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Latent Class Analysis of Students' Mathematics Learning Strategies and the Relationship between Learning Strategy and Mathematical Literacy

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Abstract This study investigated how various mathematics learning strategies affect the mathematical literacy of students. The data for this study were obtained from the 2012 Programme for International Student Assessment (PISA) data of Taiwan. The PISA learning strategy survey contains three types of learning strategies: elaboration, control, and memorization. To objectively classify students' learning strategies, we conducted a latent class analysis (LCA) to determine the optimal fitting latent class model of students' performances on a learning strategy assessment and to explore the mathematical literacy of students who used various learning strategies. The result of the LCA showed that, among the models of two to five classes, the model of four classes was the optimally fitted model. The results of this study provide crucial information for mathematics educators regarding the achievement-related and strategy-related outcomes of schooling. By combining information from the assessment of mathematical literacy and survey of learning strategies that predispose students to using their mathematical literacy, a more complete picture of mathematics teaching and learning emerges. The implications of these results are discussed.

Keywords Mathematical Literacy, Latent Class Analysis, Learning Strategy

1. Introduction

Mathematics is fundamental in life and academics. Mathematics skills are required in the workplace and in everyday life to solve problems.

In recent years, since the Organization for Economic Cooperation and Development's (OECD's) Programme for International Student Assessment (PISA) results were released, discussion regarding mathematics education increased because the PISA evaluates student preparedness for the future, which is an indicator of national progress, and the PISA results can be compared among various countries because the PISA is an international assessment. The PISA assesses mathematical literacy in students by examining how effectively students can formulate, employ, and interpret mathematics problems that correspond to everyday life. The PISA data can provide information on the school systems of participating countries and have a profound impact on mathematics education policies.

This study explored the relationship between learning strategies and mathematical literacy. A total of 65 countries or economies participated in the PISA 2012. Shanghai–China ranked first in mathematics performance followed by Singapore and Hong Kong–China. Taiwan first participated in the PISA 2012. According to the PISA 2012, Taiwan's average mathematical literacy rank was the fourth, which appears favorable. However, Taiwan had a standard deviation of 116, which was substantially higher than the OECD average (standard deviation= 92), indicating a substantial gap in the mathematics performance of Taiwanese students. Although Taiwan's standard deviation was high, several countries and economies had higher standard deviations, too. Hence, reducing the gap in students' mathematical literacy is crucial in Taiwan and these countries and economies. Exploring student attitudes, engagement, and learning strategies should be useful for understanding the mathematics achievement gap.

Learning strategies may be critical to academic performance. When learning mathematics, students typically acquire learning strategies through the guidance of teachers or peers to improve their learning efficiency. Students with outstanding mathematics performance typically adopt learning strategies appropriately to manage their learning. By contrast, students with relatively poor mathematics performance typically cannot apply effective learning strategies to solve problems or monitor their learning [1].

Mathematics learning strategies were measured in the PISA 2012 and 2003, but these instruments used distinct measurement approaches. Mathematics learning strategies were measured using Likert scales in the PISA 2003, but forced-choice questions were used for measurement in the PISA 2012. Investigating results on the basis of these two types of measurement is necessary to develop appropriate and efficient methods for assessing mathematics learning strategies. However, the results of assessment of learning strategies and the relationship between learning strategy and mathematical literacy were not discussed in the international reports of the PISA 2012. Hence, students' learning strategies and the relationship between learning strategy and mathematical literacy should be further investigated. Students use a variety of strategies to solve mathematics problems. To objectively classify students' learning strategies, we conducted a latent class analysis (LCA) to determine the optimal fitting latent class model of students' performances on a learning strategy assessment and explored the mathematical literacy of students who used various learning strategies.

1.1. Mathematical Literacy

According to the PISA 2012, mathematical literacy is "an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematics concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It assists individuals in recognizing the role that mathematics plays in the world and in making the well-founded judgements and decisions needed by constructive, engaged, and reflective citizens"[2]. The assessment approach in the PISA, which emphasizes the application of knowledge and skills in real-life situations, is distinct from that of school mathematics. Under the PISA approach, students' ability to reason quantitatively and represent relationships or dependencies is more crucial than the ability to answer familiar textbook questions in determining students' practical mathematics ability.

