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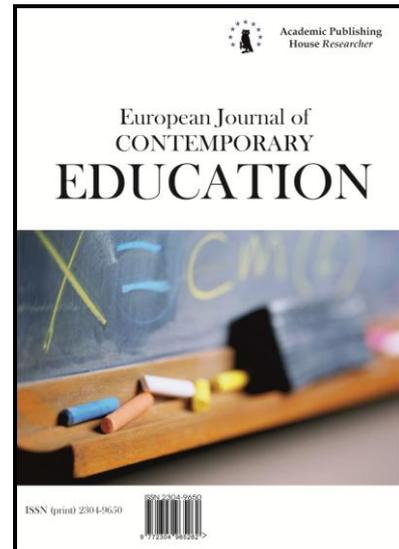
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### **Computer Mathematical Tools: Practical Experience of Learning to use them**

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#### **Abstract**

The article contains general information about the use of specialized mathematics software in the preparation of math teachers. The authors indicate the reasons to study the mathematics software. In particular, they analyze the possibility of presenting basic mathematical courses using mathematical computer tools from both a teacher and a student, argue in favour of using software instead of traditional paper-and-board style. The special course of the study of mathematics software (main tasks of the course are disclosed; knowledge and skills that students should possess on completion of their studies are dedicated; content of computer workshop is detailed) is briefly described. Also the authors give a detailed solution and its methodic basis of typical problem with the use of the SCM Maple. The typical errors which made by students in the use of specialized mathematics packages are characterized. Author's own experience of teaching computer workshop of the study of mathematical computer tools by the students of Sumy State Pedagogical Makarenko University is presented and analyzed.

**Keywords:** mathematics; computer mathematics; mathematics computer tools; special courses on computer mathematics.

**Introduction**

Modern training, as in pure mathematics, and in the field of teaching mathematics, involves creating the ability to use the professional hardware and software. It means not only the use of software to represent mathematical knowledge, but also the understanding of the possibilities of using modern information technologies for solving professional tasks, the ability to analyze the experience of using mathematics software by other members of the scientific community, the ability to model the mathematical objects, consciously use the computer mathematical tools for solving different classes of problems, etc.

These and other skills acquired during the training and the professional activity of a math teacher.

**Results and their Discussion**

**1. The Foundation of the Studing of the Mathematics Software.**

Understanding the importance of formation of skills to use mathematical computer tools, the main mathematical courses in Sumy State Pedagogical Makarenko University is conducted with the support of various information systems, including the computer mathematics software (table 1). The results of the analysis on the use of specialized software shells during the preparation of math teachers in the educational process show the following.

Teachers of the older generation are more likely to use office software packages (MS PowerPoint, MS Excel), rather than specialized mathematics software. This is explained by different factors, among which we highlight the lack of time to study mathematics packages, unwillingness to study something unfamiliar in IT or strong opinion about unnecessarily of the use of information resources in the system of modern mathematical education.

Table 1: The Use of Software in the Preparation of Bachelors of Mathematics

Courses	Software (SCM)	The Use of SCM by Teacher	The Use of SCM by Student	Notes
Linear algebra and analytical geometry	Maple	-	-	1,2-th year of studies, 162 class hours
	MS Excel	-	+	
	Gran	-	-	
	DG	-	-	
	GeoGebra	-	-	
Mathematical analysis	Live Mathematics	-	-	1,2-th year of studies, 286 class hours
	Maple	-	-	
	MatCad	-	-	
	MathLab	-	-	
	Mathematica	-	-	
Projective geometry	Maxima	-	-	2-th year of studies, 70 class hours
	MS Excel	-	+	
Mathematical programming	Live Mathematics	+	-	2-th year of studies, 72 class hours
	GeoGebra	+	+	
Numerical methods	MS Excel	+	+	3-th year of studies, 60 class hours
	GeoNext	+	-	
Methods of teaching mathematics	Maple	+	-	3,4-th year of studies, 72 class hours
	MS Excel	+	+	
	Gran (Gran1, Gran2d, Gran3d)	+	+	
Theory of Probability and Mathematical Statistic	Open Mathematics	+	-	4-th year of studies, 94 class hours
	MS Excel	+	+	
Computer applications	Gran	+	+	4-th year of studies,

in mathematics education	DG GeoGebra Live Mathematics MathKit	+	+	50 class hours
		+	+	
Computer practical work	Maple GeoGebra Live Mathematics	+	+	4-th year of studies, 54 class hours
		+	+	

Also for some reason, they believe that the study of classical courses of mathematical analysis, algebra, etc. have to be done with paper book, notebook and pencil, and not using specialized computer tools.

