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Strengthening pre-service teachers' mathematical content knowledge

Introduction

Over the past two decades there has been a growing body of literature concerning the preparation of pre-service mathematics teachers. Research efforts have focused principally on approaches conducted by tertiary educators to adequately prepare pre-service teachers (PST) for the mathematics classroom. In particular, such efforts have emphasised the importance of *pedagogical content knowledge* (PCK) (Beswick & Goos 2012; Shulman 1986; Shulman 1987) and *mathematical content knowledge* (MCK) (Meany & Lange 2012; Ponte & Chapman, 2008) in teacher preparation programs. Specifically, researchers have reported on ways to best support pre-service primary and secondary teachers' PCK (Aguirre, del Rosario Zavala & Katanyoutanant 2012; Kennedy, Ball & McDiarmid 1993) and MCK (Ma 1999; Stohlmann, Moore & Cramer 2013), the effects of mathematical content units on PST (Matthews, Rech & Grandgenett 2010), and the effects of mathematical pedagogy units on PST (Sowder 2007). Other research has drawn attention to PST confidence levels and attitudes towards mathematics (Hamlett 2009). Despite the extensive literature there is no consensus on how to adequately train PST of mathematics (Ball, Hill & Bass 2003; Chapman 2005). However, a growing number of scholars recommend teacher educators focus their efforts on *mathematical knowledge for teaching* (MKT), or teaching both for procedural understanding and mathematical fluency (Delaney Ball, Hill, Schilling & Zopf 2008).

Research aims and significance

There are two specific aims of this research project. The first is to investigate the self-perceptions of pre-service primary and secondary teachers enrolled in a mathematics education unit as they engage with and consolidate their mathematics content. The second aim is to explore how these pre-service teachers understand and perceive their 'readiness' to undertake such a task, based on their recent tertiary training. Both aims will be investigated during the two stages of the research project. The significance of this research lies in the belief that the unit *ED2315: Mathematical Learning for Early Adolescents* adequately prepares students' mathematical content knowledge in conjunction with their pedagogical content knowledge, and that research into this area can strengthen future efforts in preparing pre-service teachers. Specifically, the unit has the potential to influence the way pre-service mathematics teachers are professionally prepared to teach mathematical content in the classroom. This study seeks to build upon the extant literature by describing the self-perceptions of pre-service teachers preparing to teach mathematics to upper primary and lower secondary students.

Literature review

Preparation of pre-service mathematics teachers

The preparation of elementary teachers in mathematics is an increasingly critical topic for teacher preparation programs (Matthews, Rech & Grandgenett 2010). Preparing teachers to teach mathematics effectively is one of the most urgent problems facing those who wish to improve students' learning (Morris, Hiebert & Spitzer 2009). Unfortunately, it cannot be assumed that pre-service primary teachers entering Australian universities will be competent in mathematics (Hamlett 2009). Hutchinson (1997) extends this notion, reporting how graduate teachers faced many problems which were largely due to their inadequate preparation in primary school mathematics content knowledge. According to Hungerford (1994) the Mathematical Association of America has registered a similar sentiment: "the mathematical preparation of elementary school teachers is perhaps the weakest link in our nation's entire system of mathematics education". More

than 60 years ago, Glennon (1949) reported “those preparing to teach mathematics in the elementary grades understand approximately 50% of the computational processes taught in grades one to six” (cited in Rech, Hatzell & Stephens 1993). Acknowledging that it is unrealistic to expect graduates of teacher preparation programs to enter the classroom as expert teachers, Hiebert, Morris, Berk and Jansen (2007) proposed that PST should acquire knowledge, skills, and dispositions that would enable them to study their teaching and gradually improve over time. After the Australian Academy of Science identified mathematics as a critical skill for Australia’s future, it was recommended that “all mathematics teachers in Australian schools have appropriate testing in the disciplines of mathematics and statistics” with “national accreditation standards for teachers of mathematics at all levels of schooling... and... appropriate programs to ensure that future teachers meet those standards” (Rubinstein 2006, p. 15). Clear evidence indicates a strong relationship between teachers’ mathematical content knowledge and their ability to teach well in classrooms (Ball, Hill & Bass 2003; Chapman 2005). Additionally, research in the United States has shown that the quality and the rigour of the mathematics curriculum are strongly correlated to the mathematical content knowledge of the teachers (Schmidt 2002).