Because PISA questions are based on real situations, they typically involve a variety of processes, content, and contexts. According to the PISA, the four overarching concepts that relate to algebra, geometry, numbers, and statistics are "change and relationships," "space and shape," "quantity," and "uncertainty and data." The three process categories are "Formulating situations mathematically"; "Employing mathematical concepts, facts, procedures, and reasoning"; and "Interpreting, applying, and evaluating mathematical outcomes." The four context categories in the PISA identify the broad areas of life in which mathematics problems may arise: personal, societal, occupational, and scientific.

1.2. Learning Strategy

Students adopt strategies for mathematics learning. By consciously adopting effective learning strategies, students learn more effectively than if they only followed teachers' instructions.

The word "strategy"is derived from the ancient Greek word "strategia", which describes steps or actions taken for the purpose of winning a war. The war connotation of strategia has not persisted, but the term still signifies control and goaldirectedness[3]. Schunk and Zimmerman [4]reviewed research showing that students with high self-efficacy perceive difficult reading tasks as challenging and work diligently to master them, using their cognitive strategies productively.

Theories suggest that these student-level inputs can affect the processes of learning, thinking, and test-taking, which influence learning motivation and educational aspirations. Pintrich [5]proposed a model of self-regulated learning that includes three general categories of strategies: (1) cognitive learning strategies, (2) self-regulatory strategies to control cognition, and (3) resource management strategies. In terms of cognitive learning strategies, following the work of Weinstein and Mayer rehearsal, elaboration, and organizational strategies were identified as crucial cognitive strategies related to academic performance in the classroom (e.g.,[5]).

The PISA focuses on three kinds of learning strategies: memorization, elaboration, and control strategies. Furthermore, on the PISA, these learning strategies are applied for a variety of purposes, including problem-solving and test-taking. Memorization strategies include rote learning of facts and rehearsal of examples, whereas elaboration strategies involve relating new material to material that is known. Control strategies involve determining what students have learned to ascertain what they are still required to learn.

2. Materials and Methods

2.1. Data

The data of this study were based on the PISA 2012 data for Taiwan from the PISA database website http://pisa2012.acer.edu.au/. The PISA is a triennial international survey, which aims to evaluate education systems worldwide by testing the skills and knowledge of 15-year-old students in mathematics, reading, and science. The primary focus of the PISA 2012 was mathematical literacy. For the PISA 2012, three forms of student questionnaire were designed to obtain comprehensive information on student performance. Thus, not every student who participated in the PISA 2012 filled out the same questionnaire scales. After excluding the students who did not respond to the learning strategy assessment and weighted to ensure that each sampled student appropriately represents the correct number of students in the full PISA population,192,819 Taiwanese students participated in this study.

2.2. Learning Strategy Survey

The PISA learning strategy assessment used forced-choice questions, as opposed to rating-scale questions, to ascertain which learning strategy students preferred. The PISA mathematics learning strategy assessment contained four items, with three options for each item. Each option referred to a different learning strategy. For example, the first option of Item 1, which was related to control strategies, was: "When I study for a mathematics test, I try to work out what the most important parts to learn are." The second option of Item 1, which was related to elaboration strategies, was: "When I study for a mathematics test, I try to understand new concepts by relating them to things I already know." The third option of Item 1, which was related tomemorization strategies, was: "When I study for a mathematics test, I learn as much as I can by heart."

Students were instructed to choose the option with which they most agreed. The responses to this assessment are categorical variables, and there is no difference in degree or strength among options. Hence, this assessment is suitable for using LCA to identify categories of mathematics learning strategies.

2.3. Analysis Methods

Conceptually related to cluster analysis, LCA is a multivariate method designed to identify unobserved (or latent) subpopulations of individuals on the basis of multiple measures. LCA uses maximum likelihood estimation to fit a hypothesized model in which membership in a specified number of latent classes is related to performance on the included measures and to produce fitted probabilities of class membership for individuals. The observed measures can be categorical or continuous. Each latent class can be interpreted as a subpopulation with homogenous profiles on the multiple observed measures included in the analysis, whereas the differences between the latent classes indicate heterogeneity in the population studied [6]. For LCA models with categorical outcomes, the item parameters correspond to the conditional item probabilities. These item probabilities are specific to a given class and provide information on the probability of an individual in that class to endorse the item. The class probability parameters specify the relative prevalence (size) of each class [7].

