

# **Preservice Teachers' Perceptions About Teaching Mathematics Through Music**

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This study examined preservice teachers' perceptions about teaching elementary level mathematics lessons integrated with music. It also sought to determine how preservice teachers would strategize the integration of music activities when introducing elementary level mathematics lessons. The participants, 53 undergraduate preservice teachers at a large public university in a southern metropolitan area, were provided a series of six 40-minute interactive music-math integrated activities. Results of qualitative data analyses revealed that the majority of preservice teachers provided positive feedback about music-integrated pedagogy for teaching mathematics because it allowed them opportunities to escape from the limitations of traditional mathematics instruction.

*Keywords:* music-mathematics interdisciplinary curriculum, interdisciplinary mathematics instruction, preservice teachers' perceptions, pedagogical content knowledge.

Mathematics is a subject content area that can be easily integrated with other subject content areas. However, one of the drawbacks to this notion is that students think school disciplines are independent to each other and that school education is irrelevant to real life (Cumming, 1994). An instructional strategy that can be used to change students' outlook on education is the integrated curriculum, which is an approach that offers students the opportunity to understand and apply school subjects through multiple approaches (Fiske, 1999).

Interdisciplinary instruction enhances students' holistic thinking skills by developing knowledge through interdisciplinary connections (Hargreaves, Earl, Moore, & Manning, 2002); facilitating their creativity through the opportunities to fulfill situated, socially-constructed, and culturally intervening experiences (Marshall, 2005); and providing learning experiences that can intellectually and emotionally motivate their understanding of academic concepts from multiple perspectives, as well as transfer of learning (Chrysostomou, 2004). Teachers can also benefit from integrated instruction. Such instruction can provide teachers with opportunities to address important issues that may be difficult to investigate in individual subjects and it can lead to a broader view of the curriculum thereby reducing redundancy of content (Carrier, Wiebe, Gray, & Teachout, 2011).

Moreover, interdisciplinary education can provide students with a learning environment that enables them to have a better social relationship with their everyday experiences. It can also engage them in a reflection and inquiry process (Parsons, 2004) to help them better understand the core subjects with rich connections among school subjects and out-of-school experiences (Hargreaves, et al., 2002).

Reports of improvement in students' achievement through the integrated curriculum has led many organizations, including the National Arts Education Association [NAEA] (1994), the American Association for the Advancement of Science [AAAS], (1998), the National Research Council [NRC], 1996), and the National Council of Teachers of Mathematics [NCTM] (2000) to adopt national standards and constructivist teaching approaches. In fact, the NCTM explicitly acknowledges that students are required to have opportunities to identify and apply knowledge associated with other school subjects and real life experiences.

Unlike many curriculum integration designs that link two or more school subjects together (e.g., mathematics-science), the arts integrated curriculum, with its unique function of transferring student learning from the arts to non-arts content, has the potential to be integrated with all school subjects (Catterall, 2005). Several studies have found that the arts can provide students and teachers learning experiences that can motivate understanding (Chrysostomou, 2004; Mansilla, 2005), engage learners through self-reflection and active inquiry (Parsons, 2004), and provide an enjoyable and collaborative learning environment (Fiske, 1999).

Music, an essential category of the arts, has been developed with different curriculum elements and instructional strategies to supplement other school subjects, especially mathematics (Cornett, 2007; Robertson & Lesser, 2013). The natural similarities between music and mathematics (such as patterns, structures, and symbols) offer rich overlapping knowledge areas for mathematics educators to develop K-16 mathematics lessons integrated with various music concepts such as music composition and musical instrument designs (Fauvel, Flood, & Wilson, 2006; Loy, 2006).

By designing appropriate music integrated mathematics lessons, students can understand, analyze, and interpret mathematics through different routes (An, Capraro, & Tillman, 2013; An, Kulm, & Ma, 2008; Brown, 2013). For example, elementary school teachers have utilized advanced digital manufacturing technologies, including 3D-printers, to involve students in music instrument design and fabrication activities based on mathematical concepts such as measurement and geometry (Tillman, 2013). However, existing research on teaching mathematics through music is usually focused on the superficial relationship between mathematics and music (Rogers, 2004).

Although more connections need to be explored, such as the connections between music and mathematics during the composition of rhythm with repeating beats per minute patterns and internal rhythmic structures with underlying mathematical foundations (An, Ma, & Capraro, 2011), more research needs to be done on how teachers develop their abilities to teach mathematics concepts through contextualized music activities. The purpose of the study was to examine preservice teachers' perceptions about teaching elementary level mathematics topics integrated with music and to determine how they would strategize integrating music activities in lessons to address elementary mathematics topics.

## Conceptual Framework

The conceptual framework for this study was based on a synthesis of three areas of music-mathematics connections – the cognitive structures and levels of the students, the emotional and attitudinal impact of mathematics on student behavior, and the pedagogical methods of teachers. Researchers (e.g., Rauscher, Shaw, & Ky, 1993; Rideout & Laubach, 1996) synthesized these themes for using music as a context for mathematics education by focusing on the impact of music activities (such as the Mozart Effect) on students' abilities in performing mathematics tasks.

### *The Cognitive Structures and Levels of the Students*

In 1993, Rauscher and his colleagues published a groundbreaking study about the “Mozart Effect” entitled *Music and Spatial Task Performance*. In this study, they reported that the group of participants who listened to Mozart's music demonstrated significantly higher IQ scores than participants in the other two groups who listened to either relaxing music or silence. However, this advantage only lasted about 10 to 15 minutes after the treatment. Although many researchers have replicated the “Mozart Effect” with participants of various ages and backgrounds and have found positive impacts on various mathematical tasks (e.g., Rauscher et al., 1995; Rideout & Laubach, 1996; Nantais & Schellenberg, 1999; Wilson & Brown, 1997), it is theorized that a possible explanation for these findings is that music stimulates neuron activity in certain areas of the brain that is also responsible for mathematical related reasoning.