Pedagogical content knowledge (PCK)

Pedagogical content knowledge (PCK) is concerned with the most useful ways of representing and formulating mathematics that make it comprehensible to others (Beswick & Goos 2012). PCK is also necessary for teachers to understand student misconceptions, to know how topics are organised and taught, as well as to influence the ability to adjust lessons catering for all learners (Shulman 1987). Drawing upon the original ideas of Shulman (1986), many scholars have developed theoretically and empirically-based approaches to enhancing pedagogical approaches. For instance, Kennedy, Ball and McDiarmid (1993) have suggested that during the teaching ‘act’, teachers draw upon knowledge of: learners, learning, pedagogy, the teacher’s role, curriculum, and subject matter. In her work in teacher development, Grossman (1990) delineated four specific components of this specialised form of teaching knowledge:

1. An overarching knowledge and belief about teaching a subject at specific grade levels;
2. Knowledge of students’ understandings, conceptions, and potential misunderstandings of particular topics of a subject;
3. Knowledge of curriculum and curricular materials, including horizontal and vertical directions within a subject;
4. Knowledge of the instructional strategies and representations for teaching particular topics.

In relation to mathematics teaching, Sowder (2007) argued that Grossman’s delineation of PCK into four components “are helpful for those developing teacher education programs and professional development opportunities for mathematics teachers” (p. 164). Moreover, Aguirre, del Rosario Zavala and Katanyoutanant (2012) highlighted how mathematics PSTs require intellectual tools to support and extend PCK in rich and rigorous ways. One tool is through the exercise of reflective practice from a culturally responsive mathematics teaching perspective, whereby PSTs can develop robust forms of PCK to help them become effective mathematics teachers.

Mathematical content knowledge (MCK)

Mathematical content knowledge (MCK) has been described as a comprehensive understanding of mathematics which has breadth, depth, connectedness, and thoroughness (Ma 1999). MCK has also been described using a variety of terms, including: numeracy (Kemp & Hogan 2009),

quantitative literacy (Latiolais, Baloch & Loewi 2003; Steen 2001) and computational fluency (Flowers 2003) as well as mathematical literacy. As articulated by PISA (Programme for International Student Assessment), mathematical literacy is viewed as “an individual’s capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgements and to engage in mathematics in ways that meet the needs of that individual’s current and future life as a constructive, concerned and reflective citizen” (cited in Steen 2001, p. 7). Mathematics understanding is understood as “the dynamic, constructed, and reconstructed process of sense making by the learner” and “learning to represent or communicate mathematical ideas or interpret mathematical representations through the use of language, diagrams, pictures, manipulatives, and other tools” (Heaton 2000, p. 4). As such, it is important for PST to have a firm grasp of the mathematics they will teach in order to facilitate their students’ explorations and investigations of mathematics concepts (Whittington 2002).

Pre-service teachers' content knowledge is an essential focus for properly preparing teachers (Darling Hammond 2000; Stohlmann, Moore & Cramer 2013). The AAMT *Standards for Excellence in Teaching Mathematics in Australian Schools* underscored the importance of teachers’ MCK: “excellent teachers of mathematics have a sound, coherent knowledge of the mathematics appropriate to the student level they teach” (2002). However, no consensus exists on what mathematical content knowledge is needed to teach well (Ball et al. 2001). Research in the United States has shown that elementary teachers have room for improvement in robust content knowledge (Ball, Lubienski & Mewborn 2001; Ma 1999). Several researchers have highlighted how many pre-service teachers - even those who have completed a major in mathematics (Bryan 1999) - have gaps in their content knowledge or deficiencies in knowing how to apply and teach the mathematics addressed across the secondary school mathematics courses (Ball & Wilson 1990; Mansfield 1985). Without sound mathematical knowledge many pedagogical processes are of little benefit (Southwell & Penglase 2005).

