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A Story of African American Students as Mathematics Learners

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Abstract

Educational systems throughout the world serve students from diverse populations. Often students from minority populations (i.e. racial, ethnic, linguistic, cultural, economic) face unique challenges when learning in contexts based on the cultural traditions and learning theories of the majority population. These challenges often leave minority populations labeled as incompetent, unmotivated, and cognitively deficit. In the United States, African American female students are among minority populations who are often positioned as deficit when compared to the majority White population. This study investigates middle school African American female perceptions of themselves as learners and students' knowledge of the meaning of ratio, proportionality, and how to apply and explain their application of proportionality concepts by examining written problem solving strategies over a three-year period. Students' responses are analyzed according to the strategies they used to reach their final solution. The categories of strategies include no-response, guess and check, additive build up with and without a pictorial representation, and multiplicative. The majority of students in this study 86.5%, 69.2%, and 68.6% did not attempt or demonstrated no understanding in year one, two, and three respectively. Additionally, participants reported positive dispositions about themselves as mathematics learners.

Key words: Problem solving strategies, African American female students, Middle school, proportional reasoning.

Introduction

In countries around the world, schools serve very diverse populations. Diversity of population can serve as an asset within any school when each learner and the learners' knowledge is valued and integrated into the learning process. All to often, major disconnects exist between the curriculum and environment within schools and the lived experiences of minority learners (Malloy & Malloy, 1998; Martin, 2009). In the United States, this is true for all African American students but this paper focuses on the experiences of African American female students who are described as a 'double minority' because of their gender and race (Gray & Katada, 2008). Understanding the mathematical learning experiences of minority students can be instructive for other countries striving to meet the educational needs of *all* learners and pushing *all* learners to their maximum potential in mathematics education.

Disparities between White and African American students in mathematics achievement and instruction are widely considered a national dilemma (Johnson & Kritsonis, 2006) but there has been a lack of research examining the mathematical thinking of African American students (Lubienski & Bowen, 2000). In the research available, African American students are positioned as deficit, underachieving, unmotivated learners with inferior skills when compared to their White and Asian peers (Ladson-Billings, 1995; Reyes & Stanic, 1988; Tate & Rousseau, 2002). This positioning of African American student has fostered the image that failure among these students is the norm (Berry et al., 2011). This article is an attempt to legitimize the mathematical thinking of African American females through providing a description of their mathematical problem solving strategies and their perceptions of themselves as mathematics learners. Malloy and Jones (1998) found that studies focusing on mathematical problem solving have traditionally examined white students and typically do not report for African American students. Thus, there is little research addressing African American female students' mathematical learning or mathematical problem solving and hence the need for this study. As the mathematics education community moves forward, it is imperative that we recognize that we cannot talk about curriculum development, educational reform for all, or the needs and challenges in mathematics without

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empirical research that addresses and includes *all* students. We have much to learn from *all* students that sit within our classrooms.

Perspectives on Problem Solving and Problem Solving Strategies

Problem solving

Researchers and educators agree that problem solving is a crucial part of mathematical understanding and the substance of the mathematics discipline (Polya, 1981; NCTM, 2000). Problem solving is more than completing numerical operations; it is a way of thinking and involves using one's knowledge to generate new knowledge and to solve a problem that does not have an obvious solution. Problem solving is an important tool in students' development as mathematics learners, and successful problem solving is necessary for the development of mathematical understanding (NCTM, 2000). When students are allowed to grapple with situations that involve important mathematical concepts, they construct a clearer understanding of the mathematics (Kroll & Miller, 1993).

Problem solving strategies

Problem solving strategies serve as a guide in the problem solving process; although they do not guarantee a solution, they may provide a pathway to solutions (Gick, 1986). Problem solving strategies are "rules of thumb for successful problem solving, general suggestions to help an individual to understand a problem better or make progress towards a solution" (Schoenfeld, 1985 p.23). In essence, problem-solving strategies are "cognitive or behavioral activities" (Siegler, 1998, p. 191) employed by students to reach a problem solution. While some strategies are labeled as more sophisticated than others, the strategies students used and the successful application of these strategies provide insight into students' mathematical thinking and level of understanding (Cai, 2000). The literature (Hembree, 1992; English, 1993; Malloy & Jones, 1998; Silver, Shapiro, & Deutsch, 1993) documents a variety of strategies employed by students during mathematical problem solving. The development of mathematically sophisticated strategies is connected to students' ability to solve difficult tasks (Steinthorsdottir & Sriraman, 2009). Strategies employed by students during problem solving episodes provide vital information about students' reasoning process and their errors provide information about the characteristics of students' misconceptions and inability to reach a correct response (Cai, 2000; Erbas & Okur, 2012).

