

The Intersection of Mathematics and Language in the Post-Secondary Environment: Implications for English Language Learners

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Given the increasing number of English Language Learners (ELLs) in post-secondary environments (Roessingh & Douglas, 2012), educational practices such as availability of language support for mathematics should be assessed to ensure that all students' needs are met. To explore the effects of language on mathematics in ELLs, mathematical test items were presented in four language contexts: vocabulary knowledge, negation, preposition use, and atypical sentence structure. Sixty students enrolled in mathematics courses volunteered to complete the mathematics task. Results suggest that math items falling into each of the four language contexts disadvantage ELLs, highlighting that the needs of ELLs should be considered at all levels, from classroom practices to educational policy.

The number of English Language Learners (ELLs) in post-secondary environments (PSE) is increasing (Roessingh & Douglas, 2012), and this trend is no different at Humber College in Toronto, where the present study was conducted. In one academic school at Humber, 34% of the students were ESL-streamed¹ in

2010 (Humber College, 2011). Given this change in the student population, educational practices, such as use of language support in mathematics, should be assessed to ensure that all students' needs are met. Although math tests ostensibly measure mathematical skills, language is a significant factor (Brown, 2005), which may result in

¹ Humber College has two writing streams, ESL and COMM, into which students are streamed by means of an English placement test. "If the writing sample displays ESL characteristics, the student will be placed in an appropriate level ESL course" ("Entrance/Placement Testing," n.d.). Students streamed into the ESL branch of the program complete their course work within the ESL stream for the remainder of their program.

artificially depressed math scores for ELLs. This study will explore whether language in mathematical items has disproportionately negative effects on ELLs' math performances, compared to non-ELLs.²

Many aspects of English impact mathematical performances, including vocabulary choice, negation, preposition meaning, and atypical sentence structure. Consider the use of technical terms such as 'denominator.' If students are unfamiliar with the term, they may be unable to solve the item due to a lack of English language knowledge, rather than mathematical skill. This can be expected, given that a determinant of reading comprehension is vocabulary knowledge (Grabe, 2008). Terminology, therefore, causes problems in ELLs dealing with mathematics in English (Moschkovich, 2002). When words such as 'lowest terms' and 'product' are repurposed within mathematics, students may be confused. The general meaning of the word 'product' is the result of a process, though within mathematics, product refers to multiplication, and not addition. Similarly, although 'lowest,' and 'terms' are within the top 2000 word families of English (Nation, 2004), the meaning of the 'lowest terms' is not obvious. It is, moreover, rare, occurring only 3 times in the 425 million words of the Corpus of Current American English (Davies, 2008).

Another language-related area that is known to cause problems is negation (Just & Carpenter, 1971). Negation, both clausal (e.g., *it does not equal*) and non-clausal (e.g., *except*), may affect ELLs more than non-ELLs. More linguistically complex math items are deemed to be more challenging for ELLs compared to non-ELLs (Abedi & Hejri, 2004). More subtle language areas such as the phrasing of mathematical problems, also may pose a greater problem for ELLs. These include the use of certain prepositions as well as the use of atypical sentence structures. Although the teaching of prepositions is standard in English language classes, they are typically treated within the streams of grammar, writing, and speaking rather than reading. Consider the statement: "In 2000, 12% of children were obese, but the rate has increased to 25%." If "to" is clearly understood to be the endpoint or goal (Huddleston & Pullum, 2002), the reader should understand that no calculations are required. The numbers presented suggest that the students interpret "to"

to signify the extent of the difference, as would be expected with "by" (Huddleston & Pullum, 2002). Thus some students may add $\frac{1}{4}$ of the original rate (i.e., the size of the change is $25\% \times 12\% = 3\%$, so the new rate is $12\% + 3\% = 15\%$) while others may simply add the two percentages (i.e., $12\% + 25\% = 37\%$), rather than realizing that the final percent of obese children is 25%. These errors are due to a misunderstanding of the meaning of the preposition "to" or a failure to attend to the preposition.