Regarding educational implications for the Mozart Effect, Shaw (2004) pointed out that there are two kinds of mathematics reasoning – spatial-temporal reasoning and language-analytic reasoning. Spatial-temporal reasoning involves the mental rotation of objects in space and time, searching sequences and patterns, and thinking in advance to reason through a problem. Language-analytic reasoning expects students to receive the necessary information with the goal of answering questions. The traditional approach to education focuses more on language-analytic reasoning within a lecture-type environment where information and solutions are of a quantitative nature, which may neglect the mental visualization process in the conceptual understanding of mathematics.

### *Emotional and Attitudinal Impact of Mathematics on Student Behavior*

An important facet of student learning involves emotion. Emotion is essential to students' learning because positive emotions tend to lead to higher levels of motivation that facilitate learning. Miller and Mitchell (1994) suggest that teachers create a highly motivating environment for learning, free from tension and other possible causes of embarrassment or humiliation. Music, with its aesthetical features, can provide students with a highly motivating environment with less prejudice and violence, and it can help them become better risk takers and communicators (Langer, 1997). Simply stated, by creating a highly motivating learning environment where students can be engaged to participate in mathematics tasks with less anxiety, music can be used as a sugarcoating for learning explicit concepts in mathematics (An et al., 2011).

Mathematics anxiety is also a facet of behavior that commonly exists among teachers and students of all grade levels (Zettle & Raines, 2002). It has been reported that preservice teachers have higher levels of mathematics anxiety and more negative mathematics attitudes than their peers in other majors (Bursal & Paznokas, 2006). Not only do preservice teachers report feeling nervous and uncomfortable when involved in mathematical-related tasks, they also tend to transmit these negative dispositions toward their prospective students when they are in field-based learning environments (Furner & Berman, 2005).

### *Pedagogical Methods of Teachers*

Traditional mathematics curricula and instructional methods that promote one way to demonstrate mathematics, thereby neglecting conceptual understanding, are considered the key factors that cause mathematics anxiety among students (Furner & Berman, 2005). Offering preservice teachers multiple opportunities to build their pedagogical content knowledge and experience with innovative teaching strategies (e.g., using problem-solving activities, simulations, discoveries, contextualized challenges, and games) have been postured as effective solutions to reshaping their teaching beliefs and attitudes, which may in turn positively influence their students (Bursal & Paznokas, 2006).

Since most elementary preservice teachers lack sufficient classroom teaching experiences, one of the most important areas they need to strengthen in their pedagogical content knowledge is mastering strategies on how to explain mathematics concepts to young children (van Driel, Verloop, & de Vos, 1998). Teaching mathematics with connections (or integrated with other subjects such as music) provides a different way to present and apply mathematics knowledge. To effectively deliver intellectual communications, as well as reduce the redundant content across different disciplines, preservice teachers, especially elementary teachers, should be able to design and implement lessons in interdisciplinary ways (Catterall, 2005).

Developing integrated instructional abilities are often difficult because synthesizing processes are required during lesson preparation and higher order cognitive procedures are required to combine different pieces of subject knowledge into one part (Stein, Connell, & Gardner, 2008). Thus, implementing an integrated curriculum or lesson is impractical for most teachers because they lack the strategies, resources, and supports to prepare lessons that foster connections among subjects (Czerniak, Weber, Sandmann, & Ahern, 1999; Pang & Good, 2000). Because of curriculum design limitations at the elementary grade levels (e.g., lack of instructional time, weak curriculum organization, ambiguity of content focus) most teacher education programs fail to offer courses that prepare preservice teachers to develop integrated teaching methods (Zhou & Kim 2010).

### **Method**

The current study was characterized by a sequence of classroom activities aimed at understanding preservice teachers' perceptions about the integration of music with mathematics instruction and their development of pedagogical content knowledge related to teaching mathematics integrated with music. The two research questions addressed were:

1. What are elementary preservice teachers' perceptions of the relative merits of music-mathematics integrated approaches to the teaching and learning of elementary mathematics?
2. How do elementary preservice teachers plan to integrate music activities addressing elementary mathematics topics for their future mathematics teaching practice?

### *Participants*

The participants were 53 undergraduate preservice teachers (47 females; 6 males) pursuing an elementary education degree. Most (N=49) were Hispanic and ranged in age from 22 to 43 years. Thirty-five (35) were enrolled in the K-4 generalist certificate program and 18 were enrolled in the K-4 bilingual generalist certificate program. All participants were either in their third or fourth year of study, and were split between two mathematics methods courses during a regular academic semester.

Consent for participation in the study was obtained by the graduate teaching assistant and conducted in a manner that ensured that preservice teachers understood their rights, the purpose of the study, and did not feel coerced into signing the consent form. The graduate teaching assistant also safeguarded the anonymity of the participants so that the instructor would not know which preservice teachers were participating in the study.

### *Setting*

The research took place at a large public university in a southern bilingual metropolitan area. Approximately 23,000 students attend this university. The student body is 77% Hispanic and 54% female. At the time of this study, slightly more than 250 preservice teachers were enrolled in the K-4 generalist certificate program and approximately 150 were enrolled in the K-4 bilingual generalist certificate program.

### *Intervention*

The intervention consisted of a series of six 40-minute interactive music-mathematics integrated activities, which were introduced to preservice teachers over a six-weeks period within a regular academic semester. Table 1 shows a list of the music activities integrated into the mathematics content areas.

