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The Effect of Learning Environments Based on Problem Solving on Students' Achievements of Problem Solving

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Abstract

Problem solving is recognized as an important life skill involving a range of processes including analyzing, interpreting, reasoning, predicting, evaluating and reflecting. For that reason educating students as efficient problem solvers is an important role of mathematics education. Problem solving skill is the centre of mathematics curriculum. Students' gaining of that skill in school mathematics is closely related with the learning environment to be formed and the roles given to the students. The aim of this study is to create a problem solving based learning environment to enhance the students' problem solving skill. Within this scope, students' practiced activities and problems that provide them to proceed in Polya (1945)'s problem solving phases and throughout the study, students' success in problem solving have been evaluated. While experimental group students received problem solving based learning environment performed, control group students have continued their present program in this quise-experimental study. Eleven problem solving activities were given to the students at the beginning, middle and end of the study and the students' performances were analyzed based on problem solving phases. The findings illustrated that the experimental group students' success in problem solving activities has increased while the control group students' success has not changed significantly.

Keywords: Mathematics Education, Problem Solving, Polya's Problem Solving Phases.

Introduction

An In our everyday lives, we use problem-solving skills. Also, most of us have to make daily plan, make decisions in our business and manage our budget. All of these events require logical thinking and also problem solving skill (Weidemann, 1995). Problem solving is

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recognized as an important life skill involving a range of processes including analyzing, interpreting, reasoning, predicting, evaluating and reflecting (Anderson, 2009). For these reasons one of the aims of mathematics teaching is to educate students as efficient problem solvers (Baki, 2008). Therefore, problem solving is considered as a central to school mathematics. It is highlighted in reform documents by National Council of Teachers of Mathematics (NCTM, 1989, 2000) as a key factor of change in mathematics education. NCTM (2000) states that students should be given chance to apply and adapt a variety of appropriate strategies to solve problems; and monitor and reflect on the process of mathematical problem solving in instructional programs during the problem solving process. Similarly, Kilpatrick, Swafford, and Findell (2001) argue that problem solving provides an important context for students to learn numbers and other mathematical terms and problem-solving ability is enhanced when students have opportunities to solve problems themselves and to see problems being solved. Thus, problem solving is important as a way of doing, learning and teaching mathematics. Therefore, preparing mathematics curriculum in the centre of problem solving appears to be important.

Problem Solving in the Reform Movement

Over the past decades, there have been many changes in mathematics teaching. There are some foundations in that period of change such as the National Council of Teachers of Mathematics (NCTM, 2000), the National Research Council (NRC, 1989) as well as the Third International Mathematics and Science Study (TIMSS) and the Ontario Mathematics Curriculum (Ontario Ministry of Education and Training, 1997). These foundations put an emphasis on problem solving in mathematics learning. In compliance with NCTM, problem solving is an integral part of all mathematics learning. So, problem solving should not be an isolated part of the curriculum" (NCTM, 2000). At the same time, the teacher has an important role in the development of students' problem solving skill and the teachers must choose problems that engage students (NCTM, 2000). Similarly, NRC states in its report which was published in 2001 that problem solving ability is enhanced when students have opportunities to solve problems themselves and see problems being solved. Problem solving also provides opportunities with teachers to assess students' performance (Kilpatric, et al., 2001).

TIMSS, providing trend data on students' mathematics and science achievement from an international perspective, gives mathematics educators many educational implications. TIMSS data show higher mathematics achievement when teacher emphasize reasoning and problem solving activities (Mullis, et al., 2000). According to TIMSS, the students in Japan are more successful than the students in US and Canada. The factor behind that difference is that while 49% of the teachers in Japan emphasize reasoning and problem solving, this rate for the teachers in US and Canada is 18% and 13% successively (Mullis et al., 2000). Therefore, a correlation between problem-solving and students' achievement in mathematics is seen clearly.

Based on NCTM standards, problem solving has been emphasized in the curriculum that has been prepared in Ontario State in Canada. That curriculum describes problem solving as a skill that should be along with mathematics teaching. However, in the curriculum students should use problem solving methods not only in problem solving task in mathematics, but in other appropriate circumstances. They should also use problem solving methods extensively as a means of developing the full range of mathematical skills and knowledge in all strands (Ontario Ministry of Education and Training, 1997).