There is clear evidence on the relationship between teachers’ MCK and their ability to teach well in classrooms (Ball, Hill & Bass 2003; Chapman 2005). Mathematical content courses are an effective way of enhancing the mathematical knowledge that elementary teachers might require for their own classroom instruction of mathematics (Matthews, Rech & Grandgenett 2010). Moreover, such content courses can help new teachers to become more prepared to teach and represent the increasingly important discipline of mathematics to their students. Many states in the United States require prospective secondary mathematics teachers to pass the *Praxis II: Mathematics Content Knowledge* (10061) test before becoming certified to teach (Wilburne & Long 2010). While this test focuses on measuring PST undergraduate MCK, it does not assess their PCK, nor their understanding of the mathematics they will teach in a secondary classroom. Many mathematics educators, including Usiskin (2001), emphasise the need for PST to acquire content knowledge different from the kind they normally receive in university level instruction. Policy documents including *A Call For Change*, (Leitzel 1991) and *The Mathematical Education of Teachers* (Conference Board of the Mathematical Sciences 2001) recommend PST undertake courses that deepen and enhance current knowledge and conceptual understanding of the mathematics they will teach (Wilburne & Long 2010). In support of this recommendation, teachers’ MCK is not dependent on the number of university level courses, their grade point average, or their scores on a standardised test (Even 1993; Ball 1990). Furthermore, Monk (1994) found no correlation between the number of undergraduate mathematics courses taken by secondary mathematics teachers and their students' achievement. According to the Conference Board of Mathematical Sciences (2001), few university level mathematics courses actually address the mathematics that pre-service secondary teachers will teach. Instead, teacher educators need to demonstrate the importance of

well-structured content knowledge activities (Stohlmann, Moore & Cramer 2013) to improve MCK without having a negative impact on PST confidence (Hamlett 2009).

Mathematical knowledge for teaching (MKT)

Following Shulman's (1987) advocacy of knowledge categories for effective teaching, Delaney, Ball, Hill, Schilling and Zopf (2008) actively analysed and developed fundamental knowledge categories relevant to mathematics teachers. Instead of taking into account the multiple facets of PSTs' knowledge and beliefs, there appears to be a tendency among teacher educators to view PSTs as simply lacking particular knowledge (Delaney et al. 2008). Furthermore, although some PSTs are able to successfully solve mathematical problems, many are unable to explain the concepts and procedures they perform (Mewborn 2001). Significantly, Ball, Hill and Bass (2005) found a correlation between a teacher's mathematical knowledge and student achievement. However, these researchers concluded that teaching PSTs more content knowledge is not the best way to prepare teachers; rather, teaching for understanding is required. In addition to the content (i.e. the 'what' of mathematics), Delaney et al. (2008) asserted that teachers also need to know 'how' to teach mathematics, and at the same time coined the term *mathematical knowledge for teaching* (MKT). Following research into MKT, certain scholars believe that implications for translating the content matter of mathematics into effective pedagogical practice are paramount in raising the profile of mathematics (Butterfield & Chinnappan 2010). Others assert that MKT provides the most promising current answer to the longstanding question of what kind of content knowledge is needed to teach mathematics well (Morris, Hiebert & Spitzer 2009). Various studies at the elementary school level provide initial data linking teachers' MKT with the mathematical quality of instruction (Hill et al. 2008) and the level of students' achievement (Hill, Rowan & Ball 2005). In support of developing MCK in PSTs, Ball, Thames, Bass, Sleep, Lewis and Phelps (2009) contended that teachers must be able to understand why particular content is taught and how the content should be developed. Additionally, teachers must be able to use their mathematical knowledge in teaching for identifying a range of solutions and mathematical connections when they are teaching students, planning lessons and evaluating students' work (Ball et al. 2009; Ball et al. 2008). To use MKT effectively, teachers must be able to access a wider range of knowledge such as procedural knowledge and fluency, concepts and connections (Ball, Hill & Bass 2003).

Methodology

Context

At The University of Notre Dame Australia the unit *ED2315: Mathematical Learning for Early Adolescents* is offered to pre-service primary and secondary teachers completing a Bachelor of Education degree. Enrolled students can use the unit towards a specialisation or major in mathematics education. The unit is run over thirteen weeks for a total of 39 hours of contact time, and it is worth 25 credit points. During contact hours, pre-service teachers engage with middle school mathematical content (suitable both for upper primary and lower secondary students), receive exposure to best pedagogical approaches in teaching that content, review key curriculum and policy documents, and investigate best-practice approaches regarding planning, assessment, technology, and resources. Within the unit, students complete three assessments: an Analytical Paper (AP), Mid-Semester Examination (MSE) and a Final Summative Examination (FSE). The AP consists of two parts, where students must (i) complete 10 algebraic problems showing full working out and a final solution, and (ii) after choosing one of the problems completed in (i), write a 1,500-word analytical paper articulating best pedagogical approaches in teaching this problem to

a middle school class. The MSE is a 75-minute assessment that requires students to demonstrate competency in the mathematical content covered in Weeks 1 to 7. The FSE is a 130-minute assessment consisting of two parts, where students must (i) write extended responses to two of five key topics in middle school mathematics, and (ii) demonstrate competency in the mathematical content covered in Weeks 7 to 13. The enrolment for this unit is approximately 40 students per semester.