In the present study, we combined the statistical evidence provided by the Akaike information criterion (AIC), Bayesian information criterion (BIC), and adjusted BIC. If these statistics decrease as additional classes are added, the conclusion that the additional subgroups found exist in the population is further supported. In other words, the model with smaller values based on AIC, BIC, and adjusted BIC measures was the most effective option.

Our data analysis was based on LCA and required the use of Mplus software. The analysis of variance, which was used to test the difference in average mathematical literacy among students using various learning strategies in our study, was performed using the Statistical Package for Social Sciences (SPSS) software.

3. Results

3.1. Optimal Fit Model

The results of the LCA are presented in Table 1. These results indicate that the AIC was smallest for the five-class model, but the BIC and adjusted BIC were smallest for the four-class model. Because the sample size of this study was large, the BIC is a suitable indicator for model selection. Consequently, a four-class model is the preferred model in this study.

Table 1. Summary of LCA Criteria in Each Class Model

Model	AIC	BIC	Adjusted BIC
2-class	32257	32363.94	32309.92
3-class	32099.93	32263.48	32180.87
4-class	32027.37	**32247.53**	**32136.32**
5-class	**31996.58**	32273.36	32133.54

The class probabilities of the four-class model are shown in Table 2. For Class 1, the class probability was 0.17, indicating that 17% of students were classified as Class 1. For Class 2, Class 3, and Class 4, the category probability was 0.36, 0.24, and 0.23, respectively. In other words, 36% of students were assigned to Class 2, 24% of students were assigned to Class 3, and the remaining 23% of students were classified as Class 4.

Figure 1 and Table 2 present the conditional probability of students in each class for individual items and options. The conditional probability for Class 1 students responding to the first cell in Table 2 was 0.305, indicating a 30.5% probability that Class 1 students chose "I try to work out what the most important parts to learn are."

The probability in each category for individual indicators can be used to assign a label to each class. From the distribution of conditional probability presented in Table 2 and Figure 1, Class 1 students, who prepared for the mathematics exam, studied mathematics, and solved mathematics problems, tended to use memorization strategies. Class 1 was thus labeled the "memorization strategy" group.

Class 2 students tended to use control strategies for studying mathematics, employ control and elaboration strategies in preparation for mathematics exams, and apply memorization and elaboration strategies to learn problem-solving methods and ascertain information. Class 2 was thus termed the "multiple-strategies" group.

Class 3 students tended to use control strategies to prepare for mathematics exams and study mathematics. Class 3 was therefore termed the "control strategy" group.

Class 4 students tended to use elaboration strategies in preparing for mathematics exams and applied elaboration and control strategies in the study of mathematics. Class 4 was thus termed the "elaboration and control strategies" group.

Table 2. Conditional Probabilities and Latent Class Probability on the Mathematics Learning-Strategy Scale for the 4-Class Model

Item	Strategy	Class 1	Class 2	Class 3	Class 4
A.1. When I study for a mathematics test, I try to work out what the most important parts to learn are.	control	0.305	0.468	0.701	0.210
A.2. When I study for a mathematics test, I try to understand new concepts by relating them to things I already know.	elaboration	0.262	0.482	0.095	0.698
A.3. When I study for a mathematics test, I learn as much as I can off by heart.	memorization	0.433	0.05	0.205	0.092
B.1. When I study mathematics, I try to figure out which concepts I still have not understood properly.	control	0.241	0.768	0.541	0.465
B.2. When I study mathematics, I think of new ways to get the answer.	elaboration	0.082	0.087	0.060	0.403
B.3. When I study mathematics, I make myself check to see if I remember the work I have already done	memorization	0.677	0.145	0.400	0.132
C.1. When I study mathematics, I try to relate the work to things I have learnt in other subjects.	elaboration	0.184	0.241	0.351	0.607
C.2. When I study mathematics, I start by working out exactly what I need to learn.	control	0.325	0.604	0.641	0.299
C.3.When I study mathematics, I go over some problems so often that I feel as if I could solve them in my sleep.	memorization	0.491	0.155	0.008	0.094
D.1. In order to remember the method for solving a mathematics problem, I go through examples again and again.	memorization	0.756	0.473	0.496	0.118
D.2. I think about how the mathematics I have learnt can be used in everyday life.	elaboration	0.079	0.045	0.354	0.512
D.3. When I cannot understand something in mathematics, I always search for more information to clarify the problem.	control	0.165	0.481	0.150	0.370
Class Probability		0.17	0.36	0.23	0.24