**TABLE 1**  
**Intervention Activities for Teaching Mathematics Through Music**

	<b>Mathematics Content Area(s)</b>	<b>Music Activity</b>
Week 1	Numbers and Operations	Music Playing Activities
Week 2	Data Analysis, Algebra	Music Composition Activities
Week 3	Geometry	Musical Instrument Designing Activities
Week 4	Probability	Music Composition Activities
Week 5	Measurement	Musical Instrument Designing Activities
Week 6	Numbers and Operations, Algebra	Music Composition Activities

## Procedure

The intervention activities were implemented in two mathematics methods courses during regular class meetings over the course of six-weeks. In each of the courses, one music-mathematics integrated activity was introduced each week (see Figure 1) that focused on one or more major mathematics content areas corresponding with that week's topic. After each activity, preservice teachers were introduced to demonstrations and research studies about how and why integrating mathematics with music might improve students' mathematics learning. Although participants had different schedules for presenting their lessons, the researchers took diligence in trying to ensure that both class sessions received equivalent activities during the intervention. Following the intervention activities, the researchers introduced two discussion forums. The first forum was introduced in week seven; the second forum, week eight. It should be noted that the same instructor taught both sections of the course, and though the participants were roughly equivalent demographically, they were not chosen based on any distinguishing factors.

## Data Collection

Data collection occurred during one academic semester over a period of three weeks. After the first six weeks of demonstrations involving integrated music-mathematics activities, preservice teachers participated in an online interactive discussion by writing a reflective essay and making comments on each other's essays. Two discussion forums were created that asked students about: (1) their experiences in participating in the music-mathematics integrated activities and (2) their plans to design and implement a mathematics lesson with music activities in their future classroom.

In weeks seven and eight, preservice teachers were asked to write a disposition essay with multiple paragraphs for each topic of the discussion forum. In week nine, preservice teachers were asked to provide meaningful comments with one or more paragraphs about their classmates' essays for both discussion forum topics. The first discussion forum topic yielded a total of 148 writing pieces (53 disposition essays; 95 follow-up comments; 109 writing pieces). The second discussion forum topic yielded 53 disposition essays and 56 follow-up comments. Although data were collected during the semester, they were not reviewed until after final grades were calculated. Figure 1 outlines the data sources and data collection timelines.

**Figure 1.** *Intervention and Data Collection Timelines*

Week 1 – Week 6	Week 7	Week 8	Week 9
Demonstration of Music-Mathematics Integrated Activities	Discussion 1	Discussion 2	Comments

## Data Analysis

For data analysis, a grounded theory approach (Corbin & Strauss, 2008) was used which posits “systematic, yet flexible guidelines for collecting and analyzing qualitative data to construct

theories grounded in the data themselves” (Charmaz, 2006, p. 2). The coding process was comprised of two main steps. The first step involved open and selective coding, which focused on creating categories and their properties. The second step involved theoretical coding, which connected the substantive codes together into a complete hypothesis and theory.

The constant comparative method was used to code the data. First, the writings were compared instance to instance to generate categories. Then, new instances were compared to the categories, which resulted in the formation of new categories from the comparisons. This process allowed for an initial and broad list of categories to be created from the first reading of the data. In the second phase, the data were grouped based on a series of questions such as “What category does this instance indicate? What is actually happening in the data?” (Glaser, 1978, p. 57). This allowed us to collapse the categories and establish themes that were immersed in the data. After the initial themes were established, a randomly selected subset of the data was reviewed and coded based on these themes to determine if the data were saturated or if additional categories needed to be created. Once the coding of subsets was completed, the remaining data were coded using categories previously developed.

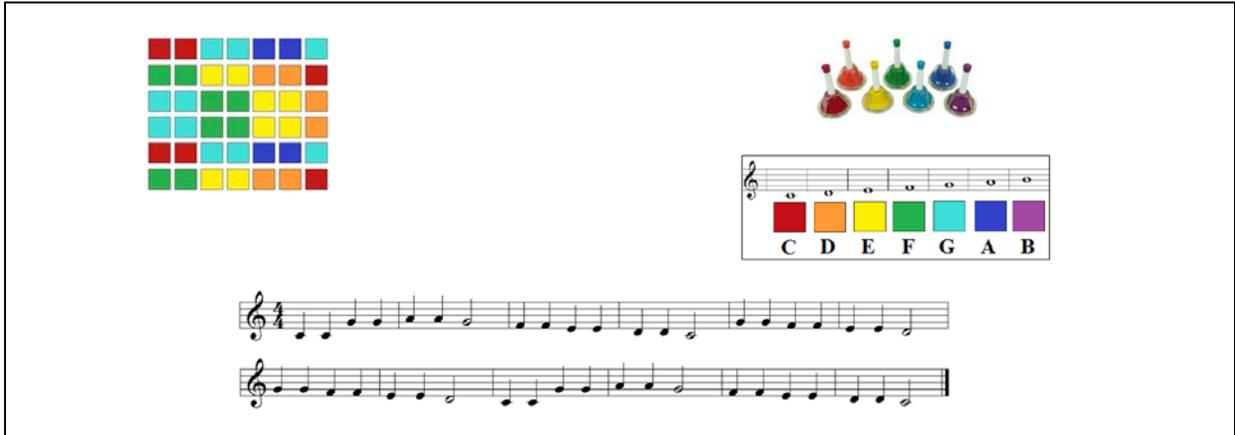
## Results

This study addressed two primary research questions. The first question examined elementary preservice teachers’ perceptions of the relative merits of music-mathematics integrated approaches to teaching and learning elementary mathematics. The second question attempted to determine how elementary preservice teachers planned to integrate music activities into elementary mathematics topics for their future mathematics teaching practice?