Method

This study was interpretive in nature, and primarily used qualitative research methods to collect and analyse data about how pre-service teachers perceived their readiness to teach middle school mathematics. Participants recorded self-perceptions through three qualitative questions, and one seven-item, five-point, Likert scale question. The researcher used two online, qualitative surveys to collect data from research participants. In Stage 1 of the project, participants were asked to respond to four research questions prior to commencing a thirteen-week tertiary unit based on mathematical content. During Stage 2 (immediately following the completion of this unit), the participants will be asked the same research questions. The research questions are included in Appendix 1.

Participants

Stage 1

The entire student population enrolled in the tertiary unit *ED2315: Mathematical Learning for Early Adolescents* was invited to participate in the research. Half of the students enrolled in this unit (20 of 40) comprised a self-selected sample for Stage 1 and completed a pre-unit survey. All students had the opportunity to review the ED2315 unit outline prior to giving informed consent to participate in the research. From the 20 participants, 6 were male and 14 were female; 12 participants were completing a BEd (Primary) degree, 4 a BEd (Secondary) degree, and 4 a BHPE degree (Table 1).

Table 1: Project participants (Stage 1)

	Bachelor of Education (Primary)	Bachelor of Education (Secondary)	Bachelor of Health and Physical Education	Total
Male	1	2	3	6
Female	10	2	2	14
Total	11	4	5	20

Data analysis

Qualitative data from the 20 pre-unit surveys were analysed and explored for common themes. When analysing these data, this project adhered to the framework and guidelines offered by Miles and Huberman (1994). This framework assisted the researcher in identifying relationships among social phenomena, based on the similarities and differences that connect these phenomena. The approach itself is comprised of three main components: data reduction, data display, and drawing and verifying conclusions. These components themselves involve three main operations: coding, memoing, and developing propositions. Codes, as Miles and Huberman (1994) explain, “are tags or labels for assigning units of meaning to the descriptive or inferential information compiled during a study” (p. 56). These codes were attached to the data gathered through qualitative surveys, and were selected from those data based on their meaning. The researcher then used memoing to synthesise coded data together so that they formed a recognisable cluster grounded

within one general concept. The memoing process also captured the ongoing thoughts of the researcher as the process of coding took place. Lastly, as a study proceeds, there is a greater need to “formalise and systematise the researcher’s thinking into a coherent set of explanations” (Miles & Huberman 1994, p. 75). For Stage 1 of this project, the researcher generated propositions about connected sets of statements made by participants, reflected on the findings, and drew preliminary conclusions from the study.

Presentation of findings

Self-perceptions of readiness to teach mathematical content

All of the participants (20 of 20) indicated the extent to which they felt ready to teach mathematical content to upper primary and lower secondary students. For the seven content topics available (Fractions, decimals and percentages; Algebra; Equations and formulas; Perimeter, area, volume and capacity; Calculating probability; Using statistics; Venn diagrams) participants provided a Likert-scale rating from 1 - 5 (with 1 representing feeling least ready, and 5 representing feeling completely ready). Overall, participants shared they felt most ready to teach the topic *Perimeter, area, volume and capacity* (mean = 3.85) and least ready to teach the topic *Using statistics* (mean = 3.15). The mean score for each topic indicated a feeling of readiness among participants (3.15 < mean < 3.85), although not overwhelmingly so. These data are presented in Table 2.