Perspectives on Proportional Reasoning

Proportional reasoning involves watershed concepts that lie at the center of middle school mathematics curriculum in the United States and is an important form of mathematical reasoning (Cai & Sun, 2002). Proportional reasoning is a prerequisite for the development of algebraic and other higher-level mathematical thinking (Hierbert & Behr, 1988; Inhelder & Piaget, 1958, Resnick & Signer, 1993). However, students are slow to attain mastery of these concepts (Steinthorsdottir & Sriraman, 2009). Students have difficulty recognizing and distinguishing the relationships that exist between ratios, rates, proportions and rational numbers (Behr, Harel, Post, & Lesh, 1992; Lamon, 1993; NRC, 2001; Lobato, J. et al., 2010). Students not attaining mastery of these concepts may serve as a roadblock to the development of higher-level mathematical thinking and hinder access to higher-level mathematics courses. Proportional reasoning is defined as more than the ability to set up missing value problems and solve them with mechanical strategies. Proportional reasoning involves students' ability to discriminate between proportional and non-proportional situations and recognize multiplicative relationships *within* and *between* ratios (In this paper ratios are defined as the relationship between two quantities that have two different measure space units) (Behr et al., 1992). People often identify proportional reasoning with the use of the cross-multiplication strategy but proportional reasoning involves much more. It involves a robust sense of rational numbers and proficiency in ratio sense, partitioning, and unitizing (Gomez, et al., 2002). Studies of students' development of proportional reasoning have identified three levels of strategy use, qualitative, build-up and multiplicative (Inhelder & Piaget, 1958; Resnick & Signer, 1993). Steinthorsdottir & Sriraman, 2009, conducted a study of the development of Icelandic girls proportional reasoning development. In their paper, they used to following tasks as a context to describe each strategy level.

> *It is lunchtime at the Humane Society. The staff has found that 8 eats eat 5 large cans of cat food. How many large cans of cat food would the staff members need to feed 48 cats? (In an algebraic equation: 8/5 = 48/x)* (p. 8).

A student using a qualitative strategy could reason that more than 5 cans are needed because 48 is more than 8 but would not arrive at a specific answer. Build-up is an additive strategy that involves identifying a ratio and extending that to another ratio by adding. For the example a student using a build-up strategy would add 8/5 to 8/5 until reaching 48/30. This strategy is often used when students are unable to recognize the multiplicative relationship between rational expressions or measure spaces (Lamon, 1993; Tourniare & Pulos, 1985). The multiplicative strategy involves understanding the multiplicative relationship ***between*** and ***within*** ratios. A student solving the problem *8/5 = 48/x* may recognize that 8*6 = 48 therefore 5 *6 = 30. In addition to the three strategies discussed above, cross multiplication is a fourth strategy discussed in the literature. Cross-multiplication is an effective and mechanical model, but the use of the cross-multiplication method without understanding could hinder the development of proportional reasoning (Cramer & Post, 1993; Cramer, Post, & Currier, 1993; Lamon, 2005).

Perspectives on Conceptual Understanding

Student proficiency in mathematics involves not only a facility with factual and procedural knowledge, but it involves the attainment of conceptual understanding (Carpenter & Lehrer, 1999; NCTM, 2000). Kroll and Miller (1993) suggest that successful problem solving hinges on students' possessing the knowledge and their ability to apply that knowledge to new problem situations. Conceptual understanding for this study is defined according to students' ability to (a) to apply concepts to new situations; (b) to connect new concepts with existing information; and (c) to use mathematical principles to explain and justify problem solutions. This definition is directly related to the five forms of mental activity of Carpenter and Lehrer (1999) that are important in attaining conceptual understanding of mathematics. They characterize understanding not as a static attribute but as emerging in learners as they engage in the following activities: (a) constructing relationships, (b) extending and applying mathematical knowledge, (c) reflecting about experiences, (d) articulating what one knows, and (e) making mathematical knowledge one's own and thus available for use in future situations (p. 20). In this study, involving students in activities that allow them to use mathematical principles to explain and justify their problem solutions supports students learning for conceptual understanding.

For the purposes of this study, students' conceptual understanding was measured in relation to students understanding of fraction always representing part-to-whole relationships and a ratio, representing part-to-part or part-to-whole relationships. It also assessed students' understanding and application of proportional reasoning in scaling. These domains for conceptual understanding were selected based on mathematical domains stressed in middle school content strands of the Principles and Standards of School Mathematics (NCTM, 2000), the State Standard Course of Study for Mathematic, and the district's pacing guides for middle grades mathematics. For this paper I will investigate:

1. What strategies do African American female students employ during mathematical problem solving?
2. How do African American female students understand proportionality concepts?
3. How do African American female students perceive themselves as mathematics learners?

Method

Procedures and Participants

The current study is a secondary analysis of qualitative and quantitative data from the Mathematical Identity Development and Learning project (MIDDLE). The MIDDLE research project was a three-year longitudinal, cross-sectional design with the following purposes:

1. To better understand how mathematics reform affects students' development as mathematics knowers and learners
2. To provide a longitudinal analysis of students' mathematical development during the middle school years
3. To identify the processes that explains changes in students' mathematical learning and self-conceptions.