In the light of changes that have occurred in mathematics teaching, in Turkey mathematics curriculum of primary and secondary schools has been renewed in 2005 with the reforms in education. The mathematics curriculum bases on the principle that every child can learn mathematics and lays stress on basic mathematical skills such as problem solving, communication and reasoning. As one of the most important goals, the national curriculum by Turkish Ministry of Education (MEB) defines problem solving as not a subject matter to be taught but a process helping students to gain essential skills to solve problems. How the students solve the problem, which data contribute to that solving, how they represent that problem (table, figure, concrete object etc.), how the strategy that they chose and representation manner make the solution easier, and how the students explain the solution to their peers should be emphasized in that curriculum (MEB, 2006).

Role of Problem Solving in Mathematics Education

Problem solving has an important role in mathematics teaching and it is also been the centre of mathematics programs (NCTM, 1989; NCTM, 2000, MEB, 2006; Howland, 2001). Thus improving the students' problem solving skills have been emphasized in the program of mathematical studies. Problem solving enables students to do mathematics and to comprehend mathematics meaningfully (Van de Walle, 2001).

When it is taken into account that permanent learning takes places at social surroundings, Artzt and Armour-Thomas (1992) state that problem solving settings that based on class discussion gives students a chance to analyze their thought, students can share and compare their thought with their peers in that setting and that setting also makes discussion of different ideas possible. It is pointed out that problem solving has some advantages such as developing students' responsibility, directing them to searching, raise their interest for learning, providing students with permanent learning, increasing students' motivation etc. (Fisher, 1990). Therefore, process of learning should help students to develop a sense of being responsible for their learning. An effective learning process also helps students to grow their interest in learning and promote students to share ideas to each other and finally make learning as stable as possible

Teachers have many opportunities to build knowledge about teaching problem solving and using problems as a focus of learning in mathematics (Cai, 2003). When used as methods for instructional method, it allows students their own understanding and takes some ownership for their learning. Additionally, students perceive an active role in problem solving activities by which their thoughts and ideas become a focus of learning activities (Annable, 2006). In addition, Schoenfeld (1992) advocated that problem solving based learning environments enables students to have deep mathematics knowledge and gives them the opportunity of pursue their own mathematics learning enthusiasm. Hiebert and Wearne (2003) point out that the process of problem solving improves and enrich students' mathematical perception.

Annable (2006) has taught mathematics to6th grade students on the basis of problem solving so as to enhance their problem solving and critical thinking skills. He also revealed that when problem solving strategies are stressed in the learning environment, and the students discuss the problems with their peers, students' skill of problem solving advances. Similarly, Perveen (2010) has carried out a study on the effect of problem solving on the success of 10th graders. For that experimental study based on Polya (1945) (heuristic phases of the problem-solving approach) was performed and the students in experimental group was taught by problem solving approach, thereafter the study revealed that the academic achievement of the students in experimental group is much better than the students in control group. Schoenfeld (1989) also performed a study that based on development of high school students' metacognitive skills. According to the result of that study, class discussion oriented teaching gives students opportunity to express their ideas and share those ideas with their peers. Therefore, it is necessary to design learning environment which is suitable for the development of students' problem solving skill. Moreover, it is relatively important for students to share their thoughts with their peers in problem solving environment.

Purpose of the study

The purpose of this study is to develop students' problem solving skill by designing learning environment that based upon problem solving. Within this scope, learning environments where the students have the chance to pace Polya (1945)'s problem solving phases have been designed and students' problem solving and their progress have been evaluated.

Method

Participants

This study's sample consists of 53 7th grade students. 27 of them are experiment group and 26 of them are control group. These students' 6th grade mathematics final exam results were compared and there was no significant difference ($t_{(51)} = 1.298$, $p>.5$) between the groups. Thus, the experimental and control group students' math competencies were observed to be similar before the study. Students in experimental and control groups were taught by the same teacher.

Study context

Turkish education is compulsory for every Turkish citizen from the age of six to the age of eighteen, regardless of gender and socio-economic status, and is free of charge in state schools across the country. The education system comprised four years of primary school, four years of middle schools, and four years of secondary schools. The secondary school where the study carried out is located in an area where middle income families live. Furthermore, mathematics teacher who takes part in this study has a master and degree in mathematics teaching and currently she was pursuing her doctorate degree by the time of the study. The teacher also participated in professional development programs and has qualifications that can make him an expert in his area. The teacher graduated from mathematics teaching programme in 1999 and has been working as a secondary school teacher since that date. Thus, the teacher can practice mathematics curriculum efficiently in that process. The teacher in that study is regarded to be close to current development in mathematics teaching area and can adapt them easily as he continues to doctorate programme. For all of these reasons, teacher's readiness for innovation is really important for the study to be performed effectively. The teacher gives importance to student centered education and gives place to students' thought in the class.