Finally, atypical sentence structures may selectively cause problems for ELLs. Open questions in English typically start with an interrogative phrase containing an interrogative word (e.g., *who, what, where, why*). The interrogative phrase in such questions is usually fronted (Huddleston & Pullum, 2002). But English also allows a less common interrogative clause structure with non-fronted interrogative phrases. For example, the question: "What volume is 36 litres $\frac{3}{8}$ of?" can be restructured as: "Thirty six litres is $\frac{3}{8}$ of what volume?" with a non-fronted interrogative phrase.

In order to examine the aforementioned language issues, mathematical test items were constructed within each of the four language contexts: vocabulary knowledge, negation, preposition use, and atypical sentence structure. Although language and math issues have been investigated previously, the vast majority of research has focused on elementary students (e.g., Barwell, 2005; Ockey, 2007), neglecting PSEs. It is, however, becoming an increasingly important issue given the rising proportion of ELL students in post-secondary environments. Considering that: a) ELLs constitute a growing segment in the post-secondary population; b) certain language forms are known to cause problems interpreting math test items; and c) these issues have not been widely studied in PSEs, this study seeks to answer the following question: To what extent, if any, are ELLs disadvantaged by mathematical language in math at the post-secondary level?

Method

Participants

Sixty students enrolled in mathematics courses at Humber College volunteered to participate: 28 ELLs ($M = 21.86$

² We use 'non-ELL' here to include all students who write English at a proficient level without demonstrating the ESL characteristics described in Footnote 1 above and, as a result, were not streamed into ESL classes at Humber College.

years, $SD = 4.09$) and 32 non-ELLs ($M = 20.94$ years, $SD = 3.08$). Their status was determined by the results of Humber College’s English Placement Test, as self-reported in the demographic questionnaire. Each participant was given a \$25 gift certificate from the campus bookstore as an honorarium and signed consent forms stating that they could withdraw without consequence at any time, and assuring them of confidentiality and anonymity.

Materials and procedure

The study was conducted after research ethics approval was granted. A demographic questionnaire and a 25-item math task were administered. Participants were permitted to use calculators. The math task consisted of multiple-choice items focusing on four specific language issues that were expected to disproportionately affect ELLs: preposition use (5 items), vocabulary knowledge (5 items), negation (4 items), and sentence structure (3 items). Another five math items were designed to require a minimal amount of language. Three other items appearing in the task were not used in the analysis. See Table 1 for examples from each type.

The math items assessed college-level mathematics in two primary areas, algebra and arithmetic, and measured the mathematical skills all first-year students entering the business and technical programs at Humber were assumed to possess. Mathematical concepts included in the task were operations of addition, subtraction, multiplication and division of algebraic expressions and fractions; simplifying equivalent fractions; the relationship between fractions, decimals, percents, and rates; and solving fractional equations including those with an unknown in the denominator. The math items were randomly ordered throughout the task.

Results

The Cronbach alpha of the math items was 0.72, which is considered an acceptable level of reliability (George & Mallery, 2003). An alpha level of .05 was used for all statistical tests.

The average number of correct responses for each type of math items was calculated, and t -test analyses were performed (Table 2).

Differences were found between the ELLs and non-ELLs in the average number of items correctly solved in four of the five types of math items. With the exception of math items in which language played the minimal role, non-ELLs outperformed ELLs. In all cases where significant differences were found, the effect size, represented by Cohen’s d , was medium (Cohen, 1988).

Of the top three items that discriminated the ELLs and non-ELLs, two focused on negation, and the third addressed atypical sentence structure. Specifically, the first item shown is “Which of the following does not equal 36% of X?” an example of negation. Similarly, the second item is also an example of negation: “Which of the following does not equal 6 ¼%?” The third item, “\$195 is 53 ½ % of what?” is an example of an atypical sentence structure.

Discussion

This study found empirical evidence for possible language biases in mathematical problems at the post-secondary level disadvantaging ELLs. Only math items requiring minimal language showed no statistically significant difference between the two groups. For example, Solve. ($12 \frac{3}{4} \div 3 \frac{1}{2}$) could be completed with very little consideration

Table 1
Examples of Math Questions as a Function of Item Type

Type of Math Item	Example Question
Preposition Use	<i>Pierre’s house insurance bill for 2008 was \$879 How much did he pay in 2009, if it increased by \$190?</i>
Vocabulary Knowledge	<i>Change 3.55 to an <u>equivalent</u> fraction</i>
Minimal Language	<i>Solve $12 \frac{3}{4} \div 3 \frac{1}{2}$</i>
Negation	<i>Which of the following <u>does not equal</u> 36% of X?</i>
Atypical Sentence Structure	<i>32 is 40 percent of <u>what number?</u></i>