MIDDLE research was conducted in middle schools located in southeastern United States with a diverse population. Of the 39,000 students in the district, 54% were African American, 24.3% white, 15.7% White, 3.4% multiracial, 2.4% Asian, and 0.2 % Native American (Malloy, 2009).

MIDDLE data was collected at two levels: Level I looked at classrooms and Level II looked at individual students. Level I data consisted of conceptual understanding problem used to assess students' levels of understanding. Tasks for MIDDLE were selected from released International Mathematics and Science Study (TIMSS) (1994) and National Assessment of Educational Progress (NAEP) (1990, 1992) tasks and then modified into tasks that have multiple solution paths that lead to a single correct solution. The measures for Level II included observations of students, individual student interviews, and student mathematical autobiographies. Level II data provided an in-depth examination of students' mathematical understanding, motivation, identity, peer pressures, and mathematical experiences.

A secondary analysis of a subset of MIDDDLE longitudinal dataset was conducted to address the research questions of the current study. The current study's sample is composed of 52 African American female 6th, 7th, and 8th grade students who participated in the MIDDLE project for three years. Each student had level I data consisting of the proportionality conceptual understanding problem and quantified scores for each year of the study. Students were asked to complete the following task.

> **Problem 7**
> A class has 28 students. The ratio of girls to boys is 4 to 3.
> How many girls are in the class?
> **Explain <u>why</u> you think your answer is correct.**

This MIDDLE conceptual understanding item measured students' understanding of proportionality, a major concept within the middle school mathematics curriculum; it assessed students understanding of a fraction always representing part-whole relationships and a ratio representing part-to-part or part-to-whole relationships. Additionally this item assessed students' understanding and application of proportional reasoning in scaling.

A team of five to seven people scored the problem. Each person in the team scored the problem independently. A second rater scored 25 percent of the papers to gain inter-rater agreement, which allowed for the calculation of percent agreement using the more conservative Cohen's kappa measure of agreement. The reliability score for the proportionality problem was .855 for year one and .609 for year two. A reliability score was not calculated for year three because the reliability was established in a prior year with the same scorers. When interpreting Cohen's kappa, a 0.40 is a minimum acceptable value and 0.60 or above indicates a good to excellent reliability (Landis & Knoch, 1977).

All conceptual understanding data was collected via students' written responses because "[w]riting has been viewed as 'thinking-aloud' on paper" (Pugalee, 2004, p.29). In order to understand students' cognitive processes during mathematical problem solving, researchers often use "think-aloud" protocols. Verbal protocols are powerful to gain information about students' cognitive processes, but research has shown the feasibility and validity of using written responses from open-ended tasks to assess students' cognitive processes during mathematical problem solving (Cai, 1997).

The problem was scored using a rubric with a scale ranging from 0 to 4, a score of 0 indicates "no attempt", 1 indicates "no understanding", and a score of 2 indicates "some understanding"-- (students understands different representations of ratios) with scores of 3 and 4 indicating " procedural"-- (understands the meaning of ratio and proportionality) and "conceptual"-- (understands the meaning of ratio, proportionality, and how to apply and explain their application) understanding respectively see Appendix A for item specific rubric. This numerical score represented the student's level of conceptual understanding of the concepts assessed by the task (Morton, 2008). After completing the problem students were asked to rate their level of confidence that their answer was correct on a likert scale ranging from 0 to 6 where 0 was "not at all confident" and 6 was "very sure".

Nine of the 52 females had Level II data consisting of narrative autobiographies and individual student interviews. The mathematical autobiographies provided a sketch of students' mathematical experience and their sense of self as mathematics learners. In the interviews, students were asked to reflect on how they learn mathematics, their classroom structure, and their classroom experiences.

Analysis

This study was based on a secondary analysis of a subset of data from the Mathematics Identity and Development (MIDDLE) Project. The analysis of student strategy use and level of understanding was based on students' written responses. Research has shown the feasibility and validity of using written responses form open-ended tasks to assess students' problem solving approaches, levels of understanding and misconceptions of mathematical concepts (Moskal, B. & Magone, M., 2000). Participants' responses on the mathematics task were analyzed in three ways. First, responses were sorted according to their final response as correct, incorrect, or no solution/attempt (including blank tasks and those stating some variation of "don't know" or "don't understand"). Secondly, students' written solutions were analyzed to determine the solution plan (problem-solving strategies) they devised and carried out to solve the task. Thirdly, student's level of understanding, as indicated by solution strategies and written explanations, were coded according to the item-specific conceptual understanding rubric using the following categories: "no understanding", "understands difference representation of ratio", "understands the meaning of the ratio and proportionality", "understand the meaning of the ratio, proportionality, and how to apply and explain their application". See Appendix A for a detailed description of each category. Student interviews and autobiographies were used to create profiles of students' perceptions. These profiles include their (1) description of what it means to understand mathematics, (2) mathematical learning preferences, (3) when they feel they do well in mathematics, (4) why it is important to do well in mathematics and (5) do they feel confident in their ability to problem solve.

