985) also found that with appropriate instructions through an extended period of time, most children have the ability to develop suitable strategies to compare and order fractions.

Sowder (1992a) believed that if children want to compare decimal numbers accurately, then they must have a good understanding of whole number place value. Hiebert and Weame (1986) found that nearly half of sixth and seventh graders chose 0.1814 as the largest decimal among 0.09, 0.385, 0.3, and 0.1814 by using the method of more digits makes bigger. This finding supports the research of Sackur-Grisvard & Leonard (1985) that children choose 3.63 as the larger when giving 3.63 and 3.8, because 63 is larger than 8. Children incorrectly based their judgement of the relative magnitude of numbers on the number and value of the places in the decimal position of the number, paying little attention to the digits in these places or to the overall meaning of the number (Resnick, 1987).

Relative magnitude of numbers is the ability to compare and to order numbers (Sowder, 1992b). Several research studies have shown that, particularly for rational numbers, children have an inadequate ability or understanding to determine relative size (Peck & Jencks, 1981; Hiebert & Weame, 1986; Sackur-Grisvard & Leonard, 1985). Sowder and Wheeler (1987) found that most students before tenth grade were unable to correctly compare 5/6 and 5/9. The study results of Peck and Jencks (1981) demonstrate similar poor performance for comparing fractions such as 2/3 and 3/4. In fact, some students thought the two fractions were equal, because " there are the same number of pieces left over ” (p. 344). A frequent strategy cited by students to compare fractions is to compare numerators if the denominators are equal, and denominators if the numerators are equal (Behr, Wachsmuth, Post, & Lesh, 1984; Sowder & Markovits, 1989; Sowder & Wheeler, 1987 ; Reys ,Reys, Nohda & Emori ,1995).

Sowder (1992a) posits that it is difficult for children to make sense of large numbers if they do not have opportunities in an instructional setting to explore big numbers. Saxe (1988) found that unschooled Brazilian children could manage large numbers with the Brazilian money system. Zaslavsky (2001) stated that students can begin to appreciate the Hindu-Arabic numeration system better as they learn about, and experience, the many ways that people have counted and recorded numbers throughout the development of society over a period of thousands of years. Students not only gained in their understanding of other mathematical systems but also developed greater appreciation of the fact that all people count Zaslavsky (2001).

Benchmarks are described by the phrase, “ a compass provides a valuable tool for navigation, numerical benchmarks provides essential mental referents for thinking about numbers ” (McIntosh, Reys, & Reys ,1992 , p. 6), For example, using 1 as a benchmark, the sum of 6/7 and 14/15 should be near 2 and less than 2, because both of the fractions are a little less than 1. McIntosh, Reys, & Reys (1992) state that “ benchmarks are often used to judge the size of an answer or to round a number so that it is easier to mentally process ” (p. 6).

Trafton (1989) believes that percentages should be taught with an early emphasis on the meaning of percent using estimation and mental computation as vehicles for developing the concept of percents. In particular, Trafton (1989) posits that particular attention should be given to establishing key benchmarks such as 10% and 50%, performing mentally computations with these benchmarks, estimating percent using benchmarks, and finally calculating exact percents.

McIntosh, Reys, & Reys (1992) stated that “ solving real world problems which require reasoning with numbers involves making a variety of decisions including : determining what type of answer is appropriate (exact or approximate), determining what computational tool is efficient and/or accessible(calculator, mental computation, computation estimation, etc.) choosing a strategy, applying a strategy, reviewing the data, and result for reasonableness” . The researchers see the use of mental computation and estimation as necessary.

MENTAL COMPUTATION

Researchers (NCTM, 1989; Reys, B., 1985; Rey, R., 1984; Sowder, 1990; 1992; Reys, Reys, Nohda and Emori 1995; McIntosh, Reys, B. & Reys, R., 1997; Calvert, 1999) have highlighted the importance of mental computation in elementary school and viewed it as a good way to develop an understanding of the structure of the number system. Students need to foster the use of a wide variety of computation and estimation skills, to improve mental calculation, because these skills are needed in order to prepare for the 21st century (NCTM, 2000).

Number sense is a broad domain and includes both estimation and mental computations (McIntosh, Reys, B. & Reys, R., 1997). Estimation is used when an exact answer is not needed. Most often the first step informing the estimate involves mental computation (McIntosh, Reys, B. & Reys, R., 1997). Two ways of viewing mental computation are presented by Reys, Reys, Nohda and Emori (1995). The first is to look at it as a basic skill. This point of view understands mental computation as a set of procedures that are applied mentally. It calls for direct teaching and practice of these procedures. "On the other hand, when students generate their own computation strategies, mental computation can be seen as higher-order thinking" (Reys, Reys, Nohda and Emori 1995, p305). Reys, B. (1985) highlights that "mental computation promotes an understanding of the base-ten number system as well as of basic number properties" and "mental computation rewards flexibility in dealing with various forms of numbers" (pp. 45-46). Reys, R. (1984) believes that mental computation “ promotes greater understanding of the structure of number and their properties" and "promotes creative and independent thinking and encourages students to create ingenious ways of handling numbers ” (p.549). Mental computation assists in developing number sense because it makes students think (Reys and Barger, 1994).