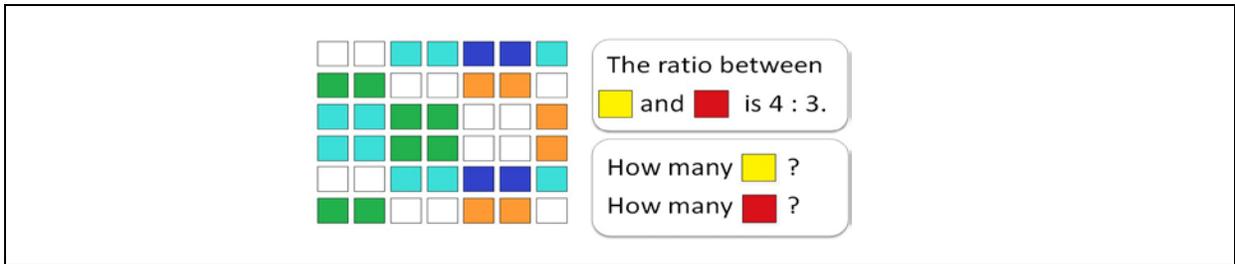
### *Interactive Music-Mathematics Activities*

Of the six interactive music-mathematics activities (see Table 1), the two activities experienced most by preservice teachers were “music composition” and “musical instrument design”. In these activities, preservice teachers learned how to use graphic notation (e.g., music color cards) and a variety of musical instruments (e.g., handbells, drums, keyboards) as manipulatives to teach mathematics as well as to represent music through statistical methods (see Figures 2-4). During these activities, preservice teachers were given musical pieces to help them understand how to use mathematical methods to analyze the pieces based on music theories, and they were given opportunities to experiment, practice, and apply various mathematics concepts and skills through the series of music-mathematics integrated lessons. For example, during the mathematics lesson that incorporated a music composition activity, music composition color cards were provided to the preservice teachers as a creative music composition tool. Preservice teachers used the color patterns, number patterns, and letter patterns written on the cards to compose the music, which was played by using hand bells; and they were asked to solve a number of mathematics word problems based on their musical works.

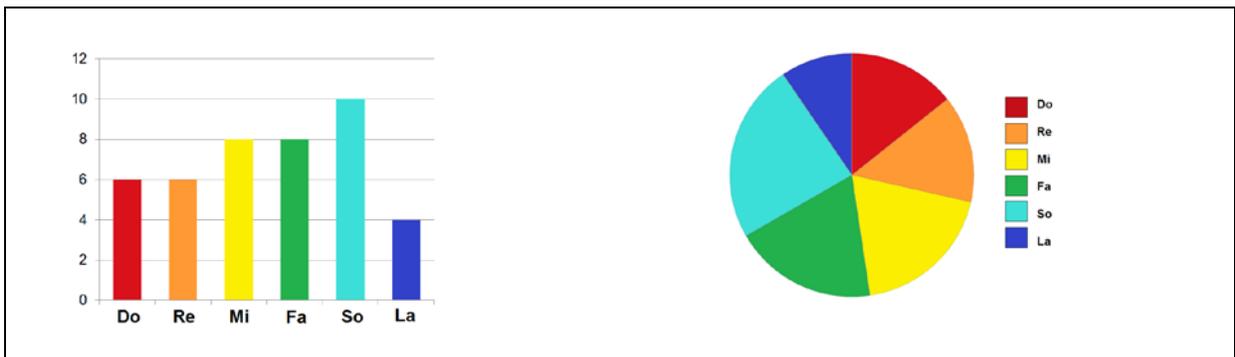
**Figure 2.** Handbells, Graphical Notation, and Staff Notation of “Twinkle Twinkle Little Star”



**Figure 3.** Sample of Mathematics Problems Based on “Twinkle Twinkle Little Star”



**Figure 4.** Statistical Graphs Based on “Twinkle Twinkle Little Star”



Further, the results indicated that preservice teachers expressed a variety of ideas about their perceptions of teaching mathematics integrated with music and teaching strategies for music-mathematics integration.

**Research Question 1:** *What are elementary preservice teachers' perceptions of the relative merits of music-mathematics integrated approaches to teaching and learning elementary mathematics?*

An analysis of preservice teachers' reflective essays from the first discussion forum (the topic, which was extricated from Research Question 1) revealed that preservice teachers' perceptions about the music-mathematics integrated approach to teaching mathematics yielded four main themes — fixing common mathematics education problems (33.78%), improving student academic achievement (43.92%), engaging math education with enjoyable experiences (94.59%), and developing creativity in mathematics and learning (30.41%) — and 21 subthemes. These themes and subthemes were generated from preservice teachers' writings on the discussion forum, which emerged from 53 disposition essays and 95 follow-up comments (see Table 3).

**TABLE 3**

**Themes Derived from Preservice Teachers' Perceptions Toward the Teaching and Learning of Mathematics Through Integrated Music-Mathematics Instruction**

General Themes	Subthemes	Response Count (n=148*)	Response Rate
1. Fixing Common Mathematics Education Problems	<ul style="list-style-type: none"> <li>• Ease teaching process</li> <li>• Reducing anxiety</li> <li>• Accommodating different learning styles</li> <li>• Appropriate challenge</li> <li>• Alternative understanding and evaluation</li> <li>• Remove language barrier</li> </ul>	14 15 9 7 3 2	50 (33.78%)
2. Improving Student Academic Achievement	<ul style="list-style-type: none"> <li>• Facilitate learning and development</li> <li>• Connecting and recalling information</li> <li>• Improve understanding</li> <li>• Broad content coverage</li> </ul>	15 22 22 7	65 (43.92%)
3. Engaging Mathematics Education with Enjoy-able Experiences	<ul style="list-style-type: none"> <li>• Foster engagement</li> <li>• Entertaining and fun</li> <li>• Motivation and interest</li> </ul>	28 75 37	140 (94.59%)
4. Developing Creativity in Math Teaching and Learning	<ul style="list-style-type: none"> <li>• Innovative pedagogy</li> <li>• Multiple approaches</li> <li>• Dynamic learning process</li> </ul>	30 8 7	45 (30.41%)

\* Note: Responses computed based on participants' total writing pieces.