Study limitations

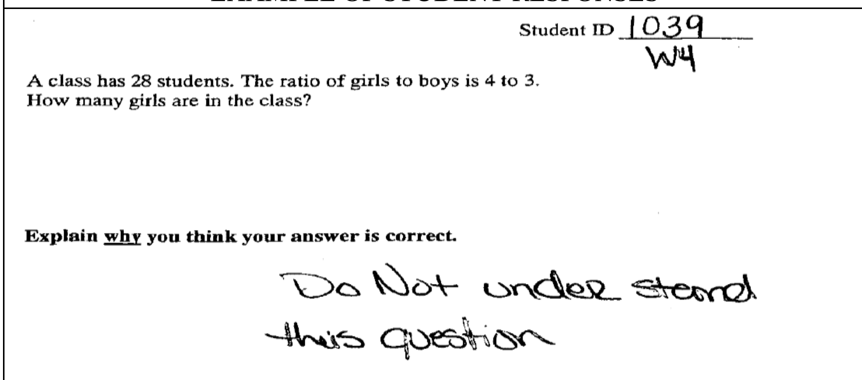
This generalizability of this study results is limited by the sample size and relying on written responses instead of task-based interviews. Since, these students are providing responses to one task this study can only speak to the written responses of the specific students on this particular task.

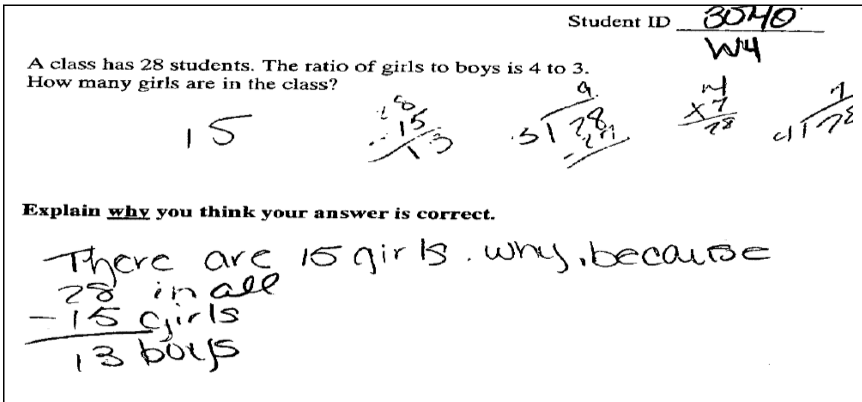
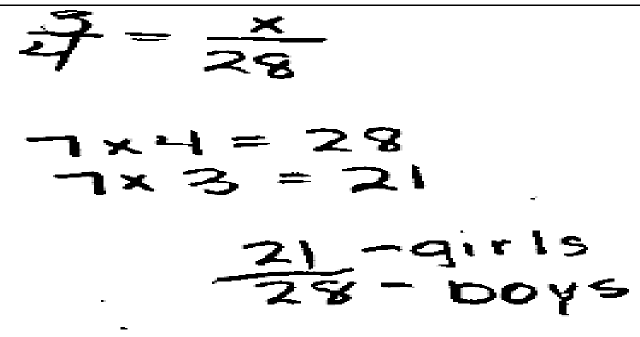
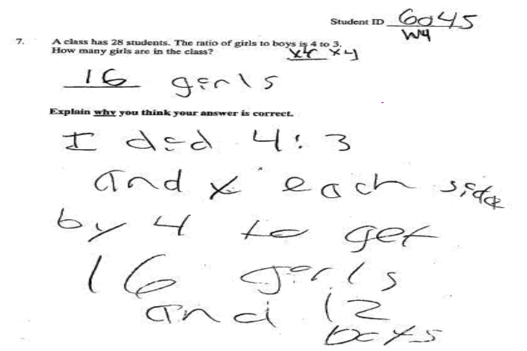
Results and Discussion

Strategy Use

In year one, seven students reached a correct solution followed by 10 students in year two and 12 students in year three. Participants in this study employed various strategies when solving the proportionality task. Strategies leading to a correct solution included (1) guess and check-students found two numbers that sum to 28, (2) build up strategies: students build up from the ratio 4:3 to the desired ratio 16:12, (3) build up strategy with pictorial representation: students draw a pictorial representation to illustrate building up from the ratio 4:3 to the desired ratio 16:12 and (4) multiplicative relationship: students found the multiplicative relationship between ratios. The most commonly used strategy was the additive build up strategy. Figure 1 show four sample responses, also visible are a classification of their understanding associated with each strategy and justification.