Procedure

Experimental Group. As the learning environment is very important for the development of students' problem solving skills, how the learning environment should be was decided first. It was necessary to determine problems that the students would discuss and solve in that learning environment. The teacher who would teach control and experimental groups was chosen and the gains of the curriculum for 7th graders were defined and how they should be performed in the class were decided. How the problems should be dealt within the classroom was discussed with the teacher who would teach these participating groups. The worksheets that would to be given the students and the phases of problem solving were arranged and the instructions in those worksheets were outlined. Five pilot studies were done on teacher's gaining experience, context of worksheets, deciding on the problems that would be used in the learning environment and researcher's gaining experience. In pilot study, researcher made observation and took observation notes for teacher to use the problems effectively in the classroom. In parallel with observational data of researcher and opinion of participating teacher, final shape of the problems that would be used in the learning environment was structured. Nine problems used in the learning environment were determined with the teacher. The problems were prepared considering their being applicable to multiple ways of solutions and results, directing students to discussion, involving problem solving skills and being suitable for concept learning. A sample problem used in the study was given in figure 1.

Miss Yasemin goes to the grocery store to supply her three months rice need. She wants to buy 16 kg rice. However, rice is sold in bags of different sizes, as shown in the figure below. Decide on which bags Miss Yasemin should buy to make the most effective shopping.

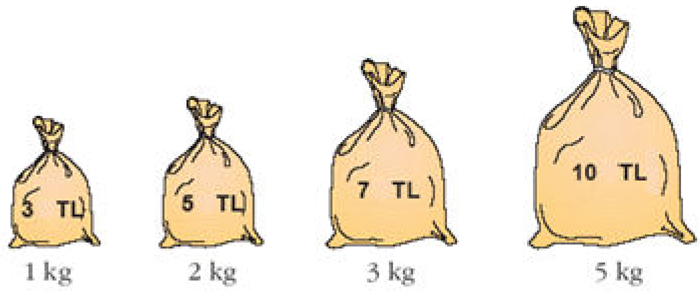

Figure 1. A sample problem (TL represents Turkish liras).

In the second part of the study, nine problems were carried out on experimental group students in parallel with Polya's (1945) heuristic phases of the problem-solving approach to improve students' problem solving skills. The mathematics teacher in the experiment group implemented the Connected Mathematics Project's (CMP) instructional model: "*launch, explore and summarize*" in their teaching. This model of instruction involves three main phases. In the *launch* phase, the teacher explained the problem to whole class and tried to make students understand and raise their interests for the problem situation. In the *explore* phase, students search and try to find possible solutions for the problem situations either individually or in groups during that process. In that phase, while students were dealing with solutions of the problems, the teacher were observing them and giving them tips in some situations. The teacher tried to form the basis for the class discussion by analyzing the solutions and process. In the *summary* phase, students discuss their solutions and share their strategies they used to reach a solution. Students decide on the most appropriate way of solution and solve the problems with the help of their teacher. They will also appreciate other approaches proposed by their peers to the problem, and can see ways to enhance their own strategies. The teacher also offers guidance and suggestions for a deeper understanding of the concepts and more effective and efficient problem solving strategies (Reys, Reys, Lappan, Holliday, & Wasman, 2003).

Control group. The existing mathematics curriculum was applied to the control group. That curriculum aims to develop students' problem-solving, reasoning, communication skills by means of activity-based approach (MEB, 2006). Teaching was preceded in accordance with the examples in teacher's book. Both experimental and control groups were taught by the same teacher. Teaching process was carried out by providing the terms and definitions in mathematics teaching curriculum.

Data collection instrument

In order to evaluate students' problem solving performance 11 problems that involve the subjects in 7th grade curriculum were used as a means of collecting data. The problems which would give use chance to evaluate their problem solving skills and that process were carried out in line with the teacher's view. The problems were separated into three different groups and were distributed to experimental and control groups and students' solving were taken under review. The problems that were prepared with participating teacher have been considered to contain a different solution methods and problem-solving skills. Both experimental and control groups practiced three of the problems in initial phase, four of them in middle phase and the other four problems in final phase. These problems were applied to the students and were assessed. Among the problems used as a means of data collecting tool chocolate problem is shown in table 1.

Table 1. *Sample Problems in data collection instrument*

Chocolate Problem

Different sizes of chocolate sold in the supermarket are presented below. Prices in the table are determined by the number of packet type and chocolate. Think about which package you choose as a customer? Please do and explain the problem.