Some research evidence reveals that the skill of mental computation is closely related to an understanding of the structure of the number system and flexible use of numbers. In the case study by Hope (1987), evidence shows that a highly skilled mental calculator could use various ingenious calculation methods, including distributing and factoring. Hope and Sherrill (1987) discuss the differences of mental computation strategies used by skilled and unskilled mental calculators among secondary students. The unskilled students were more likely to use standard written methods than self-developed strategies, and they paid no attention to the most obvious number properties. However, the skilled students used a variety of mental strategies, including different types of distribution and factoring. They used more efficient and useful methods than unskilled students. Skilled students tended to "eliminate the need for a carry operation, proceed in a left-to-right manner, and progressively incorporate each interim calculation into a single result" (Hope & Sherrill, 1987, p. 108).

Reys and Barger (1994) posit that students who are good mental calculators use number properties and equivalent representations of numbers to perform transformations on numbers. Students who are less proficient tend to rely on mental versions of standard algorithms. Carraher, Carraher, and Schliemann (1987) concluded that the oral mathematics used to solve everyday problems provides evidence about students' understanding of the decimal system. Furthermore, students tend to use strategies based upon decomposition and grouping that reflect their understanding of number properties.

Reys, B. (1985) categorized the strategies used by high and middle-ability seventh and eighth grade students. The high-ability students could widely use different number properties, including the forms of commutativity, associativity and distributivity. The high-ability students flexibly utilized the knowledge of place value, connections between operations, and translation method. On the other side, middle-ability students tended to use the standard written algorithms and seemed unwilling to create invented strategies mentally (Reys, 1985).
Mental computation is useful in understanding the system of numbers and operations in addition to playing an important role in developing number sense. Reys, B. (1985) states that " mental computation nurtures the development of keen number sense " (p. 46). Even though there is no clear evidence to what extent the role of mental computation plays in the development of number sense, improving the mental computation skills through exploring the structure of the number system and relations between numbers is one of the ways of developing number sense (Sowder, 1990, 1992a).

COMPUTATIONAL ESTIMATION

According to NAEP data, the need for developing students' estimation skill is also reflected in the Agenda for Action (NCTM, 1980). One of the specific recommendations is that teachers " incorporate estimation activities into all areas of the school program on a regular and sustaining basis " (p. 7). The reason for stressing estimation is its usefulness in every situation involving numbers (Carpenter, Coburn, Reys, & Wilson, 1976). Previous research (Hall, 1977;Sowder & Wheeler, 1987, 1989) has focused mostly on elementary or junior high school students; little research has been conducted on undergraduate college students.

Levine (1982) interviewed college students of varying mathematical backgrounds to identify strategies used to estimate. Each student was asked to estimate the answers to 20 questions. In addition, each student was given a quantitative ability test. The results revealed that students who had higher scores on the quantitative ability test could use more and varied estimation strategies, and were better in computational estimation skills than students who had lower scores on the quantitative ability test. Levine (1982) found that poor estimators preferred to look for the exact computation, then round to find an estimate. She states that this method did not " require the individual to sense any relationships or to have any 'number sense' to carry it out " (p. 358). Good estimators in Levine's study used more strategies and appeared to be more flexible in their thinking than poor estimators (Levine, 1982). Levine (1982) found eight principal strategies:

1. using the fraction relationship,
2. rewriting numbers as exponents,
3. rounding both numbers involved to multiple of 10,
4. rounding only one number to a multiple of 10,

5. replacing a number with a power of 10,
6. replacing a number with "known" numbers,
7. pairing off parts of numbers, and
8. using a standard algorithm to roughly calculate and combine partial answers.

The study conducted by Reys, Rybolt, Bestgen, and Wyatt (1980, 1982) was designed to identify and characterize the computational processes used in estimation by school students and adults. The researchers identified three key processes characteristically used by good estimators. The first key process is called reformulation: " the process of altering numerical data to produce a more mentally manageable form. This process leaves the structure of the problem intact." (p. 187) . The second key process is identified as translation: " the process of changing the mathematical structure of problem to a more mentally manageable form " (p.188). The third key process used by good estimators is compensation: " adjustments made to reflect numerical variation that come about as a result of translation or reformulation of the problem. These adjustments were typically a function of the amount of time available to make a response but were also influenced by the ability to manage the numerical data, context of the problem, and the individual's tolerance for error." (p. 189). Throughout the interviews, the researchers (Reys, Rybolt, Bestgen, and Wyatt , 1982) also found that the strategies used by the good estimators included the experiential background of individual, the mathematical operation being performed, the size of numbers, and the relationship of numbers within a given problem.

The 12- week study by Reys, Rybolt, Bestgen, and Wyatt (1980) , conducted with preservice elementary teachers evaluated practice with and without instruction on specific estimation strategies. One group of preservice teachers received 5 minutes of instruction per week for ten weeks and a weekly quiz. After each quiz, the four quiz problems for which the strategy of that week was most appropriate were discussed. Another treatment group received only the weekly quizzes. The result was consistent improvement in estimation performance in all experimental groups, with the greatest gain in multiplication and decimal numbers (Reys, et al., 1980). Other research also found that intermediate grade students who were taught estimation skills outperformed students in a control group (Reys, Trafton, Reys, & Zawojewski,1984). Though instruction was brief, the message is clear that frequent practice can improve estimation skills. Research also showed that these students had an improved understanding of number concepts.