Table 2: Self-perceptions on teaching content (1 = least ready, 5 = completely ready)

Topic	1	2	3	4	5	Total	Mean	S.D.
Fractions, decimals and percentages	3	1	6	7	3	20	3.30	1.23
Algebra	1	2	8	7	2	20	3.35	0.96
Equations and formulas	1	1	9	7	2	20	3.40	0.92
Perimeter, area, volume and capacity	0	3	4	6	7	20	3.85	1.06
Calculating probability	3	2	5	5	5	20	3.35	1.35
Using statistics	2	2	10	3	3	20	3.15	1.11
Venn diagrams	2	4	4	4	6	20	3.40	1.36

Readiness to teach students

All participants (20 of 20) reported a variety of responses regarding their self-perceived readiness to teach mathematical content to upper primary and lower secondary students. Qualitative responses were classified by the researcher as: *confident, uncertain, unconfident, or undecided*. Eight of twenty participants communicated they felt confident to teach mathematical content. Various participant comments included “I feel completely ready”; “My core knowledge is strong”; “As I excelled in mathematics in school I believe I believe I am ready to teach any level of maths”; and “I completed the highest level of maths in Year 12. I also currently tutor high school students in mathematics.” Nine of twenty participants shared that they felt uncertain about teaching; for a majority of these responses participants articulated that they required further MCK, PCK and MKT, to varying degrees. To illustrate, one participant shared “I believe I have thorough content knowledge and skills, but would need to work on how I deliver [those] to the class.” Another participant stated that “I would be able to confidently extend a fair amount of knowledge to students, but cautious to receive questions and queries.” A comparatively smaller number of participants intimated that they felt either unconfident (1 of 20) or undecided (2 of 20) to teach

mathematics. One of these participants offered that “My mathematical involvements during school were very poor and I feel myself being very unconfident in this subject. Therefore, I think my readiness to teach the subject is very low until I know the content strongly enough to teach it.”

Mathematical content knowledge support needed

A majority of participants (18 of 20) asserted that they required further MCK in order to adequately teach upper primary and lower secondary students. In addition, these participants identified particular Australian Curriculum strands where they felt additional MCK was needed. Specifically, over half of these participants (12 of 18) reported they required support in the strand *Number and algebra*. One participant commented that support in *Number and algebra* could include “Recapping [sic] a lot of formulas as most have been forgotten leaving myself stuck in situations; long expanded word questions; I need to get familiar with algebra again as I have forgotten plenty which leaves me unsure if I’m doing it correctly”. A comparatively smaller proportion of participants reported that they needed further MCK from Australian Curriculum strands *Statistics and probability* (4 of 18) and *Measurement and geometry* (2 of 18). From the 2 of 20 who registered a feeling of ‘unsure’ one of them stated “None yet, but I’m sure we’ll find some.”

Getting the most out of the unit

All participants (20 of 20) predicted that completing the unit would positively contribute to their preparation as mathematics teachers. In particular, over half of the participants (14 of 20) mentioned that this unit would assist in both strengthening their MCK and PCK. One student was hopeful the unit would

Reinforce and clarify basic mathematics principles. Confirm that the way I do and think about them are correct and that I am not missing an easier or more certain method or understanding of them. I could probably do many of the examples at this level using a calculator, pen and paper but explaining the why of maths and the relationships between principles is something I hope to gain a clearer understanding of as well as the principles for teaching maths. The more times I work through it or have it explained the more ideas I hope to have for how to work through it clearly and logically with students.

Another student reinforced the link between PCK and MCK, stating that “I feel that it will provide me with pedagogy but also with the ability to calculate and solve mathematics problems effectively and accurately”. In addition to such comments, a number of participants (4 of 20) shared that the unit would be effective in refreshing mathematical content. One participant stated “this unit will refresh my own prior knowledge from middle school and also provide me with some new content knowledge, such as key terminology and strategies specific to the teaching of middle school mathematics.” Other participants (4 of 20) stressed that this unit would provide them with confidence to teach mathematics effectively. Here in one participant noted that “[the unit] will give me an understanding of the thinking processes during mathematics; it should give me an added confidence of delivering content to students.”

Discussion

The purpose of this research was to investigate the self-perceptions of pre-service primary and secondary teachers enrolled in a mathematics education unit as they engage with and consolidate their mathematics content. In addition to these self-perceptions, the researcher explored how the

pre-service teachers understood and perceived their 'readiness' to teach middle school mathematics content, based on their recent tertiary training. The collected data from the initial surveys (Phase 1) were categorised under two conceptual themes. These themes are: *valuable professional learning*, and *the need to strengthen mathematical content*. Both of these themes are now considered in light of the literature pertaining to pre-service mathematics teacher development.