Package type	Numberof Chocolate	Price
Small Package	2 items	36 kuruş
Middle Package	5 items	1 TL
Family Size Package	12 items	2.2 TL
King Size Package	18 items	3 TL

Data analysis

Solutions that the students found out for 11 problems consisted of the data. Therefore, there have been 11 different problem worksheets for both control and experimental groups at the end of the study. The students' problem worksheets were scored with the help of a scale prepared by Northwest Regional Educational Laboratory Mathematics and Science Education Center (NWREL) and used to evaluate other problem solving studies (URL, 2006). According to the scale, problem solving phases were given digital numbers as one, two, three, and four points (understanding the problem, developing a plan, carrying out the plan, and looking back).

In the problem phase of understanding the problem, if the students understand the problem completely, they are valued with four points. If they understand a little, they are valued with three points. If they don't understand, they are valued with two points. If they don't pay any effort to understand the problem, they are valued with one point. In the phase of developing a plan, if the students choose a suitable strategy to lead them the solution, they are given four points. If they choose only a piece of the strategy that helps them solve the problem, they are given three points. If they choose an unsuitable strategy, they are given two points. If they don't choose any strategies, they are given one point. In the phase of carrying out the plan, if the students reach the correct solution, they are given four points. If they find some parts of the solution correctly, they are given three points. If they find out an incorrect solution, they are given two points. If they don't reach any solutions, they are given one point. In the phase of looking back, if the students confirm the results logically, they get four points. If the students confirm the result partially, they get three points. If they don't know how to confirm the result, they get two points. If they don't confirm the result, they get one point.

Consequently, the total score that the students get from 11 problems and the scores from each phases of study were used to assess each student's problem solving skills. To determine the students' problem solving success, the points that the students got from the problems applied to them in three different times were calculated. The average of each student's points that they got from the first three of the problems were calculated as the points that they got from the first phase. The average of fourth, fifth and sixth problems' points were calculated as second phase points and the average of the points taken from the last four of the problems were regarded as third phase points. So, the points which each student got from three stages were obtained. To compare the results of control and experimental groups, t test and analysis of covariance (ANCOVA) test were applied.

Results

In this section, the development of problem-solving success of the experimental and control group students who participated in the study was examined. Students' results from total of 11 problems used as a data collection tool were calculated separately for each problem. Problems were applied to the students as separate groups like three, four and problem four. Scores of students that they got from the each group of problem were calculated and students' problem solving scores were obtained. Problem-solving scores of students in the experimental and control groups were compared using independent t test and ANCOVA.

Experimental and control groups students' scores of first and second application problems were compared using the t test for independent groups obtained data were summarized in table 2.

Table 2. *The results of paired sample t test between experimental and control groups of students' total scores in first and second application.*

Variables	Groups	*N*	$\overline{X}$	*SD*	*df*	*t*	*p*
First application	Experimental	26	8.942	2.421	51	0.836	.407
	Control	27	8.481	1.501			
Second application	Experimental	26	9.346	1.547	51	3.011	.004
	Control	27	8.157	1.321			

According to the data in the table; while there was no significant difference between experimental and control groups students' scores that they got from the problems in the first practice ($t_{(51)}$=.836, p>.05), in the second practice differences were found between the scores of the student ($t_{(51)}$=3.011, p>.05). For that reason, an explicit difference between experimental and control groups students emerged towards the end of the study. ANCOVA test was carried out in order to evaluate experimental and control group's students' success of problem-solving at the beginning and end of the study. ANCOVA results that the students obtained from the first and third performance are shown in table 3.

Table 3. *Descriptive analysis of experimental and control groups of students' scores in first and third application.*

Groups	*N*	$\overline{X}_{\text{first aplication}}$	$\overline{X}_{\text{third application}}$	$SD_{\text{third application}}$	$\overline{X}_{\text{corrected third application}}$
Experimental	26	8.942	10.012	0.989	9.903
Control	27	8.481	8.197	1.114	8.303

According to the data in the table; for the first practice applied in the beginning of the study, experimental group students' score average is $\overline{x}$ = 8.942; on the other hand control group students' score average is $\overline{x}$ = 8.482. However, for the third performance applied at the end of the study experimental groups students' score average is $\overline{x}$ = 10.012 and the control group students' score average is $\overline{x}$ = 8.197. These results show that while experimental group students' problem solving success has increased, control group students' problem solving success has decreased. In order to evaluate the groups' success of problem solving in first and third practice, ANCOVA test was applied and the results are given in the table 4.