LeFever, Greenham & Waheed (1993) studied the computational estimation skills of 20 adults and several groups of elementary school children, comparing the strategies employed at different age levels according to item. Their research suggested alternative computational strategies help in the understanding and application of mathematics. Dowker (1992) gave forty-four academically pure mathematicians an estimation test, which called for an explanation of the strategies they used. "The mathematicians were accurate estimators, and they used a great variety of strategies, as many as 23 for a single problem"(Dowker, 1992, p. 45). Dowker (1992) used Levine's items to test the computational estimation skill strategies of professional mathematicians. As should be expected, Dowker (1992) found a high level of accuracy among the mathematicians and they used a greater range of strategies than Levines non-mathematician college students used. In further studies, Dowker, Flood, Griffiths, Harriss, and Hook (1996) addressed a positive correlation between estimation performance and number of strategies employed in making the estimation.

The effects of instruction on estimation was the focus of strategy used by a good estimator. Some researchers (DeCorte & Somer, 1982 Markovits, 1990) preceded an instructional unit on computational estimation in elementary and middle school students with instruction on mental computation and on comparing and ordering fractional and decimal numbers. In a study by Sowder and Wheeler (1989), twelve students at each of grades 3, 5, 7, and 9 were individually given computational estimation tasks. The researchers found that students were willing to accept that there could be multiple strategies for finding an estimate, each producing a different answer, but students were reluctant to accept more than one " right answer ". Students at all grade levels preferred computing-then-rounding to estimate rather than the rounding-then-computing, because they believed that computing-then-rounding method was a good way to get a correct answer to the estimation task. As grade levels increased, the use of the rounding method also increased. Sowder and Wheeler (1989) conjecture that perhaps this is the accumulative effect of school instruction on rounding and emphasis on unique answers.

Threadgill-Sowder (1984) posed 12 National Assessment of Educational Progress (NAEP) items to students of grades six through nine. They were asked to respond to these questions within an interview format. She found that multiple-choice items were frequently answered correctly by incorrect reasoning. This study implies that results regarding estimation from the NAEP are probably even worse than reported. Threadgill-Sowder (1984) also found that "students who gave acceptable responses consistently demonstrated this quantitative intuition, or number sense, whereas those who gave unacceptable responses seemed to have little feel for the numbers represented" (p. 335). The results of the study indicate that estimation skills are highly dependent upon a student's number sense. Threadgill-Sowder (1984) theorized that good estimators have a good understanding of basic facts, place value, and arithmetic properties, are skilled at mental computation, demonstrate tolerance for error, can flexibly use a variety of strategies, and display self-confidence. The results of this study have been useful and helpful in guiding the research of number sense. Edwards (1984) believed that improving the teaching of computational estimation is related to encourage the development of number sense.

The message that comes from the studies which focused on computational estimation (Levine, 1982; Reys, Bestgen, Rybolt, & Wyatt, 1980, 1982; R. Reys, B. Reys, Nohda, Ishida, Yoshikawa, & Shimizu, 1991; Rubenstein, 1985; Sowder, & Wheeler, 1989; Sowder, 1994; Threadgill-Sowder, 1984, Lefever et al.,1993) is summarized by Sowder : "good estimators are flexible in their thinking, and they use a variety of strategies. They demonstrate a deep understanding of numbers and operations, and they continually draw upon that understanding" (p. 375). Researchers have identified three general ways in which people estimate answers to computational problems: reformulation, translation, and compensation (Reys, Bestgen, Rybolt, & Wyatt, 1982; R. Reys, B. Reys, Nohda, Ishida, Yoshikawa, & Shimizu, 1991; Sowder, & Wheeler, 1989; Sowder, 1994; Lefever et al.,1993).

Estimation is not only a valuable skill in its own right, it can also be a valuable pedagogical tool used in the development of other important skills (Buchanan, 1980). The Levine (1982) study noted quantitative ability is related to estimation; Hall (1977) concluded that estimation ability is related to problem-solving ability. Paull (1972) found that estimation of numerical computation is significantly correlated with problem solving, mathematical ability, and verbal ability, and the ability to compute rapidly was related to the ability to estimate numerical computation.

ATTITUDES/ BELIEFS

A person's attitudes/ beliefs are developed over a long period of time through a process of assimilation and accommodation, and are a product of experience and reflection (Shealy et al.,1993). Raymond (1993) found that teachers' mathematics teaching practices do not always match their mathematics attitudes/beliefs. Understanding where one's attitudes/beliefs come from and the relationship between these attitudes/beliefs and teaching practices can help teachers identify and resolve conflicts between the two. Teachers in Raymond's (1993) study offered a list of explanations for their own inconsistencies between mathematics beliefs and practices. This list included: time constraints, lack of resources, classroom management problems, and state standardized testing requirements. Underhill (1988) provides further explanations for these discrepancies: attitudes/beliefs may represent ideals instead of reality; attitudes/beliefs may represent what a teacher wants but cannot achieve due to a lack of knowledge or skill; and attitudes/beliefs may be hierarchically arranged. This last explanation concerns how teachers resolve conflict. Often teachers must compromise between attitudes/beliefs and actual classroom practice- causing them to move down in their list of hierarchical attitudes/beliefs until the problem in the classroom is resolved.