*Fixing Common Education Problems*

The first theme, fixing common mathematics education problems, emerged from preservice teachers' views of the integrated music-mathematics approach as being an effective method to

engage students to focus on mathematics learning and participate in mathematics explorations. Collectively, teachers listed the enjoyable and exciting experiences solving musically contextualized mathematics problems, representing and reasoning through mathematics in diverse ways with musical connections, and cooperating with classmates to create and analyze mathematics through music as activity components that have the potential to motivate students to attempt more challenging mathematics tasks. Preservice Teacher A remarked that mathematics-music interdisciplinary teaching and learning experiences were engaging through the dynamic and interactive nature of the activities, and described how this pedagogical approach can help students experience mathematics learning with excitement.

Math it is a subject which the majority of the students think it is so difficult to learn. By using dynamic activities the students will be more focused and excited to participate and learn. Children love the sound of music, the beat, the rhythm is what catches their attention at the very first listen. Honestly I am impressed in how different kind of activities and content areas can we adjust using music-mathematics to teach them. In the class I was so excited to participate and be part of the lesson, I can imagine how more excited the young kids will be

### *Improving Student Academic Achievement*

The second theme, improving student academic achievement, emerged from preservice teachers' comments about how teaching mathematics integrated with music activities can improve students' achievement in mathematics. Various statements were provided to explain how this improvement could occur. For example, it would: (a) stimulate the brain as well as cognitive development, (b) help students to retain and recall information more effectively by making more connections, and (c) allow teachers to introduce different mathematics contents in relation to music. Preservice Teacher B remarked:

Music is the key to the soul. Music is used for celebration, expression, connections to others, and simple enjoyment. So why wouldn't music be good for education? I think that integrating music into learning is pure genius! Through the use of rhythm, the brain makes connections that otherwise would not be made. One method to help improve students' mathematical performance is to change the approach and to integrate mathematics with other academic subjects; music gives students a practical approach to learning and using math. It is important to prepare activities which can attract the students' attention, in that way the students will be able to remember what they are learning.

### *Engaging Mathematics Education With Enjoyable Experiences*

The third theme, engaging mathematics education with enjoyable experiences, materialized from Preservice teachers' shared instances about how music can help teachers solve or alleviate current issues in mathematics education, including mathematics anxiety, difficulty in reaching all students, language-culture barriers, and lack of techniques to provide alternative ways to represent and assess mathematics concepts. In general, preservice teachers reported that using music as a context to develop mathematics lessons benefits both teachers and their students.

Specifically, they noted that teaching mathematics through music is a way to present math concepts that make sense to students and it can provide more flexibility in presenting mathematics tasks that accommodate students with different learning styles and academic levels. Additionally, music-mathematics instruction can foster a more positive attitude and less anxiety among students, as well as facilitate communication with their peers through the universal language of music without language-culture barriers. Preservice Teacher C described his perspective of the value of music for reducing students' anxiety in learning mathematics in the following manner:

Learning how music can be integrated into math throughout this semester was interesting as well as fun. We as future educators have to create a classroom that has low anxiety and stress levels. Students often get frustrated when trying to understand math and with music-math activities I think students will not only enjoy it but it will help them comprehend the material better. Music can help to keep the levels of tension and stress to a minimum as well as have the power to keep students engaged. Using music activities in my future lessons can help by making the day more alive and interesting as well as the lesson.

### *Developing Creativity in Mathematics Teaching and Learning*

The final theme, developing creativity in mathematics teaching and learning, emerged from preservice teachers' perceptions of music being a meaningful context to combine with mathematics. Preservice teachers perceived this approach as having benefits not only for teachers (by providing them with new ways to demonstrate mathematics concepts) but also for students in helping them understand and apply mathematics in non-routine ways through active exploration processes and problem-based learning experiences. For example, Preservice Teacher D noted that many teachers refuse to teach mathematics in conjunction with the other school subjects because they believe they do not know enough about the educational connections among the different subject areas. Nonetheless, this participant stated that the music-mathematics integrated lessons was an innovative way to show teachers a new approach to teach contextualized mathematics and possibly learn with their students at the same time:

Fear keeps many teachers from trying to teach mathematics through music. Teachers often think they don't know enough about the relationships between these subjects. The relationship is not that mysterious and help does exist. Moreover, showing students how an adult goes about learning a new subject may well be one of the most important lessons a teacher can pass along. Admitting to students that we don't know something can be a daunting task for teachers, but the lessons learned from this experience can stay with students for a lifetime. This semester, we have seen many different strategies that are being used in classrooms. Since students have different ways to learn this is also helpful for those who struggle in learning the traditional way.

**Research Question 2:** *How do elementary preservice teachers plan to integrate music activities addressing elementary mathematics topics for their future mathematics teaching practice?*

An analysis of preservice teachers' reflective essays from the second discussion forum (the topic, which was extricated from Research Question 2), revealed 18 specific mathematics lesson topics and five mathematics content areas in which they proposed to integrate music activities (see Table 4). Numbers and Operations (75.22%) was the most prevalent content area followed by algebra (55.96%), geometry (37.61%), probability and data analysis (32.11%), and measurement (15.60%).

**TABLE 4**  
**Preservice Teachers' Plans for Integrating Mathematics Content Activities in Future Mathematics Teaching Practices**

Mathematics Areas	Mathematics Content/Lesson Topics	(n=109*)	
		Response Count	Response Rate
1. Numbers and Operations	Counting	14	82 (75.22%)
	Number Relationships	5	
	Concept of Fraction	20	
	Real Number (Negative Number)	3	
	Whole Number Computation	26	
	Basic Facts	8	
	Fraction Computation	6	
2. Algebra	Algebraic Patterns	46	61 (55.96%)
	Algebraic Expression	4	
	Ratio/Proportion	7	
	Representation of function	4	
3. Geometry	Shapes and Prosperities	22	41 (37.61%)
	Geometric Transformation	19	
4. Probability and Data Analysis	Concept of Chances	8	35 (32.11%)
	Independent Events	4	
	Statistical graph	21	
	Statistical Relationship	2	
5. Measurement	Time	11	17 (15.60%)
	Length	4	
	Area	2	

\* Note: Response computed based on participants' total writing pieces.