Valuable professional learning

All research participants (20 of 20) highlighted that the unit ED2315 would be valuable in their professional development as pre-service mathematics teachers. In addition, many participants were able to articulate the extent to which the unit would be useful to them in their professional roles. For instance, a majority of participants described how they felt the unit would assist them in strengthening their MCK, PCK, or MKT, or any combination of these three knowledge categories. Additional benefits included being "a good content refresher", while others mentioned the unit helping to give them confidence in consolidating and teaching mathematics. This finding is consistent with literature underscoring the value of mathematics units for pre-service teachers (Butterfield & Chinnappan 2010; Matthews, Rech & Grandgenett 2010). Half of the Stage 1 participants (10 of 20) asserted that they felt either uncertain or unconfident in teaching upper primary or lower secondary mathematics, with another 2 participants sharing that they were undecided. This assertion, together with a majority of the surveyed population (18 of 20) reporting they feel they require further MCK, suggests that this unit is of considerable value to the preparation of undergraduate mathematics teachers. Even those participants who expressed a degree of confidence in their MCK (2 of 20) suggested the unit would strengthen their MKT or PCK or both. These self-reported participant claims concerning the value of the unit resonate closely with those of various scholars (Ball, Thames, Bass, Sleep, Lewis & Phelps 2009; Whittington, 2002; Wilburne & Long 2010).

The need to strengthen mathematical content knowledge

As a corollary to research participants avowing the value of undertaking this unit, collected testimony frequently highlighted the self-reported need for pre-service teachers to consolidate their mathematical content knowledge. Half of the Stage 1 participants (10 of 20) asserted that they felt either uncertain or unconfident in teaching upper primary or lower secondary mathematics, with another 2 participants sharing that they were undecided. A majority of participants indicated that they felt least confident in their mathematical content knowledge first, then they required assistance in learning how to teach mathematics to students second. These assertions reinforce claims that educators must know how to apply and teach mathematics addressed in schools (Ball & Wilson 1990; Mansfield 1985), and that MCK is required for PCK to have any demonstrable impact (Southwell & Penglase 2005). Over half those who admitted needing further assistance in strengthening their MCK (12 of 18) specified that the Australian Curriculum Strand they required assistance in was *Number and algebra*. Although current literature suggests a need for mathematics teachers to improve their PCK, MKT and MCK overall, the research participants placed particular emphasis on the need to consolidate the MCK they require to teach students confidently and competently. Such emphasis accords with various scholars who posit that teachers require a firm grasp on MCK in order to facilitate student learning (Stohlmann, Moore & Cramer 2013; Whittington 2002; Wilburne & Long 2010).

Conclusion

This paper explored the self-perceptions of pre-service primary and secondary teachers enrolled in a mathematics education unit as they prepared to engage with and consolidate their mathematics content. In addition, the self-reported understanding and perception of pre-service teachers' 'readiness' to undertake such a task was interrogated, based on their recent tertiary training. In addition to the body of literature already suggesting that pre-service mathematics teachers require additional support in developing their MCK, the testimony of the research participants reinforces that claim. At the same time the collected data from Phase 1 indicate the extent to which pre-service teachers believe taking a mathematical content unit will be valuable for their future professional roles. High value was ascribed to the acquisition of MCK, MKT, PCK, or any combination of these knowledge categories. In particular, all participants expressed a view that this unit will be valuable for them as they strengthen and consolidate their MCK, especially with key topics in the Australian Curriculum strand *Number and algebra*.

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Appendix

Appendix 1: Pre-Unit Survey

1. How ready do you feel to teach the following topics? Complete each item by selecting a number (1 indicates the least ready, 5 indicates the most ready).
 - (a) Fractions, decimals, percentages
 - (b) Algebra
 - (c) Equations and formulas
 - (d) Perimeter, area, volume, capacity
 - (e) Calculating probability
 - (f) Using statistics
 - (g) Venn diagrams

2. Describe your readiness to teach mathematics to upper primary/lower secondary students in terms of the mathematical content knowledge and skills you currently possess.

3. In what area(s) of mathematical content knowledge do you feel you require further learning?

4. How do you feel this unit will contribute to your grasp of middle school mathematical content?