Table 4. *ANCOVA results of experimental and control groups' scores in the third application which was organized according to the first application.*

Source of variance	Sum of Squares	*df*	Mean Square	*F*	Sig.(*p*)
First application	22.074	1	22.074	31.817	.000
Group	33.021	1	33.021	47.597	.000
Error	34.688	50	.694		
Total	4477.819	53			

According to the data given in the table it seen that adjusted difference between problem-solving scores of problem-solving groups is statistically significant ($F_{(1\text{-}50)}$ =47.597, $p<.05$). It shows that problem solving based method of instruction improves the students' problem solving success positively.

As shown in table 5, multiple and repetitive tests were carried out in order to observe the change depending on the effects of problem solving based method of instruction on problem solving success.

Table 5. *The results of repeated measures analysis about the success of problem solving*

Impact	Wilks' λ	*F*	*SD*	*p*	ω^2	Strength
Time	.91	2.42	2	.10	.09	.47
Time*Experiment	.84	4.82	2	.01	.16	.77

According to the data in the table; it is seen that there is not any significant difference on interference program for time, at the level of Wilks' λ = .91, F = 2.42, $p>.05$.On the other hand, it is observed that for same variance time* experiment effect is significant at the level of Wilks' λ = .84, F = 4.82, $p<.05$. As regards these results, even if some or all of the students in control group were in experimental group and some or all of the students in experimental group were in control group, the improvement in problem solving success of the students who were taught in the environment depending on problem solving has progressed remarkably. Therefore, if the study had continued in the same way, while there wouldn't be any change on the problem solving success of the students in control group, the rise in the success of the students in experimental group would continue.

In the process of students' problem solving, students' scores in each step were calculated and evaluated statistically in order to compare them according to Polya's problem solving phases. The scores that the students in control and experimental groups took in the process of understanding the problem were compared with the help of independent t test. Obtained results were given in table 6.

Table 6. *The results of paired sample t test between experimental and control groups of students' scores in first and second application about the phase of understanding the problem*

Variables	Groups	*N*	$\overline{X}$	*SD*	*df*	*t*	*P*
First application	Experimental	26	2.756	.467	51	.175	.862
	Control	27	2.775	.308			
Second application	Experimental	26	2.878	.209	51	1.925	.060
	Control	27	2.731	.329			

Regarding the results in the table; there isn't any significant difference between control and experimental groups students' scores that the students got for understanding the problem phase in the first application ($t_{(51)}$= .175, p>.05). Similarly, there isn't any significant difference between control and experimental groups students' scores that the students got for understanding the problem phase in second application ($t_{(51)}$= 1.925, p>.05). However, when the scores that the students in control and experimental groups got in comprehension phase were examined, control group students' average is $\overline{x}$ = 2.736 for the first application and the raise for the second application is $\overline{x}$ = 2.878, and in the control group $\overline{x}$ = 2.775 for the first application and $\overline{x}$ = 2.731 for the second application it noticed that even if it is just a bit, there is a drop in the scores.

Descriptive statistics of scores that the students got from the first and third application in understanding the problem phase are given in table 7.

Table 7. *Descriptive analysis of experimental and control groups of students' scores in first and third application about understanding the problem*

Groups	*N*	$\overline{X}_{\text{first aplication}}$	$\overline{X}_{\text{third application}}$	$SD_{\text{third application}}$	$\overline{X}_{\textit{corrected}\text{ third application}}$
Experimental	26	2.756	2.951	.15842	2.952
Control	27	2.775	2.722	.25737	2.722

According to data in the table; while the control group students' average scores taken from the problems in the first application is $\overline{x}$ = 2.756 for understanding the problem phase, control group students average score is $\bar{x}$ = 2.775.At the end of the study average scores of the problems in the third group for experimental groupis $\bar{x}$ = 2.951, on the other hand, control group students' average is $\bar{x}$ = 2.722. All these findings show that experimental group students who were taught in the environment based on problem solving made a progress in understanding the problem phase. ANCOVA results for the third application are given in table 8.

Table 8. *ANCOVA results of experimental and control groups' scores in the third application which was organized according to the first application about understanding the problem*

Source of variance	Sum of Squares	*df*	Mean Square	*F*	Sig.(*p*)
First application	.004	1	.004	.080	.778
Group	.702	1	.702	14.955	.000
Error	2.346	50	.047		
Total	428.993	53			

According to data given in table; it is clear that there is a significant difference between adjusted problem solving scores of the groups related with groups' understanding the problem phase($F_{(1\text{-}50)}$=14.955, p<.05). This result indicates that the students' success related to understanding the problem phase has improved positively throughout the study.