The study of teachers' beliefs has helped change the emphasis of educational research "from a behavioral conception of teaching towards a conception that takes account of teachers as rational beings. " (Thompson, 1992, p.142) This has contributed to reform of the educational research agenda (Thompson,1992). Understanding teaching from a teacher's perspective has enriched our understanding of learning from a student's perspective. However, more research is needed concerning the connections between teacher and student perspectives. Through research on teachers' /beliefs about mathematics and mathematics teaching and learning, teacher educators can learn how best to address these issues. Attitudes/beliefs are an important part of education of pre-service teachers and must be explicitly addressed in teacher education programs (Raymond, 1993). Pre-service teachers need to explore their mathematics attitudes/beliefs. If they learn to reflect about their attitudes/ beliefs and practices, conflicts can more easily be identified and resolved; and this will enable them to become better teachers.

METHODOLOGY

Population and Sample

The population of this study consists of preservice elementary school teachers at a mid-sized, four-year, state university in a mid-sized town in the Rocky Mountain region. The sample was composed of students in six intact entry-level mathematics sections of a course populated by preservice elementary school teachers. 155 participants from these six classes completed data collection tasks during the Spring 2002 semester for the study.

Data Collection Procedures

In the beginning of the semester, instruments used to collect data was the components of the Fennema-Sherman Mathematics Attitudes Scale (FMAS), the Number Sense Test (NST), the Mental Computation Test (MCT) and the Written Computation Test (WCT). Calculator use was allowed. At the beginning of the semester, the Fennema-Sherman Mathematics Attitude Scale, the Number Sense Test, the Mental Computation Test and the Written Computation Test were given to all classes. The scores from each of the three tests was analyzed.

Number- Sense Test

During the first week, a 25-item Number Sense Test was given to the students. Students were given a copy of the NST and instructed not to begin work until told to do so by the researcher. The researcher and instructors were provided with general instructions and answer questions from students. Students were asked to obey the rules of this test: timing per item is about 45 seconds and students were told not to spend too much time on any one question.

Mental Computation Test

The Mental Computation Test (MCT) was individually administered by the researcher/ instructors with twenty-five second intervals per item. The time of 30 seconds for displaying each item of the mental computation test was chosen based on the research of McIntosh, Reys, & Reys (1997). Oral items were read twice with a short pause (2-3 seconds) between readings followed by a 30 second waiting period between items to let students answer the questions (McIntosh, Reys, & Reys, 1997). The MCT contains 15 items (see Table 1). Students were given an answer sheet that contains a limited amount of space to record answers. The examiner was read the general instructions aloud for the MCT and answer any questions from students. Students were advised to listen carefully because each oral question will be read aloud and repeated only once. Immediately after these instructions, two practice questions were provided.

Table 1. The Frame of Number Domain and Four basic Operations on MCT

	Addition	Subtraction	Multiplication	Division
Whole Number	1. 72+ 45+128	2. 85+39-26-39	3. 38 ×50	4. 1200 ÷ 40
Decimals	5. 6.5 + 1.9	6. 4.5- 2.6	7. 95 ×.01 8. 60 ×1.5	9. 6.5 ÷ 0.5
Fraction	10. 1/2+3/4	11. 5 (1/4)-3(3/4)	12. 4×3 ½ 13. ½× 6(½)	14. 90 ÷ ½ 15. 1/2 ÷1/5

Written Computation Test

Immediately following the Mental Computation Test, the Written Computation Test were administered. Each student was given a copy of the WCT and was told not to begin work until instructed to do so by the examiner. The examiner has read aloud the general instructions and answer questions from students. Students then worked independently on the sixteen items during the twenty minutes allowed for the test.

INSTRUMENTS

Fennema-Sherman Mathematics Attitudes Scales (FSMAS)

Fennema-Sherman Mathematics Attitudes Scales (Fennema & Sherman, 1976) are used to evaluate student mathematics attitudes. This publicly available attitude test consists of nine domain specific tests which reflect attitudes about learning mathematics; these tests may be used as separate tests or in any combination.

The domain scales are identified and described as follows:

- The Confidence in Learning Mathematics Scale (C) is intended to measure confidence in one's ability to learn and to perform well on mathematical tasks. (Fennema & Sherman, 1976, p. 4).
- The Mathematics Anxiety Scale (A) is intended to measure feelings of anxiety, dread, nervousness and associated bodily symptoms related to doing mathematics. (Fennema & Sherman, 1976, p. 4).
- The Effectance Motivation Scale in Mathematics (E) is intended to measure effectance as applied to mathematics. (Fennema & Sherman, 1976, p. 5).
- The Mathematics Usefulness Scale is designed to measure students' beliefs about the usefulness of mathematics currently and in relationship to their future education, vocation, or other activities (Fennema & Sherman, 1976, p. 5).
- This study used four of the domain scale tests: Confidence (C), Mathematics Anxiety (A), Effectance Motivation (E), and Mathematics Usefulness (U). These four tests, consisting of twelve questions per test, offered a 48 item test. This test was randomly generated by a computer program for a cumulative test. Table 2 shows three items.