Figure 1. Sample of participants' responses and classification of their understanding

EXAMPLE OF STUDENT RESPONSES	UNDERSTANDING
 Student ID 1039 W4 A class has 28 students. The ratio of girls to boys is 4 to 3. How many girls are in the class? Explain **why** you think your answer is correct. Do Not under stand this question	No Attempt

	No Understanding
	Understands different representations of ratio
	Understands and the meaning of the ratio, proportionality, and how to apply and explain their application.

An analysis of student scores (n=52) on the proportionality problem revealed that 86.5%, 69.2%, and 68.6% did not attempt or demonstrated no understanding (score of 0 and 1) in year one, two, and three of the project respectively. Of those who attempted the problem 5.8%, 13.5%, and 9.8% of students demonstrated and understanding of different representations of different ratios (score of 2), 5.8%, 5.8%, and 7.8% of students demonstrated an understanding of the meaning of the ratio and proportionality (score of 3), and 1.9%, 11.5%, and 13.7% demonstrated an understanding of the meaning of the ratio, proportionality, and how to apply and explain their application (score of 4) in years one, two, and three respectively.

This data shows that over 50% of African American students made “no attempt” or demonstrated “no understanding” each year of the project. Students’ self-report of their level of confidence in the correctness of their solution did not align with students level of understanding of concepts assessed in the problem. By this, I mean there were multiple students who expressed confidence in the correctness of their solution when their approach to the problem demonstrated no understanding of the concepts assessed in the problem. When examining students confidences level in year one 46.8% self-reported a confidence level of 4 or higher in the correctness of solving the proportional reasoning task followed by 53.4 % in year two and 51.9% in year three. It is evident from the data that there is a larger number of students with higher confidence then their level of understanding (See Table 1).

Table 1. Participants' Conceptual Understand Scores and Confidence Levels

YEAR	N	CU SCORES					CONFIDENCE LEVEL					
		0	1	2	3	4	1	2	3	4	5	6
One	52	27	18	3	3	1	28	5	4	4	3	8
Two	52	13	22	7	3	7	13	4	7	8	8	12
Three	52	12	24	5	4	7	16	7	2	4	5	18

A deeper look at students (n=52) understanding and confidence level reveals the 23% of students in year 1, 28.8% of students in year 2, and 28.8% of students in year 3 demonstrated no understanding but reported a confidence level of 4 or higher.

Student Learner Perceptions—Student Interviews and Autobiographies

Understanding of Mathematics

In this study females demonstrated various levels of conceptual understanding and utilized a variety of problem solving strategies, but there was agreement among students when asked what it means to have an understanding of mathematics. Understanding mathematics was about understanding mathematics concepts and doing mathematics. Being able to understand and do mathematics was not only for individual benefit, but for being able to explain the mathematics to someone else. One voice echoed many others when she responded that to understand mathematics was,

> …to know the concept of what you're doing, not only to do it, but to be able to explain it to somebody else.
>
> In addition to explaining mathematics to others, someone who understands mathematics should be able to apply what they know to real life situations:
> It means to me that…it means like if you get a job, you have to know how to count money. You have to calculate calculations.

Mathematical Learning Preferences

When analyzing teachers could help African American females understand mathematics and learn mathematics better, teaching style was important. African American females in this study understood mathematics better when teachers used a variety of instructional strategies such as hands-on activities and group work. Female students spoke more about the benefit of teachers modeling, providing multiple examples, providing notes, and assigning homework for practice.

> We take notes in class. And we have little sets that help us understand it more. And we have groups we work in. And if we don't understand we can ask her and she'll try to explain it more in a way that we'll understand it.

They understood better when teachers "break down" the material so they can understand and provide extra help when they are having trouble. One female summarized how teachers helped develop their understanding of mathematics when she said:

> She gives us plenty of notes on the particular item we are studying. And she also goes over it, and if half the class didn't do well on the quiz or even if they do well on the quiz, she still goes over it with us so we will know how to do it later. She also has after-school tutorial for those who need it and sometimes we do hands on activities and group work (8248).

Students preferred to learn and learn mathematics best in an environment where their teachers helped them understand mathematics through varied methods. They preferred to work in groups, participate in peer teaching, but also enjoyed taking notes and following the teacher as he/she modeled various examples.

Feeling One is Doing Well in Mathematics

The students in this study felt they did well in mathematics when they received good grades and was given positive feedback from their teachers.

> "When I get good reports every Monday, when I have good grades, and when I get compliments from the teacher, telling me I 'm doing good. Also when I'm on the board in my class and I do the problem on the board and I can explain it to everybody as well".

Importance of Doing Well in Mathematics

It is important for these students to do well in mathematics because they realized that an understanding of mathematics is essential for their future success. The students expressed that an understanding of mathematics is needed for their day-to-day living such as balancing their checking accounts and maintaining their household budget. Students also recognized that an understanding of mathematics was needed for their potential career options; some career options would position students to become trailblazers within their own family.