The scores that the control and experimental group's students got in the planning phase were compared using t test. Obtained results were given in table 9.

Table 9. *The results of paired sample t test between experimental and control groups of students' scores in first and second application about the phase of developing a plan.*

Variables	Groups	*N*	$\bar{X}$	*SD*	*df*	*T*	*p*
First application	Experimental	26	2.384	.664	51	.987	.329
	Control	27	2.224	.494			
Second application	Experimental	26	2.509	.465	51	2.864	.006
	Control	27	2.151	.445			

According to data in table; it is seen that there is no difference between the experimental and control groups students' average score in the first application of plan phase ($t_{(51)}$ = 0.987, p >.05). However, experimental and control groups students' average scores appear to be different from each other($t_{(51)}$ = 2.864, p <.05). When the experimental and control groups students' scores in plan phase were analyzed, the raise in the scores of control groups students is clear ($\bar{x}$ =2.384 for the first application and $\bar{x}$ = 2.676 for the third application).

On the other hand, there is a decrease in control groups students' scores ($\bar{x}$ = 2.224 for the first application and $\bar{x}$ = 2.197 for the third application).

Descriptive statistics of experimental and control groups students' scores from the first application carried out at the beginning and the third application carried out at the end of the study which is about plan phase can be seen in table 10.

Table 10. *Descriptive analysis of experimental and control groups of students' scores in first and third application about developing a plan*

Groups	*N*	$\bar{X}_{\text{first aplication}}$	$\bar{X}_{\text{third application}}$	$SD_{\text{third application}}$	$\bar{X}_{\textit{corrected}\text{third application}}$
Experimental	26	2.384	2.676	.26908	2.653
Control	27	2.224	2.197	.29612	2.220

According to data in the table; while control group students' average score taken from the problems in plan phase is $\bar{x}$ = 2.384, experimental group students' average score is $\bar{x}$ = 2.225. Moreover, experimental group students' average score from the problems in third application is $\bar{x}$ = 2.676; however, control group students' average score is $\bar{x}$ = 2.197 at the end of the study. This point out that the students of experimental group in which problem based learning environment is conducted showed progress in relation to the planning phase while the control group did not alter in the same way. ANCOVA results of groups' third application are given below.

Table 11. *ANCOVA results of experimental and control groups' scores in the third application which was organized according to the first application about developing a plan*

Source of variance	Sum of Squares	*df*	Mean Square	*F*	Sig.(*p*)
First application	1.237	1	1.237	21.682	.000
Group	2.436	1	2.436	42.699	.000
Error	2.853	50	.057		
Total	320.701	53			

According to data in table; it is understood that the difference between the groups' adjusted problem solving scores about developing a plan is significant ($F_{(1\text{-}50)}$ =42.699, p<.05). This shows that learning environment based on problem solving effects students' success related with developing a plan positively.

The scores that the experimental and control groups students got in the carrying out the plan phase has been compared by conducting t test. Obtained results were presented in table 12.

Table 12. *The results of paired sample t test between experimental and control groups of students' scores in first and second application about the phase of carrying out the plan*

Variables	Groups	*N*	$\overline{X}$	*SD*	*df*	*t*	*p*
First application	Experimental	26	2.224	.739	51	1.482	.145
	Control	27	1.948	.593			
Second application	Experimental	26	2.217	.573	51	3.010	.004
	Control	27	1.759	.536			

According to data in the table; in the first application experimental and control groups students didn't get significantly different scores in carrying out the plan phase ($t_{(51)}$= 1.482, p>.05). On the other hand, in the second application significantly different scores were seen between experimental and control group students. ($T_{(51)}$= 3.010, p<.05).

The descriptive statistics of scores that the control and experimental group students got from the carrying out the plan phase's problems performed in the beginning and end of the study can be seen in table 13.