Table2. Sample of Fennema-Sherman Mathematics Attitudes Scales

Statements	Circle Choice
I usually have been at ease during math tests.	A B C D E
I am no good in math	A B C D E
I like math puzzles.	A B C D E

The Fennema-Sherman tests use a Likert type scale wherein the subjects respond, on a scale of 1 to 5, to their degree of agreement with a statement. The response choices are strongly disagree (1 point), disagree (2 points), cannot decide (3 points), agree (4 points), and strongly agree (5 points). Each domain scale consists of 12 statements, 6 worded positively and 6 worded negatively. A score of 5 is given to the response that is hypothesized to have a more positive relation to learning mathematics. Scores of each domain scale, and the cumulative score of all domains, indicate student attitudes toward learning mathematics. A high score represents a positive attitude toward learning mathematics. A Cronbach alpha of reliabilities of the Fennema-Sherman Attitude Test on seven universities, college students showed the following values: The Cronback's alpha reliability coefficient for Confidence in Learning Scale (C) is 0.89 and for Mathematics Usefulness Scale (U) is 0.85 (Elliott, 1990). Betz (1978) reported the split-half reliability of Mathematics Anxiety Scale (A) is 0.92. The test-retest reliability of Effectance Motivation Scale (E) is 0.96 (Wilburn, 1997).

Fennema-Sherman (1976) established construct validity of the mathematics attitude scales by a principal components factor analysis. Although a correlation study between the scales showed some interrelation, each scale measured the construct it was designed to measure. The test may be given in any combination of scales for measurement of particular constructs.

Mental Computation Test

The Mental Computation Test (MCT) was constructed by Yang (1997) for grade 6 and 8 students in Taiwan. This research study included 15 MCT items. Yang (1997) reported that the items in the MCT were created to elicit the use of mental computation strategies possessed by students and suggested from earlier research suggested (Hope & Sherrill,1987 ; B. Reys 1985; Sowder,1990). The MCT included whole number, fraction and decimal number items as well as the four basic operations. Table 1 provides the framework for MCT items by number domain and the four basic operations.

Number Sense Test

The Number Sense Test (NST) was developed by Yang (1997) for grade 6 and 8 students in Taiwan. The 25 item NST includes whole number, fraction, and decimal items as well as the four basic operations. According to Yang, the split-half reliability of the NST is over 0.80 for both 6th and 8th grade of students. Table 4 provides the framework of NST items by number domain and four basic operations. Table 3 shows two items.

Table 3. Sample of Number Sense Test Items

Without calculating an exact answer, circle the best estimate for: $\frac{21}{36} \times \frac{7}{16}$	A. More than $\frac{21}{64}$ B. Less than $\frac{21}{64}$ C. Equal to $\frac{21}{64}$ D. Impossible to tell without working it out
Without calculating an exact answer, circle the best estimate for: $\frac{12}{13} + \frac{7}{8}$	A. 1 B. 2 C. 19 D. 21 E. I don't know

Table 4. The Framework of Number Domain and four Basic operations on the NST

	Addition	Subtraction	Multiplication	Division
Whole Number	9,12	21	11,23,25	14
Decimals	18	19	1,7,8	5,15,17
Fractions	4,6	10,13,22	2,21,24	3,20

Written Computation Test

A Written Computation Test (WST) was constructed by Yang (1997) for Grade 6 and Grade 8 in Taiwan. The WCT contains 16 written computation items that are designed to parallel the first 16 items of the Number Sense Test. According to Yang's study, the split-half reliability of the WCT is over 0.80 for both grades of students. The main purpose for this structure is to enable examination of the difference between written computation skills and number sense and the relationship between written computation skills and number sense. Refer to Table 5 for example of problem test items.

Table 5. Sample of Problem Test Items

WCT Item	**72÷0.025**
NST Item	Without calculating an exact answer Circle the best estimate for 72÷ 0.025 A. More than 36 B. Less than 36 C. Equal to 36 D. Impossible to tell without working it out.

The WCT consists of whole number, fraction, and decimal items as well as the four basic operations. Table 6 provides the framework for WCT items by number domain and the four basic operations.

Table 6. The Framework of Number Domain and four Basic operations on the WCT

	Addition	**Subtraction**	**Multiplication**	**Division**
Whole Number	9		11, 12	14
Decimals			1, 7, 8	5, 15
Fractions	4, 6	10, 13	2, 16	3

DATA ANALYSIS PROCEDURES

Scoring Data

The Fennema-Sherman Mathematics Attitude Scale uses a Likert scale wherein the subjects respond, on a scale of 1 to 5, to their degree of agreement with a statement. The response choices are strongly disagree (1 point), disagree (2 points), cannot decide (3 points), agree (4 points), and strongly agree (5 points). Each domain scale consists of 12 statements, 6 worded positively and 6 worded negatively. A score of 5 is given to the response that is hypothesized to have a more positive relation to learning mathematics. Scores of each domain scale, and the cumulative score of all domains, indicate student attitudes toward learning mathematics. A high score represents a positive attitude toward learning mathematics.

SCORING THE MCT, THE WCT, AND THE NST

Mental Computation Test Items

Since the MCT was designed to elicit the use of mental strategies and investigate the correlation between mental computation and number sense performance. Scoring is dichotomous with correct answers scoring 1 point. No point was scored if evidence of written computation methods are shown on an answer sheet even if the correct answer is given (Yang, 1997). Therefore, the total score of the MCT is 15 points.

Written Computation Test Items

The WCT was developed to ask students to find an exact answer, and to explore the correlation between written computation performance and number sense test performance. Scoring is dichotomous if an item is correct, then this item will be awarded one point. No partial credit was awarded. That is, if the answer of an item is incorrect, or there is no response, then no point is awarded to this item (Yang, 1997). The total score is 16 points for the WCT.