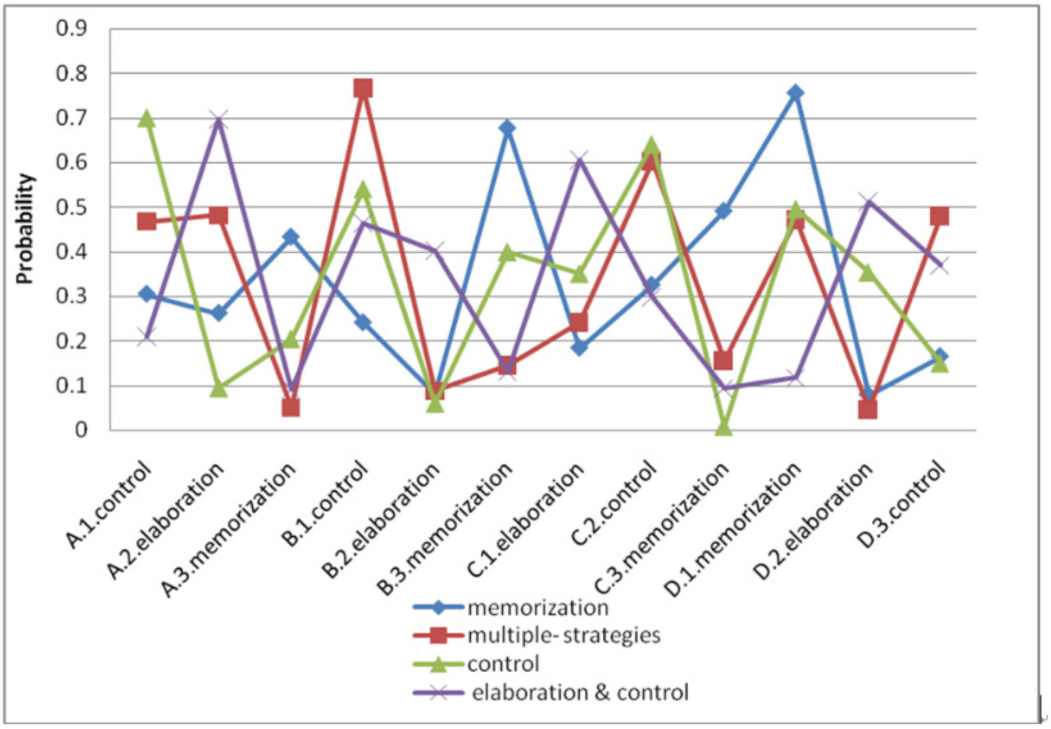

Figure 1. Distribution of Conditional Probabilities of Each Item for the 4-Class Model

Table 3. Descriptive Statistics of Mathematical Literacy for the Four Classes of Learning Strategy

Type	N	Mean	SD
Memorization strategy	32361	530.19	110.98
multiple-strategies	68789	587.05	107.21
control strategy	43826	524.90	110.95
elaboration & control strategies	47843	577.32	122.46
total	192819	560.97	115.98

Table 4. ANOVA Results of Mathematical Literacy for Different Learning Strategy Types

Source	*SS*	*df*	*MS*	*F*	*p*	η^2
Between	147346298	3	49115433	3871.856	0.000	0.057
Within	2446266521	192815	12687			
Total	63270625413	192819				