Table13. *Descriptive analysis of experimental and control groups of students' scores in first and third application about carrying out the plan*

Groups	*N*	$\overline{X}_{\text{first aplication}}$	$\overline{X}_{\text{third application}}$	$SD_{\text{third application}}$	$\overline{X}_{\text{corrected third application}}$
Experimental	26	2.224	2.461	.410	2.433
Control	27	1.948	1.910	.460	1.938

According to data in the table; in the first application experimental group students' average score taken from the problems in the carrying out the plan phase is $\overline{x}$ = 2.224; however, control group students' average score is $\overline{x}$ = 1.948. Furthermore, at the end of the study experimental group students average score taken from the problems in the third application is $\overline{x}$ = 2.461 and control group students' average score is $\overline{x}$ = 1.910.It is obvious that experimental group students who were taught in problem solving based learning environment has improved in problem application phase. In the third application conducted at the end of the study the relation between the groups about the planning phase has been described with ANCOVA test and obtained results are in table 14.

Table 14. *ANCOVA results of experimental and control groups' scores in the third application which was organized according to the first application about carrying out the plan.*

Source of variance	Sum of Squares	*df*	Mean Square	*F*	Sig.(*p*)
First application	.846	1	.846	4.763	.034
Group	3.086	1	3.086	17.380	.000
Error	8.879	50	.178		
Total	265.812	53			

According to data in the table; it is seen that the difference in the adjusted problem solving scores related with groups' carrying out the plan phase is significant ($F_{(1-50)}$ =17.380, p<.05).These points out that learning environments have a positive influence on the success of students about carrying out the plan phase and experimental group students show significant difference when compared with control group.

Experimental and control groups students' score that the students got from the looking back phase have been compared by applying independent t test. Obtained results were presented in table 15.

Table 15. *The results of paired sample t test between experimental and control groups of students' scores in first and second application about the phase of looking back*

Variables	Groups	N	$\overline{X}$	SD	df	T	p
First application	Experimental	26	1.717	0.475	51	1.120	.268
	Control	27	1.564	0.514			
Second application	Experimental	26	1.759	0.460	51	3.464	.001
	Control	27	1.376	0.336			

According to data in the table; it is apparent that there is no difference between the experimental and control groups students' scores for the first application of looking back phase ($t_{(50)}$ = 1.120, p>.05). In the second application the difference between experimental and control groups students' score is clear ($t_{(51)}$ = 3.464, p<.05).

Descriptive statistics of experimental and control groups students' looking back phase scores that they got from the problems applied at the beginning and end of the study can be seen in table 16.

Table 16. *Descriptive analysis of experimental and control groups of students' scores in first and third application about looking back*

Groups	N	$\overline{X}_{\text{first aplication}}$	$\overline{X}_{\text{third application}}$	$SD_{\text{third application}}$	$\overline{X}_{\text{corrected third application}}$
Experimental	26	1.717	1.826	.322	1.792
Control	27	1.564	1.342	.387	1.376

According to data in the table; when the scores for looking back phase have been analyzed throughout the study, it can be seen that there is not an observable difference between experimental and control groups. Experimental group students' average score from the first application of the problems is $\overline{x}$ = 1.717 whereas control group students' average score is $\overline{x}$ = 1.564. Moreover, at the end of the study experimental group students' average score from the problems of third application is $\overline{x}$ = 1.826 and control group students' score is

$\bar{x}$ = 1.342. The relation between the groups about the looking back phase is defined with ANCOVA test and is presented in table 17.

Table 17. *ANCOVA results of experimental and control groups' scores in the third application which was organized according to the first application about looking back*

Source ofvariance	Sum of Squares	df	Mean Square	F	Sig.(p)
First application	1.920	1	1.920	20.952	.000
Group	2.221	1	2.221	24.233	.000
Error	4.583	50	.092		
Total	141.951	53			

According to data in the tablet here is a significant difference between the groups' adjusted problem solving scores of looking back phase ($F_{(1\text{-}50)}$ =24.233, $p<.05$). This makes it obvious that problem solving based learning environment improves the students' success about looking back phase.

Discussions and Conclusion

This study focused on helping students to develop their problem solving skills and achievement in mathematics through a learning activity designed by Polya's (1945) heuristic phases of the problem-solving approach. The study revealed that while the experimental group students' achievements of problem solving increased, control group students' achievement on problem solving have not changed significantly. This difference might be attributed to the learning environment applied to the experimental group students. Polya's problem solving phases and problem solving strategies were discussed in that learning environment based on problem solving. In this learning environment it is aimed to provide students with a heuristic problem solving experience. Barrett and Compton (2003) emphasized that an effective problem solving experience helps students expand their thinking, encourages persistence through difficulties, and empowers them to navigate their own learning. For that reason problem solving experiences that will be provided for the students may likely cause to the development of students' problem solving success, thus their skills will improve. It is observed that both the experimental and control groups students' average scores in problem solving success test were similar at the beginning of the study, the experimental group students' average scores in problem solving test applied in the middle and end of the study have increased while the data did not show a similar change for the control group students. When we look at the findings of the relevant studies aiming to develop students' problem solving skills, we see a similar pattern. There is a consensus among these studies that problem solving strategies hold a great promise to enhance students' problem solving skills in mathematics learning (Keller, 1990; Lee, 1982; Yazgan & Bintaş, 2005; Verschaffel, De Corte, Lasure, Vaerenbergh, Bogaerts, & Ratinckx, 1999; Garnette, 1990; Altun, Memnun & Yazgan, 2007) improves and their problem solving success increases.