Number Sense Test Items

Each item of the NST is assigned a maximum of 2 points. On items where the subject gave the correct answer, the item will be awarded 2 points. If there is no response or if the response is incorrect, this item will be assigned 0 points. However, items 11, 13, 18, 21 and 25 have a possible point range of 0 to 2 points. For example, items 11and 13 require the subject to give correct answers and correct explanations. These items are assigned 2 points. If the answer is correct, but the explanation is unclear or if there is no explanation, this item will be assigned 1 point. If both the answer and reasons are incorrect, this item is assigned 0 points. Similarly, if the answer is correct, but the reasons are incorrect, the item also will be assigned 0 points (Yang, 1997). The total possible score of the NST is 50 points.

ANALYZING DATA

A factor analysis was performed to determine the appropriateness of the four subscales on the FMAS. If the subscales are not confirmed, the factor analysis was be used to determine appropriate subscales that consist of items that weigh together on the same factor. Items that do not weigh in are not used for descriptive information of the participants. All of the group administered tests scores will be analyzed with Statistical Package for Social Science (SPSS). Mean scores and standard deviations will be calculated. Correlation analysis was performed using Statistical Package for Social Science (SPSS). Students' responses were judged to decide whether the characteristics of " number sense " are correctly used by preservice teachers.

A Simple Linear Regression (SLR) will be performed to determine the possible correlations between each of the descriptive variables (mental computation skill, written computation skill and each of the resulting FMAS subscales) and number sense as measured by the NST. In addition a Multiple Linear Regression (MLR) were be performed using the same variables. Co-linearity could be present that could result in an MLR model that would mask the effect of one or more of the variables under consideration. Instead, the MLR was done in an attempt to determine the total variability in number sense that can be attributed to the independent variables being considered in the current study.

Reliability of Instruments

This researcher utilized the Statistical Package for Social Science (SPSS) to calculate the Cronbach's alpha coefficient in order to examine the reliability of the Number Sense Test (NST), Written Computation Test (WCT), Mental Computation Test (MCT), and Fennema-Sherman Mathematics Attitudes Scales (FSMAS) which were used in this study. The Cronbach's alpha coefficient reliability for the Number Sense Test (NST), Written Computation Test (WCT), and Mental Computation Test (MCT) were 0.77, 0.80 and 0.81 respectively. The Cronbach's alpha coefficient reliability of three instruments have demonstrated consistent reliability for measures of internal reliability. The reliability for the sub-scales in Fennema-Sherman Mathematics Attitudes Scales (FSMAS) were the Confidence in Learning Mathematics Scale (alpha = .95), Mathematics Anxiety Scale (alpha = .95), Effectance Motivation Scale (alpha = .87), and Mathematics Usefulness Scale (alpha = .87). The four sub-scales in FMAS have demonstrated high reliability for measures of internal reliability as well.

RESULTS

The Descriptive Statistics of the Sample

Table 7 displays the mean scores of the Number Sense Test (NST), Written Computation Test (WCT), and Mental Computation Test (MCT), in addition to a sub-scale of Fennema-Sherman Mathematics Attitudes Scale (FSMAS) for Confidence in Learning Mathematics Scale (C), Mathematics Anxiety Scale (A), Effectance Motivation Scale (E), and Mathematics Usefulness Scale (U) . The standard deviations are shown in parentheses.
The scores of the mean on the NST, MCT and WCT were 20.08, 7.08 and 7.90 respectively. The Standard Deviation of the score on the NST (8.03) was more varied than on the WCT (3.39) and the MCT (3.55).

Table 7. Mean Score (With Standard Deviations) on NST, WCT, MCT, and Sub-Scale in FAMS

	NST	MCT	WCT	C	A	E	U
	20.08 (8.03)	7.08 (3.55)	7.90 (3.39)	37.00 (11.10)	33.48 (10.74)	36.90 (8.27)	48.08 (6.64)
Possible Scores	50	15	16	60	60	60	60

The FSMAS sub-scale data for the combined six sections on the Confidence in Learning Mathematics Scale (C), Mathematics Anxiety Scale (A), Effectance Motivation Scale (E), and Mathematics Usefulness Scale (U) were 11.10, 10.74, 8.27 and 6.64, respectively. The combined data FSMAS sub-scale for Confidence in Learning Mathematics Scale (C), Mathematics Anxiety Scale (A), Effectance Motivation Scale (E), and Mathematics Usefulness Scale (U) indicated that the preservice teachers began the course with a slightly positive attitude toward Confidence in Learning Mathematics (C) and toward a more positive attitude on the Effectance Motivation Scale (E). Furthermore, the subjects had a more positive attitude according to the Mathematics Usefulness Scale (U). However, they had a slightly negative attitude toward the Mathematics Anxiety Scale (A). This result indicates that preservice elementary teachers appear to have stronger anxiety toward themselves and learning mathematics.