> "It is very important for me, because I want to make my mother proud. I feel this way because there aren't any doctors in my family, and I know if I can understand math and get the basics down, the harder things, I can make my whole family proud".

Students' Confidence in Problem Solving

In addition to students being confident in the correctness of their solution when solving the proportionality task, they were also confident in their overall ability to problem solve.

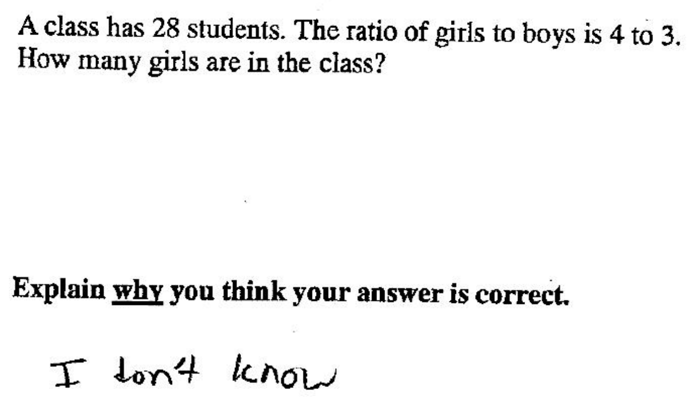
> "Sometimes, I mean most likely. Yes but I'm not going to sit up here and say I don't doubt myself when I answer my questions, because all math is not the same. I am more confident when I know something real good, that I mean I know for a fact the answer is right. I'm not worried about it".

The African American female students in this study were confident in their ability to solve mathematical problems, they were also confident in the correctness of the work they produced when problem solving. The students valued the importance of doing well in mathematics and of understanding mathematical concepts. What became troubling is that even though students valued mathematics, were confident in their ability to problem solve and the correctness of their work, these student continued to underperform on the proportionality task. In order to explore this phenomenon, I took a closer look at the data of students with Level I and Level II data. *Tiana (pseudonym for participant)* perceptions profile is similar to the majority of the subset of the sample with Level II data.

Introducing *Tiana*…

A deeper analysis of the subset of students with Level II data revealed cases similar to *Tiana*. *Tiana* participated in the study for three years. Figure 2 shows the progression of her strategy use.

Figure 2. Tiana's Proportionality Strategies

YEAR	STUDENT WORK
ONE	 A class has 28 students. The ratio of girls to boys is 4 to 3. How many girls are in the class? Explain **why** you think your answer is correct. I don't know

TWO	A class has 28 students. The ratio of girls to boys is 4 to 3. How many girls are in the class? **Explain <u>why</u> you think your answer is correct.** There are 14 girls because I add 14 and 13 to get 28 students
THREE	A class has 28 students. The ratio of girls to boys is 4 to 3. How many girls are in the class? **Explain <u>why</u> you think your answer is correct.** 14 +13 = 28 14 girls because 14 + 13 = 28.

She made "no attempt" in year one and in year two and three she demonstrated no understanding of concepts related to proportionality, but *Tiana* was confident in the correctness of her solutions in year two and three. *Tiana* ranked the correctness of her solution a 5 out of 6 in year two and 6 out of 6 in year three. She was a student who defined an understanding of mathematics as being able to do mathematics and it was important for her to do well in mathematics because she may need mathematics later in life. "Without math I wouldn't be able to do a job or something" (*Tiana*). She was confident in her abilities to solve mathematical problems because the "teacher gives me great grades and she shows the problems in a way that I can understand them. I also feel confident because I can do this and I feel more comfortable with other students so I can work with them. I don't like working with myself. I like to work with somebody else" (*Tiana*).

The responses of *Tiana* are reflective of other females in the study who were confident in their ability to problem solve and enjoyed mathematics. In addition, students recognized the importance of doing well in mathematics but when completing the conceptual understanding tasks assessing their understanding of proportionality concepts they demonstrated "little to no understanding" consistently over the three year period.

Conclusions

Analysis of students' strategy use and performance on the proportionality task revealed that students implemented strategies consistent with those utilized by their white peers. Participants who demonstrated "no understanding" were hindered by computational errors, misconceptions of the concepts assessed, and misinterpretation of the written information in the problem. The sample in this study included an alarming percentage of students who demonstrated "no understanding" or "made no attempt" consistently over the three year period.