This study examined the students' development in Polya's (1945) problem solving phases and concluded that in the understanding the problem phase, the experimental group students' success in the problems applied to them rise significantly; however, there is no difference in the success of control group students in that phase. In the understanding the problem phase the experimental group students used complex skills of dealing with shapes, tables, diagrams to solve their problems. On the other hand, control group students have only written the data and asked stable questions for the problems. That inclination of experimental group students results from the emphasis that is for the importance of understanding the problem phase in the problem solving process in the learning environment and students' discussion of different strategies for analyzing the problems. In the study of Rose (1991), it is understood that the students are not usually aware of the knowledge which helps them for the process of problem solving in understanding the problem phase. In this respect, the things that can contribute to the solution process have been discussed in the learning environment where the experimental group students were successfully employed. Therefore, the students' success in the understanding the problem phase have advanced by this research.

There was no statistically important difference between the experimental and control groups students' achievements in the problem test applied at the beginning of the study, on the other hand, in the problems applied in the middle and end of the study there was a significant difference toward the experimental group students. At this stage, the students have difficulty in the selection of strategies which help them for the solution. This result has coincided with Cmajdalka's result (1999). It can be said that in the learning environments there are various strategies to reach the problem's result and due to the discussion of specific strategies that the students use in the classroom, there is a change on behalf of experimental group students.

Similarly, in phase of carrying out the plan the experimental group students' success in the problems administrated throughout the study has progressed. At this stage, both the control group and the experimental group students have made errors in the process of solution. The students have shared the solution process and activities with their peers and the calculation errors made in the process of carrying out the plan have been emphasized in the learning environments. Thus, it is realized that towards the end of the study there were reductions in the experimental group students' errors. However, it is concluded that the students are insufficient in looking back phase. In looking back phase, when the experimental and control group students' average scores that they took from the first, second and third groups of problems were analyzed, even if it is not statistically significant, while there is an increase in the average scores of the experimental group students, the control groups' scores on this phase showed a decrease. This result can be attributed to the students' ineffective use of looking back phase and their getting further away from that tendency. Although various solution process and strategies were discussed throughout the looking back phase in the learning environment, it is seen that the student have not used looking back phase effectively. Especially, the students reaching the conclusion by setting up equations decided accuracy of the result by putting the found value in the equation in looking back phase. This situation is observed to be more common among the control group students, while it was found less among the students in the experimental group. That can result from the situation that throughout the study the experimental group students give importance to problem solving skills by using various ways for looking back phase in the learning environment. In Mubark and Zaman's study (2012) it is found out that the students had difficulty in looking back phase. The result obtained from this study matches up with Mubark and Zaman's results (2012).

This study includes problem solving activities that are applied to problem solving based learning environment and instructions that help students to successfully complete problem solving phases and also while the results were discussed, discussion regarding the problem solving phases was condensed. For this reason, at the end of the study it is emerged that the experimental group students were appeared to be more successful than control group students in preceding the problem solving phases throughout the application phase. The results in this study match up with the studies in literature (Nancarrow, 2004; Seaman, 1995; Stacey, 1992; Pouradavood, 2003; Pugalee, 2001; Diezmann, Watters & English, 2001).

Educational Implications

In the process of problem solving when the problem solving phases which Polya (1945) suggested are carried out successfully and efficiently, the students' problem solving skills and achievements improve significantly. Therefore, in mathematics education students should be provided with the activities to proceed in problem solving phases in the learning environments that are enriched with problem solving activities. The importance of taking systematic phases in problem solving process should be emphasized for the students. Besides, in this study students' various problem solving processes were discussed in the classroom and evaluations related with the proposed solutions were made. Thus, the problems that will be solved using different problem solving strategies should be discussed in the learning environment. The students should be given chance to evaluate their peers' proposed solutions in classroom discussions.

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