A Comparison between the NST Item and the WCT Item

The Written Computation Test (WCT) instrument measured students' abilities to find an exact answer through the use of paper-and pencil method. The Number Sense Test (NST) instrument measured students' abilities to recognize the relative magnitude of numbers, to decompose/recompose number suitably, to use benchmarks, to flexibly apply the strategies of mental computation or computational estimation by fully taking advantage of numbers and operations, to utilize the knowledge of the relative effect operation on numbers and to make sense of the symbols used to represent rational numbers. The first 16 items of the NST parallel the MCT items, as the same mathematics concepts are addressed, including numbers and operations. This organization facilitates comparisons between the number sense and written computation score. Table 8 displays the mean percent of correct responses for the first 16 items of the NST and the WCT. The result indicates that the WCT (53.16) had a higher percent of correct responses than parallel items on the NST (43.15). The percent of correct responses on exact computation was higher than on parallel items encouraging mental computation, estimation, or other aspects of number sense. The data provide evidence that number sense does not necessarily develop from the skill at paper-and-pencil computation (Reys & Yang, 1998).

Table 8. The Mean Percents of Correct Responses Between The NST and the Parallel WCT Items

	Mean	Standard Deviation
NST	43.85	5.63
WCT	53.16	3.38

Data Analyses

How are written computation skills, mental computation skills, and affective issues on number sense of preservice elementary school teachers enrolled in a problem-solving -based mathematics course correlated? Using regression analysis of the Written Computation Test score, the Mental Computation Test and four affective sub-scales were obtained for collection with the Number Sense Test score. Table 9 displays the results.

There were no significant correlations between the Mathematics Usefulness Scale and the Number Sense Test scores. Neither was there significant correlation between the Mathematics Usefulness Scale and the Mental Computation Test. There was a significant positive correlation between Conference Learning Mathematics, Mathematics Anxiety, Effectance Motivation of Mathematics, Mental Computation Test and the Written Computation Test. Each of the other variables in this study was significantly correlated with student work on the Number Sense Test; in particular the Mental Computation and Written Computation Test scores.

For this study, H01 stated that there is no correlation between the Mental Computation Test, Written Computation Test, or affective issues with the measures of preservice elementary teachers number sense. Using α = 0.01 as the pre-study determined level of testing, there was insufficient evidence to reject the null hypothesis regarding the Mathematics Usefulness Attitude Scale. However, the null hypothesis for the Mental Computation Test score, the Written Computation Test score, Confidence in Learning Mathematics, Mathematics Anxiety Attitude Scale, and Effectance Motivation Attitude Scale was rejected.

Table 9. Pearson Correlation Coefficient Among the NST,MCT,WCT, and Sub-scale of FMAS

	NST	MCT	WCT	A	C	E	U
NST	1.000						
MCT	0.540*	1.000					
WCT	0.593*	0.630*	1.000				
A	0.422*	0.455*	0.468*	1.000			
C	0.399*	0.402*	0.438*	0.913*	1.000		
E	0.260*	0.244*	0.334*	0.734*	0.743*	1.000	
U	0.136	0.098	0.236*	0.371*	0.444*	0.589*	1.000

*p <.01

The results from the MLR were not meant to be used as a predictor model. Instead, the MLR was initiated to determine the total variability in the Number Sense Test score that could be attributed to the variables considered in this study. Table 10 displays results from the MLR, looking at t value.

Table 10

SPSS Result of MLR Source	Standardized Coefficients	t Value	Pr >F
MCT	0.230	2.69	0.0080*
WCT	0.384	4.52	0.0001*
ATTA	0.078	0.464	0.643
ATTC	0.113	0.692	0.490
ATTE	-0.043	-0.388	0.699
ATTU	-0.031	-0.382	0.703

* $p < 0.01$

With this model, the possible source for variation in the dependent variable (NST scores) was examined to determine the effect each source can have if all other variables were already in the model before the source under consideration was added to the model. Since r^2 =0.571, approximately 57.1% of the variation in the NST score were explained with the Mental Computation Test and the Written Computation Test having the significant strongest effects.

DISCUSSION

There were five variables from the study that had strong correlations with achievement on the Number Sense Test Problems. These variables were the Mathematics Anxiety Scale (r = 0.422), the Effectance Motivation Scale (r = 0.260), the Confidence Learning Mathematics Scale (r = 0.399), the Written Computation Test (r = 0.593), and the Mental Computation Test (r = 0.540). Achievement in working number sense problems is directly correlated with stronger the Mathematics Anxiety Scale, the Confidence Learning Mathematics Scale, the Effectance Motivation Scale**,** the Written Computation Test, and the Mental Computation Test.

Unfortunately, many preservice elementary teachers in the study did not exhibit these traits. Preservice elementary teachers' confidence in their ability to learn mathematics had a mean score of 37.00. It appears that the lack of confidence in mathematics is related to the actions of preservice elementary teachers (Soodak & Podell, 1996). If a preservice elementary teacher's confidence in mathematics is low, he/she tends not to enroll in additional college mathematics courses. An example of a question with a favorable attitude, from the confidence level form, was that " Generally I have felt secure about attempting mathematics ? " An answer of disagree on six negatively stated comments indicated a favorable attitude. Examples of this include that " most subjects I can handle O.K., but I have a knack for flubbing up math " and " math has been my worst subject."