When analyzing student interview and autobiography data, it was apparent that the African American females in this study had positive dispositions as mathematics learners. Students enjoyed mathematics, desired to do well in mathematics, and recognized the importance of understanding and doing well in mathematics. Not only did students recognize the importance of doing well in mathematics for themselves but their understanding and doing well in mathematics often times dependent upon their ability to help someone else understand what was being taught in class. The nine females with level II data were all confident in their ability to problem solve. Coupling students' confident in their ability to problem solve with the higher levels of confidence in the correctness of their solution to the task, unearthed an interesting phenomenon [*Tiana*]. This phenomenon causes me to question why are African American female students who are confident in their mathematical ability, have positive dispositions towards mathematics, and values doing well in mathematics, utilizes cognitive strategies similar to their White peers continue to struggle in their mathematics performance? Thus, the results of this study direct future research towards a more vital direction: a more deep understanding of the mathematical thinking of African American students demonstrates that factors outside of student thinking impact student performance.

Experiences of young ladies like *Tiana* are not necessarily limited to the United States and can occur within any mathematic classroom. It is important for researchers and educators to understand that students underperformance in mathematics may not be associated with the learner but may be attributed to factors outside the learner such as the type of curriculum used in schools or the schooling environment itself. It is important to legitimize the mathematical thinking and experiences of learners who are often placed in a deficit light when compared to majority populations. Doing this type of work not only informs efforts to improve minority students' success in mathematics; it will increase our awareness of the importance of research studies taking diverse students perspectives into account when designing and conducting work intended to generalize to *all* students.

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References

Behr, M., Harel, G., Post,T., & Lesh, R. (1992). Rational Number, Ratio, and Proportion. In
D. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 296-333). New York: Macmillian Publishing.

Berry, R. Q., III, Thunder, K., & McClain, O. L. (2011). Counter narratives: Examining the mathematics
and racial identities of Black boys who are successful with school. *Journal of African American Males in Education*, 2(1), 10–23.

Cai, J. (1997). Beyond Computation and Correctness: Contributions of Open-Ended Tasks in Examining U.S. and Chinese Students' Mathematical Performance. *Educational Measurement: Issues and Practices, Spring*, 5-11.

Cai, J. (2000). Mathematical Thinking Involved in U.S. and Chinese Students' Solving of Process Constrained and Process-Open Problems. *Mathematical Thinking and Learning*, 2(4), 309-340.

Carpenter, T., & Lehrer, R. (1999). Teaching and learning mathematics with understanding. In E. Fennema &
T. A. Romberg (Eds.), *Classrooms that Promote Mathematical Understanding (pp. 19-32).* Mahwah, NJ: Erlbaum.

Cramer, K., & Post, T. (1993). Connecting research to teaching proportional reasoning. *Mathematics Teacher, 86*(5), 404–407.

Cramer, K., Post, T., & Currier, S. (1993). Learning and teaching ratio and proportion: Research implications. In D. Owens (Ed.), *Research ideas for the classroom* (pp. 159–178). New York: Macmillian.

Gick, M. (1986). Problem-Solving Strategies. *Educational Psychologists, 21*(1 &2), 99-120.

Gomez, C., Rousseau, C., Steinthorsdottir, O., Valentine, C., Wagner, L., & Wiles, P. (2002) Multiplicative Reasoning: Developing Student's Shared Meanings. In B. Litwiller & G. Bright (Eds.), *Making Sense of Fractions, Ratios, and Proportions* (pp. 213-223). Reston: National Council of Teachers of Mathematics.

Hiebert, J. & Behr, M. (1988), Introduction: Capturing the major themes. In J. Hibert & M. Behr (Eds.), *Number Concepts and operations in the middle grades* (pp. 1-18). Reston: National Council of Teachers of Mathematics.

Inhelder, B., & Piaget, J. (1985). *The growth of logical thinking from childhood to adolescence*. New York: Basic Books.

Johnson, C., & Kritsonis, W. A. (2006). The national dilemma of African American students:
Disparities in mathematics achievement and instruction. *National Forum of Applied Educational Research Journal, 20*(3), 1–8.

Katada, S. & Gray, C. (2008). Double-and triple-minorities in the international relations classroom. *International Studies Perspectives,* 9(4), 464-468.

Kroll, D.L., & Miller, T. (1993). Insights from research on mathematical problem solving in middle grades. In D. Owens (Ed.*), Research ideas for the classroom: Middle grades Mathematics* (pp.55-77). New York: MacMillan Publishing Company.

Ladson-Billings, G. 1997. It doesn't add up: African American students' mathematics achievement. *Journal for Research in Mathematics Education* 28: 697–708.

Lamon, S. (2005). Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers. Mahwah, NJ: Lawrence Erlbaum Associates.

Lamon, S. (1993). Ratio and Proportions: Children's Cognitive and Metacognitive Processes. In T. Carpenter, E. Fennema, & T. Romberg (Eds.), *Rational Numbers: An Integration of Research* (131-

Landis, J. & Koch, G. (1977). The Measurement of Observer Agreement for Categorical Data Models. *Biometrics*, 33,159-174.

Lobato, J. Ellis, A, Charles, R., & Zbiek, R. (2010). *Developing Essential Understanding of Ratios, Proportions, and Proportional Reasoning*. Reston, VA: NCTM.