## *Numbers and Operations*

Preservice teachers identified numbers and operations as the most frequently cited mathematics area in which they would integrate music. Among the specific lesson topics mentioned were counting, number relationships, the concept of fractions, real numbers, whole number computation, and basic facts and fraction computation. Music composition and playing activities were described as the main musical elements that would be incorporated into the lessons. For example, Preservice Teacher E proposed a mathematics lesson with the learning objective of understanding the concepts of counting, addition, and subtraction:

I think addition, subtraction, and perhaps multiplication and division could be taught using music. I would use a maximum of ten bells to be rung by individual students to make up a problem then solved by other students as a game. Have certain numbers being represented by the different color bells. I would integrate music into a math lesson by having 10 students lined up at a time with different colored bells. Each bell represents a number for example blue is one, green is two, and so on. I would say word problems to them such as “5 minus two is”...and the correct responding colored bell must ring his or her bell. If a wrong bell is rung, someone else from the class must replace their spot to attempt the activity.

This preservice teacher continued by describing how she would develop a lesson about number operations not only for students in lower grades to practice counting, addition and subtraction, but also for students in higher grade levels to learn the concept of fractions, practice fraction computations, and pose their own problems:

A math lesson that I would do integrating music would be adding by using the music notes. For younger students I would have them counts or subtract the notes on a music sheet. For older students, I would use music to teach them fractions. I would go more on depth and instead of just counting or subtracting notes I would tell them about the value of each note. For example, adding a quarter note and a half note. So we would use this kind of examples to learn about fractions. I would also have the students play the songs that they are using to answer those questions, or I would simply play it to them. I would also have the students come up with their own problems using what they learned.

## *Algebra*

Algebra was the second most frequently cited area identified by preservice teachers as an area in which they would teach music-mathematics integrated activities. Among the content lesson topics identified by preservice teachers for incorporating the music-mathematics strategy were algebraic patterns, algebraic expression, and ratio and proportion as well as representations of functions. The key musical elements cited for infusing these activities were music composition and playing activities. Preservice Teacher C plans to ask students to create their own music by using patterns, which will be embedded into the beat, rhythm, melody, and tempo of students' music. Preservice Teacher C remarked:

The type of lesson that I will integrate music into would be patterns. Patterns are a unifying theme of mathematics. Students will be able to investigate the patterns that they find in numbers, shapes and expressions as they make mathematical discoveries as well as create a variety of patterns that will help them represent mathematics in the real world. Numbers and shapes certainly offer many opportunities, but so do music, language and physical activity. Students will create a beat, rhythm, melody and a tempo as they see the pattern they are creating the music for, just like we did in class. Students will also create their own patterns by using color-music cards to compose music and play their music by using handbells.

Ratio and proportion and other algebraic concepts were also expressed by preservice teachers in their strategies for teaching a music-mathematics integrated lessons to students that introduces part-part ratios and part-whole ratios. These mathematical topics are relevant to music in several ways, one of which is that ratios are key to understanding why different notes have different tones. Likewise, the concept of ratio between different notes can be developed into algebraic questions and formulas (see Figure 2). Preservice Teacher E described this approach as follows:

Ratio proportion is one of the mathematical concepts that can be taught based on using the graphic musical notations to compose and play music. I will prepare musical composition cards for my students to compose and play music, and then I will ask them to find the ratio of different musical notes (cards with different colors) in the music that they composed by themselves. For example, what is ratio between the music note of Mi (yellow card) and Fa (Green card) in the song? What is ratio between the music note of Re (orange card) and all the music notes in the song?

### *Geometry*

Preservice teachers also generated a series of geometry and measurement lessons that can be taught with music activities. Some lessons were based on: (a) making geometric transformations such as reflection and rotation within the music composition processes, (b) musical instrument designs such as using geometrical figures to construct a guitar and taking geometrical measurements of musical instruments, and (c) using songs to help students remember geometrical shapes. As an example, Preservice Teacher F proposed a lesson using songs and singing activities to teach geometric figures:

If I am going to design a math lesson that integrates with music activities, I would focus on shapes for the lower grades. I would incorporate songs that would help my students memorize the shapes. For example, like singing the Hockey Pokey song to remember the shapes. I would cut out the shapes for every child in the class then we would start singing the song and when they hear a certain shape they need to find the shape in their pile and hold it up, like if we sing "Put your Circle in, put your circle out put your circle in and you shake it all about..." they have to hold the circle up.

Other preservice teachers like Preservice Teacher B, developed lessons using musical instruments to teach shapes. The narrative below describes how Preservice Teacher B integrated

guitar designing with triangles by using electronic guitar images and music videos with students exploring triangles by constructing their own guitar outlines:

I will let my students practice measuring angles and create the body of an electric guitar out of triangles. First, I will provide some examples about electric guitars from Google Image, and also play some video clips from Youtube.com of musicians playing electric guitars. Then I will ask students to design their own guitar by drawing triangles and make geomantic transformations about their triangles. Students will then explore the property of triangles including angles and sides.