The lowest mean score for attitudes was The Mathematics Anxiety Scale in learning mathematics at 33.84. The Mathematics Anxiety Scale is intended to measure feelings of anxiety, nervousness and associated bodily symptoms related to doing mathematics. Examples of these included " Math doesn't scare me at all " and " It wouldn't bother me at all to take more math course." Examples of those who marked agree, indicating a favorable attitude were, " Mathematics makes me feel uneasy and confused " and "Mathematics usually makes me feel uncomfortable and nervous." " The sample of preservice elementary teachers appear to have stronger anxiety toward themselves and mathematics learning. It is certainly agreed upon by educators that elementary school teachers are at a disadvantage if they possess mathematics anxiety, and to admit their fears and attempt to overcome them would not only be in their best interest, but also be in the best interest of their students. (Hadfield & McNeil, 1994)"

The mean score of the Effectance Motivation Scale in learning mathematics was 36.90. The Effectance Motivation Scale is the subject's active involvement or lack of involvement in solving mathematical problems. Examples of these responses included that, " I like math puzzles" and " When a question is left unanswered in math class, I continue to think about it afterward. " Examples of those who marked disagreement indicating a favorable attitude were " I would rather have someone give me the solution to a difficult math problem than to have to work it out for myself, " and " I am challenged by math problems I can't understand immediately." As Fennema-Sherman discussed (1976), preservice elementary teachers' focalization on exploration with confidence is lower partially due to the lack of encouragement of students to focus on problem solving skills.

Concerning beliefs about mathematics, mathematically people tend to feel helpless, fearful, insecure, inferior, and not confident about their mathematics ability (Dodd, 1992). These feeling coincided with beliefs about themselves; for example, they are just not good at mathematics and they could never work hard enough to do mathematics well. Many believe they lack an understanding of mathematics and that mathematics is not useful. There have been numerous strategies suggested for mathematics teachers to decrease their students' mathematics anxiety. Creating a supportive environment has been advocated as being of prime importance (Morris, 1981). Dodd (1992) suggested a process-oriented approach with games and activities, while other researchers saw positive benefits from the use of concrete materials (Frank, 1990). Larson (1983) advocated that instructors demonstrate the usefulness of mathematics in relation to other subjects, use diverse ways to solve problems, and use different instructional strategies.

The NCTM Standards (NCTM, 1989 b, 1991, 1995, 2000) have created a vision of what mathematics should and could be. It should help produce confident and capable problem solvers with teachers being the key to the realization of this vision. Merseth (1993) states that teachers' attitudes and beliefs can greatly influence their pedagogical practices in teaching mathematics. According to the Professional Standards (NCTM, 1995), instructors of preservice mathematics teachers should model the instructional practices and positive dispositions eventually needed by preservice teachers if reform is to occur. This includes engaging students in appropriate mathematical tasks to foster problem solving and mathematical discourse, using group work, and demonstrating beliefs and attitudes about learners and mathematics consistent with the Standards. Since three courses in the mathematics sequence for preservice elementary school teachers are probably the last opportunity to influence preservice teachers' attitudes toward mathematics, it must be designed to build confidence, help alleviate math anxiety, and promote effective teaching and learning as outlined in the Standards (NCTM, 2000)

The Number Sense Test (NST) score in this study was significantly correlated with the Mental Computation Test (MCT) score and the Written Computational Test (WCT) score. However, the mean score on the Mental Computation Test and the Written Computation Test were low when compared with sixth and eighth grade students in Taiwan (Yang, 1997). It was reflected in the preservice elementary school teachers in this study who displayed an underdeveloped number sense, particularly those who experienced difficulties with performing mental computation and written computation skills (NCTM, 1989 a ; Kaminski,1997).

We should encourage the development and practice of estimation skills. For example, have preservice teachers identify everyday and work situations for which estimates may be more appropriate than exact answers, reinforcing the notion that estimating is a valuable skill, not merely something you do when you don't know how to compute an answer the right way. Discuss reasons for need to estimate and inevitable the trade off between the benefits of estimating and the possible cost in error due to lack of precision. Stress that there are no right or wrong estimates, only ones closer or farther from a computed answer, and that the importance of the degree of exactness depends on the requirements of the situation.

We should also emphasize the use of mental mathematics as a legitimate alternative computational strategy and encourage development of mental skill by making connections between different mathematical procedures and concepts. There are two reasons for encouraging the development of mental mathematics skill: its practical usefulness and its educational benefits. Often a quick response to an everyday or workplace number-laden situation is excepted or required. Perhaps more important is the educational benefit from working with mental mathematics since it requires developing a facility for moving between equivalent representations of quantities and on understanding the connections between procedures.

Also, preservice teachers need to view computation as a tool for problem solving, not an end in itself. While the acquisition of computation skills is important, it is of little use unless students also develop the ability to determine when certain computations are appropriate and why. Time spent mastering computational skills should balanced with time spent talking about the applications of computations and enabling students to grapple with the application of their skills in both familiar and less familiar situations. By situating learning of skills in contexts, the skills mastered are generalizable and useful (Strasser, Barr, Evans & Wolf, 1991).

Another way to reinforce connections between computational skill and applications is by asking preservice teachers to write their own problem stories targeting a particular procedure. Student-generated problems can be shared with other class members, mixing problems suggesting different computation procedures so that preservice teachers will have opportunities to select appropriate solution methods.

This is an unfortunate trend, since instructors of undergraduate mathematics courses for preservice elementary teachers are overwhelmed with increasing duties and responsibilities. For example, they have to work with the curriculum and assessment demands of the course, help students learn how to compute algorithms correctly and they must work to nullify, if not change, the negative attitudes and negative effects many preservice elementary teachers have toward mathematics and teaching mathematics.

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