Lubienski, S. T., & Bowen, A. (2000). Who's counting? A survey of mathematics education research 1982–1998. *Journal for Research in Mathematics Education*, 31, 626– 633

Malloy, C. & Jones, G.M. (1998). An Investigation of African American students' mathematical problem solving. *Journal of Research in Mathematics Education*, 29, 143-163.

Malloy, C. (2009). Instructional Strategies and Dispositions of Teachers Who Help African American Students Gain Conceptual Understanding. In D. Martin (Ed.), *Mathematics Teaching, Learning, and Liberation in the Lives of Black Children* (88-122). New York, NY: Routledge.

Malloy, C. & Malloy, W. (1998). Issues of Culture in Mathematics Teaching and Learning. *Urban Review* 30(3), 245-257.

Martin, D. (2009). *Mathematics Teaching, Learning, and Liberation in the Lives of Black Children*. New York, NY: Routledge.

Morton, C.H. (2008). Making the Invisible Visible: An Examination of African American Students' Strategy Use During Mathematical Problem Solving. Unpublished doctoral dissertation, University of North Carolina at Chapel Hill.

Moskal, B. M. & Magone, M. E. (2000). Making Sense of What Students Know: Examining the Referents, Relationships, and Modes Students Displayed in Response to a Decimal Task. *Educational Studies in Mathematics, 43,* 313-335.

National Council of Teachers of Mathematics (2000). *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: The Council.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Polya, G. (1981). *Mathematical Discovery: On Understanding , Learning, and Teaching Problem Solving* (Combined Edition). New York, NY: John Wiley & Sons.

Pugalee, D. (2004). A Comparison of Verbal and Written Descriptions of Students' Problem Solving Processes. *Educational Studies in Mathematics, 55,* 27-47.

Resnick, L. & Singer, J. (1993). Protoquantitative origins of ratios constructed by two sixth-grade students. *Educational Studies in Mathematics*, 16, 181-204.

Reyes, L., and G. Stanic. 1988. Race, sex, socioeconomic status, and mathematics. *Journal for Research in Mathematics Education* 19: 26–43.

Schoenfeld, A. (1985). *Mathematical Problem Solving*. Orlando, Florida: Academic Press.

Siegler, R. (1998). *Children's Thinking* (3rd. Ed.). Upper Saddle River, New Jersey: Prentice-Hall.

Steinthorsdottir, O. & Sriraman, B. (2009). Icelandic 5th-Grade Girls' Developmental Trajectories in Proportional Reasoning. *Mathematics Education Research Journal*, 21 (1), 6-30.

Tate,W., and C. Rousseau (2002). Access and opportunity: The political and social context of mathematics education. In L. English (Ed.), *Handbook of international research in mathematics education* (271-300). Mahwah, NJ: Erlbaum.

Tourniaire, F., & Pulos, S. (1985). Proportional Reasoning: A Review of the Literature. *Educational Studies in Mathematics, 16,* 181-204.

Appendix A

Conceptual Understanding Scoring Rubric

A class has 28 students. The ratio of girls to boys is 4 to 3.
How many girls are in the class?

Concepts Assessed

Understand and apply proportional reasoning used in scaling.
Understand that a fraction always represents a part-to-whole relationship.
Understand that a ratio can represent part-to-part or part-to-whole relationships.

Scoring Rubric

Level	Identifiers	Examples of student responses	Understanding
0	No work or states they do not understand with no answer given.	"I don't understand."	No attempt
1	No evidence of understanding concepts related to fractions or proportionality.	"21 girls. I think it is right because I used my × ÷ skills." "There are 12 girls because 12 × 3 = 28 and there are 28 students."	No understanding
2	Written or symbolic explanation shows an understanding the meaning of a ratio, but does not apply the ratio to solve the problem. Correct written or drawing work but provides no explanation of how the answer was found.	Ans. 16. "1. set up proportion, 2. cross multiply, 3. reduce fractions. $\frac{4}{3} = \frac{\_\_\_}{28}$ " Ans. 16 girls. "16 girls 12 boys"	Understands different representations of ratio.
3	Explanation is accurate does not thoroughly explain the rationale used in solving the problem. The explanation is procedural rather than conceptual.	"12 boys and 16 girls equal 28 students." "There are 16 girls. If you multiply 4×4 you get 16. Then you multiply 4 ×3 to get 12. Then you add 16 & 12 to get 28."	Understands the meaning of the ratio and proportionality
4	Evidence of full understanding of proportionality either verbally or visually (scaling 4:3 or using and explaining the proportion 4/**7** = 16/**28**).	"For every 4 girls there is 3 boys, 4 3 4 3 4 3 4 3 There are 16 girls." "If you add the numbers together and multiply by 4 you get twenty-eight. So you just multiply the individual numbers and get 16:12."	Understands and the meaning of the ratio, proportionality, and how to apply and explain their application.