### *Measurement*

Measurement, the third most frequently cited category, is a concept that involves understanding and representing length, size, capacity, time, and weight. These concepts can be found in musical instrument designing and music playing processes. Preservice Teacher G introduced a strategy of teaching measurement which included the following: (a) music instruments being used as a measurement task whereby students are directed to investigate the length and size among different instruments by using different measurement units and (b) music pieces being used as measurement tasks and time as a measurement concept represented by integrating it with length:

Students could create their own musical instruments and take measurements of their materials used as they go. I will teach students what units are better for measuring several different instruments, for example, inches, meters, cm, km, depending on the size of the instrument. Students can measure the length of the instruments and compare them to other instruments. They can also measure the length of the note in a music piece to see how far they can measure, for example, whole note=1inch, and half note=1/2 inch and so forth. They can measure the strand of the music.

### *Probability and Data Analysis*

The fourth category, probably and data analysis, preservice teachers proposed a variety of activities for teaching probability and data analysis through music composition and playing. For example, Preservice Teacher C proposed a musical experiment for checking out independent events through a music composition process—e.g., after students play the music they compose, they will analyze the frequency of each musical note they used to construct a bar graph. Specifically, this preservice teacher said:

For probability, students can create a song with the handbells using the corresponding colored squares and collect data from the squares (How many blue, purple, etc.) Then they can put the squares in a bag and mix them up, and try to find the probability that a particular number will be played and this would correlate with data analysis. For data analysis, students will be introduced to different string instruments and will be asked to create a bar graph according to the number of strings in more than one instrument. For example, the guitar has 6 strings so the graph would go up to 6. The violin has 4 or so, the graph would go up to 4.

## Discussion

The results of this study provide insight into elementary preservice teachers' views about the intricacy of music-mathematics strategies for teaching mathematics. The majority of preservice teachers provided positive reflections about music-mathematics integrated pedagogy for teaching mathematics and how it allowed them to extend beyond the limitations of traditional mathematics instruction to present and apply mathematics knowledge in a more contextualized approach where students have opportunities to understand mathematics concepts in enjoyable and meaningful ways. They also proposed a variety of ways to teach mathematics with integrated music activities.

For the music-mathematics integrated instructional strategies, a total of 20 specified math lesson topics covering the five major mathematics content areas listed in the NCTM standards (NCTM, 2000) — numbers and operations, algebra, geometry, measurement, and data analysis and probability — were identified by preservice teachers. These prospective instructional strategies provided additional examples of how music activities can be developed as a way to address mathematics lessons. This finding is consistent with previous studies showing that there are multiple ways that teachers can design mathematics lessons with different topics integrated with music (An et al., 2013; Robertson & Lesser, 2013). Further, the study revealed that connections between music and mathematics could be developed as an integral part of mathematics lessons. It also showed that this connection could be used to introduce a mathematics concept at the beginning of lessons, apply mathematical knowledge in the middle of a lesson, or practice mathematical skills at the end of a lesson.

### *Benefits of the Music-Mathematics Instructional Approach*

Overall, preservice teachers' perceptions about teaching mathematics through music are consistent with existing research on arts based interdisciplinary instruction in K-12 settings and teacher education programs. This research has shown that music-mathematics integrated instruction can reduce students' mathematics anxiety (An et al., 2008; An et al., 2011; Colwell, 2008); accommodate different learning styles and allow alternative ways of thinking (Brown, 2013; Colwell, 2008); improve students' engagement and motivation in mathematics learning (Brown, 2013; Mansilla, 2005); and develop creativity in STEM teaching and learning (Carrier et al., 2011; Marshall, 2005).

The first theme, which emerged from preservice teachers' written essays about music-mathematics integrated instruction, was *fixing common mathematics education problems*. This finding was consistent with previous studies which found that music-mathematics integrated instruction can reduce common mathematics education problems, such as mathematics anxiety (An et al., 2008; An et al., 2011; Colwell, 2008). In the current study, preservice teachers envisioned that an integrated music-mathematics curriculum provided the tools to solve common mathematics education issues. They also perceived that such a curriculum could potentially fix common mathematics education problems such as math anxiety, language barriers among students, lack of challenge for high achieving students, lack of strategies to accommodate students with diverse needs, and lack of alternative ways to provide assessment. Explanations for the positive perceptions of this integrated approach may be associated with preservice

teachers' experiences, the majority of whom indicated that this approach allowed them to escape the limitations of traditional mathematics instruction that consisted mostly of presenting and applying mathematics knowledge that is disconnected from other topics. They also viewed this approach as more contextualized which allowed students opportunities to understand mathematics concepts in enjoyable, meaningful, and relevant ways.

The second theme that emerged was *improving student academic achievement*. This finding was consistent with previous studies, which have found that a music-integrated curriculum can enhance student thinking and subsequent learning performance. For example, Brown (2013) and Colwell (2008) found that an integrated curriculum, especially one that uses music as a pedagogical theme, could accommodate different learning styles and allow alternative ways of thinking. In this study, preservice teachers viewed music-mathematics (which involves listening to music, playing music, and music composition), as a medium for facilitating students' mathematics brain development and helping students memorize abstract mathematical concepts by associating mathematics facts with their favorite songs. By using music as a tool that students can use to associate with different pieces of knowledge, they have more innovative ways to retain and recall information, which may help them develop problem solving strategies by connecting different mathematics content together and using alternative ways to internalize mathematics concepts.

The third theme, *engaging mathematics education with enjoyable experiences*, was consistent with existing studies (e.g., Robertson & Lesser, 2013) which have found that a music integrated teaching method can have powerful positive effects on student-teacher engagement. In a carefully designed learning environment built on music activities, An and colleagues (2011) found that preservice teachers were aesthetically engaged. Throughout the intervention, it was reported that preservice teachers' engagement might be improved as a result of: (a) their original interests in music and curiosity of finding the mathematical patterns behind music; (b) the enjoyable experiences of composing music, playing music and designing musical instruments based on meaningful mathematics arrangement; and (c) the creative investigation of mathematics pedagogical components based on their authentic musical works. Preservice teachers were convinced that the learning environment created, based on music activities, can be used as effective mathematics teaching strategies to engage elementary students to participate more with mathematical tasks.