3.2. The Mathematical Literacy of StudentsUsing Various Types of Problem-Solving

Table 3 shows descriptive statistics regarding the mathematical literacy of students using the four classes of learning strategies. The mean scores of students using the four types of learning strategies showed that students in the multiple-strategies group performed most effectively (587.05), followed by students in the elaboration and control strategies group(577.32). The mathematics performance of the memorization strategy group and control strategy group was weak. A one-way analysis of variance (ANOVA) was further applied to investigate the mean difference. The result presented in Table 4 shows that the mean difference among the four classes was significant (F=3871.856, p<. 01), and the measure of association strength (η^2) was 5.7%. The results of the post hoc test showed that the multiplestrategies group scored significantly higher than the other three groups did. Furthermore, the elaboration and control strategiesgroup scored higher than the memorization strategy group and control strategygroup did.The memorization strategy group also scored higher than the control strategygroup did. Based on our results, learning strategy accounts for approximately 6% of the variation inmathematical literacy.

4. Conclusion and Discussion

Appropriate and effective learning strategies are crucial for positive educational outcomes. The types of learning strategies that students adopt can influence their academic performance and determine whether they are engaging in deep or surface-level learning. Effective knowledge and command of learning and problem-solving strategies, such as control, elaboration, and memorization, is acrucial outcome of and prerequisite formathematics learning because knowledge and learning strategies allow for the application of mathematics literacy to various contexts and tasks.

PISA 2003 showed that the elaboration assessment is a valid and strong predictor of mathematics performance among countries. However, memorization did not appear to be an independent and valid assessment of mathematics performance, and control scales showed mixed results[8-10]. Cognitive research has also shown that problem-solving and metacognitive strategies in particular are central to mathematical literacy. [11-14] Academically, successful students have more effective strategies for taking tests than less successful students. However, Artelt and Schneider [11] suggested that self-assessments regarding knowledge and the use of strategies are weak indicators of metacognition[15].

Based on the Taiwan data from the PISA 2012,LCA was used in this study to determine that the four-class model is the optimal fit model of learning strategy. The four groups in this study were termed the "memorization strategy," "multiple strategies," "control strategy," and "elaboration and control strategies" groups. The results of the mean difference test showed that the average mathematical literacy of the students who tended to use multiple strategies, such as the students in the "multiple strategies" and "elaboration and control strategies" groups, performed higher than average among Taiwanese students. Moreover, students who used a single learning strategy, such as the students in the "memorization" and "control" groups, performed lower than average among Taiwanese students.

Overall, this study supports the finding that learning strategies influencemathematical literacy. However, [16,17] this study further found that students who used multiple learning strategies possessed higher mathematical literacy than students who used a single learning strategy. Hence, if students are classified into the memorization or control groups, educators should teach them to use various learning strategies to improve their mathematical literacy.

Our findings should encourage others who desire to enhance mathematics learning. The analyses in the previous section suggest thatstudents who are more conscious of learning strategies have higher mathematical literacy. Because many students requireassistance to understand how to use control, memorization, and elaboration strategies, teachers must consider methods of explicitly teaching these learning strategies in the classroom. This study suggests that teachers can support students' mathematics learning by providing direct and explicit instructions about strategies for understanding mathematics and solving problems. To achieve understanding, learners must integrate new information into their prior understanding through elaboration strategies. Furthermore, learners can use control strategies to identify the most crucial material to learn and concepts they do not understand as well asto search for more information to clarify a problem.

Despite our findings, we could not elucidate the causality between students' learning strategies and their mathematical literacy. Do changes in learning strategies cause changes in students'mathematical literacy, or do changes in learning strategies occur as students increase their mathematical literacy? Although we cannot answer these questions, our analyses suggest that learning strategy and mathematical literacy are interrelated.Additionally, the use of publicly available data regarding student performance limited the results of this study. Student-level data that can be disaggregated would be useful in future studies because determining students from various countries as well as various ethnic, socioeconomic, regional, and gender groups could help to determine how various subgroups of students are affected by learning strategies. However, the use of data from Taiwan enabled this study to illustrate the effects of learning strategies on the overall student population of Taiwan. Further empirical research is needed in this area, and teachers must continue to teach mathematics and learning strategies to enable students to perform more effectively.

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