Table 2

Means (Standard Deviations) and t-tests of the Average Number of Correct Responses on the Math Task, as a Function of Language Group and Item Type

Math Item Type	ELL (n = 28)	Non ELL (n = 32)	t (58)	Cohen's d
Preposition Use	.61 (.22)	.73 (.22)	2.07*	0.55
Vocabulary	.80 (.15)	.87 (.12)	1.94*	0.51
Minimal Language	.82 (.22)	.81 (.19)	0.29	0.07
Negation	.59 (.28)	.81 (.28)	2.95*	0.79
Sentence Structure	.76 (.25)	.89 (.20)	2.10*	0.57

Note. * $p \leq .05$.

to language; one may ignore the word solve and proceed to successfully find the correct solution to the math item. This suggests that the mathematical abilities of ELLs and non-ELLs are not significantly different, supporting the notion that language is the main contributor to the differences between the two language groups. The other four types of items comprising the math task contained heavier language demands, such as the consideration of prepositions (e.g., to & by), vocabulary knowledge (e.g., equivalent & lowest terms), negation (e.g., does not equal), and atypical sentence structure, all disadvantaged the ELLs more than their non-ELL peers.

The top three items that distinguished the ELLs from non-ELLs included two phrased in a negative fashion as well as one with an atypical sentence structure. Negatively phrased math items are counter-intuitive considering how students commonly respond to math items. Normally, correct responses are given, rather than incorrect responses. ELLs who do not have the language skills to account for this may skip over this important language-based detail in the math item. "Which of the following does not equal 36% of X?" requires students to arrive at the correct response, then consider the phrasing of the problem and choose the response, which is *not* correct. ELLs, more than non-ELLs, overlooked this negation, and consequently, performed more poorly.

The third math item contained the phrase of "what?" Specifically, \$195 is 53 ½ % of what? which is an example of an atypical structure. Students must conceptualize the math item only after correctly interpreting the question using their knowledge of English sentence structure, resulting in heavy language demands. ELLs were affected more detrimentally than non-ELLs,

leading to lower math scores, likely due to language challenges rather than mathematical proficiency. This is consistent with research suggesting both that a positive correlation exists between math and reading scores and that linguistically modified items, such as those with simpler vocabulary and less complex language structures, benefit ELLs (Brown, 2005; Abedi & Lord, 2001).

Despite the consistency of this study's results compared to existing research, there are limitations that should be considered. There was a ceiling effect on the math task, which can be addressed in future studies by the addition of more math items, as well as more challenging math items in the task. Furthermore, although our sample size (N=60) was sufficient to detect differences between ELLs and non-ELLs, a larger sample size would allow a focus on more specific language issues, such as the isolation of verbal negation (e.g., does not equal to) versus non-verbal negation (e.g., except).

The most critical finding is that language aspects of mathematics items appear to selectively disadvantage ELLs, even when those ELL students have met the English language requirements for college. This may be due to the disparity between general English language proficiency needed to pass standardized language tests, such as the Test of English as a Foreign Language, and the technical language used in mathematics. Given the increasing numbers of ELLs in PSE, the practice of using specific language supports should be considered to avoid compromising the validity of mathematical tests, ensuring that the tests measure mathematical skills rather than language proficiency. This is significant, given the key findings that preposition use, vocabulary knowledge, sentence structure, and use of negation adversely affected

mathematical performances in ELLs more than non-ELLs.

These results have widespread implications on policy and curriculum. Many colleges require students to take mathematical placement tests, after an offer of admission. Math placement test scores are used to place students in program math courses or remedial math courses. Accurately placing students early in their post-secondary career in either a remedial or non-remedial program pathway improves student success and retention (Fike & Fike, 2008). If students are misplaced into the remedial stream, they are burdened with unnecessary remedial courses and required to spend extra time to catch-up on program math courses. This is just one example in which the present results may be applied in PSE. Heightened awareness of the needs of ELLs should be considered at all levels, from classroom practices to educational policy.

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Biographies

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