The fourth theme, *developing creativity in math teaching and learning*, which emerged from preservice teachers' discussion forum, centered around music-mathematics integrated instruction as a medium for developing creativity in math teaching, and learning that could provide a positive math classroom environment for students. This finding was consistent with previous literature (Carrier et al., 2011; Marshall, 2005) which suggested that offering students' learning experiences based on interdisciplinary tasks would promote creativity in both teaching and learning processes for teachers and students. In the current study, preservice teachers indicated that a music-mathematics integrated lessons would allow teachers to implement more innovative teaching methods by providing students with opportunities to think outside of the box. These strategies offer various non-drill forms of activities that have the potential to help students view mathematics from different perspectives. As a result, when students are solving mathematics problems, they have more ideas and approaches in mind for how to solve the problem.

Preservice teachers also reported that music could be explored as a framework for developing mathematics lessons by making connections within and outside of the mathematics curriculum. The interdisciplinary teaching scenarios in the music-mathematics lessons can prompt students to use meaningful strategies through wider and more flexible processes to learn mathematics.

### *Music-Mathematics Teaching Strategies*

Preservice teachers posed a variety of music-mathematics teaching strategies for the major mathematics content/lesson areas. This finding was consistent with previous studies (An et al., 2011; An et al., 2013) which have reported that when teachers have opportunities to experience exemplary mathematics lessons based on meaningful music activities, they will be enabled to explore more connections between the two subjects and design innovative mathematics lessons based on inventive musical elements. In the intervention, multiple examples of music and mathematics integrated activities were introduced to preservice teachers in music composition/playing activities and musical instrument designing activities.

From these experiences, preservice teachers were not only convinced that there are connections between music and mathematics, but also that these links can be further explored and developed as meaningful parts of mathematics lessons. Topics in music that range from basic (i.e., rhythm, intervals, and intervals) to advanced (i.e., melody, music form, and instrumentation) can be integrated into mathematics from elementary to college courses (Harkleroad, 2006). The current study suggests that preservice teachers' pedagogical content knowledge (especially knowledge of preparing mathematics lessons with meaningful connections within and outside of mathematics content) provides them with more opportunities to understand how to identify and apply educational resources to design and implement mathematics lessons in different ways.

Of the mathematics lessons proposed by preservice teachers, the two primary music activities integrated into their lessons were music composition and playing activities. Perhaps the main reason for this finding can be attributed to preservice teachers learning how to develop effective mathematics lessons from the examples provided to them during the intervention. To some extent, the intervention activity enlarged preservice teachers' pedagogical and curriculum knowledge about teaching mathematics integrated with music. Unlike common teaching methods that teachers use to teach mathematics through music by letting students count beats, rhythms or sing a song with mathematics content as lyrics (Rogers, 2004), most of the preservice teachers in this study explored the music-mathematics connections in a more profound way by developing lessons based on complex music activities such as music composition and musical instrument development.

### **Conclusion**

The current study had several limitations, among which are the sample size, which limits its generalizability; an ethnically homogenous sample with perceptions that may not reflect the perceptions of other minority preservice teachers; and the diversity in preservice teachers' interests or abilities in music and their general attitudes toward mathematics education, all of which could have been better assessed at the beginning of the course.

The findings suggest several things. First, the need for additional studies that investigate preservice teachers perceptions and attitudes of contextualized mathematics and its impact on their mathematics content knowledge. Second, the need for studies on how preservice teachers learn about music-mathematics integrated teaching strategies and the implementation of these strategies in the classroom. Thirdly, the need for knowledge on how to best integrate technology into the music-mathematics curriculum, which is a natural next step in this area that could strongly impact both teacher and student outcomes.

The findings also invite further research with pre-post assessment and control groups on the effects of using music-mathematics integrated lessons as interventions and preservice teachers' self-efficacy beliefs towards teaching mathematics. Also, additional research needs to be done on constitutional research and its impact on music-mathematics integrated curriculum and instruction on teachers' pedagogical content knowledge and their students' mathematics dispositions and achievement at various grade levels.

One of the most important goals of teacher education programs is to develop preservice teachers' comprehensive abilities and positive perceptions toward teaching and learning mathematics. However, many preservice teachers have limited opportunities to learning innovative methods for teaching mathematics. As such, mathematics teacher educators and curriculum developers need to provide a variety of workshops and other professional development opportunities to introduce preservice teachers to more effective and innovative teaching strategies. Mathematics methods courses have the potential to positively shift teachers' attitudes and beliefs towards mathematics through observing effective lessons, meaningful activities, and authentic experiences involving the development, implementation, and evaluation of mathematics lessons in innovative ways (Gresham, 2007; Knoblauch & Hoy, 2008).

By designing appropriate music activities integrated across mathematics lessons, teachers can offer more approaches for students to comprehend, investigate, and apply mathematics (An et al., 2013). With the aim of facilitating preservice teachers' understanding of effective ways of teach mathematics lessons to elementary students, teacher educators should make extra effort to improve preservice teachers' awareness and abilities for teaching mathematics contextualized with the activities that children may be engaged, such as music (Robertson & Larry, 2013).

The music-mathematics integrated teaching strategy, with its unique feature of creating a high motivational learning environment (An et al., 2011), is an effective teaching strategy that allows preservice teachers to effectively and creatively meet the needs of students. Findings from this study indicate that preservice teachers benefit from the opportunities to experience and learn different ways of teaching mathematics lessons. Teaching mathematics with connections to other disciplines, such as music, can improve students' understanding of mathematics concepts and their dispositions about mathematics (van de Walle, 2010). Consequently, we recommend that mathematics teachers take advantage of the rich connections between music and mathematics to develop a variety of activities to teach mathematics in an enjoyable and productive way.

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