Note that the younger generation of teachers implements IT and uses a virtual environment with great enthusiasm, but the lack of time does not allow them to master a greater number of specialized software, and to demonstrate the use of computer mathematical tools with their comparative analysis.

A spreadsheet is considered among students as the most "native" to solve the math problems. We explain this by the fact that it studies at school.

Students are more commonly used systems that have Russian interface (in particular, dynamic mathematics software).

Time analysis of educational plans of preparation of math teachers (data by year of study of main courses are given in table 1), the level of readiness of students to use the software, the specificity of the profession and the development of IT show the necessity of studying computer mathematical tools, starting from the second year of studies. In addition, the insufficient use of such tools by university professors on lectures and practical lessons leads to a decrease not only learning motivation, but also the future professional activity on the use of IT in teaching mathematics, which contradicts the global tasks of teacher preparation of the information society.

Many other reasons promote to necessity of use and learning of special computer mathematical tools from younger courses, among which we highlight the following:

1) active "consumption" of information technologies in educational, research and scientific activities by young generation will lead to the use of specialized professional environment;

2) wide range of software of support the mathematical research (from e-solving of problems to the modeling of complex processes and their mathematical processing) causes the problem of choosing the right product and its efficient use for the solution of certain classes of mathematics problems – the special course of computer mathematics can contribute to formation of such knowledge and skills;

3) modern information technologies allow to improve the quality of mathematical knowledge and to facilitate its submittal and processing, and the deliberate failure of IT in professional activity identifies with the "old-style" work;

4) the shift in the learning technologies in the direction of research activity and the increase of the share of self-work force all subjects of educational process to rationalize the organization of their work by searching specialized computer tools in the virtual space, which is costly and can be minimized with the compulsory study of such tools to ensure both the interest to the subject and quality of the acquired knowledge;

5) contradictions between inert bureaucratic system with an established curricula and development of modern mathematics with its popular computer tools and implemented methods of research;

6) actively used mathematical computer tools should be clear for each specialist of mathematical community.

Consideration of these factors cause the necessity of studying of the course, which integrates as familiarity with different computer tools and the study of the peculiarities of creating your own special tools, and modeling of various processes, which are described by the mathematical equivalent – equations, inequalities, geometric objects, etc.

## **2. A Special Course of the Study of Computer Mathematics Software**

We think that such course should include:

- 1) the study of mathematics packages:
  - a) to demonstrate the principal possibilities of using modern virtual shells and stipulated in them tools;
  - b) to demonstrate the capabilities and features of various software for conscious and rational choice of the computer product to solve specific types of problems;
  - c) to expand the circle of "automatically" solving problems (for example, to find the extremum, to build the intersection of the surfaces, etc.);
  - d) to simplify the construction of mathematical objects;
- 2) the realization of interdisciplinary connections, not only with academic math courses, but also, for example, with computer science through the implementation of algorithms for solving typical problems;
- 3) the formation of a "natural, consciously and fearlessly desire" to use the tool;
- 4) exploring the different ways of solutions of different types of problems (construction problems, algorithmic problems, proof problems, search problems) to demonstrate what is possible to implement in the virtual space today, and what not;
- 5) the formation of critical thinking in use of tools to reduce the probability of obtaining of false positives.

We see that the subject of study of the special course is computer tools which give an opportunity to solve different classes of problems. We mean that computer tools are as the shell, which allows to solve a math problem, and a separate command or "macros" that obtain the answer.

We distinguish among the objectives of this special course:

- the analysis of the market of modern computer tools for solving math problems;
- the demonstration of using of information technologies for solving various classes of math problems;
- the training to rational use of computer tools in solving problems of scientific, research or professional activity.

Students should know upon completion of the course:

- the modern computer tools of solutions of typical problems of the main sections of mathematics (mathematical analysis, algebra, geometry, number theory, probability theory, etc.) and ways of their application;
- the classification of computer software, which support solving of math problems;
- the stages of computer model of math problems.

Upon completion of the course students should be able:

- to perform elementary mathematical calculations, to carry out numerical and symbolic manipulations with the use of software tools;
- to model the mathematical objects and processes, including their visualization or animation, in the virtual space;
- to create procedures (macros) that describes your own computer tools;
- to choose the most convenient and efficient tool for solving concrete math problem;
- to use the computer tools for solving scientific and professional problems;
- to organize a demonstration of solving by means of information technologies.

In particular, these skills we are formed in the practical course, supported by guidelines [4] and has the following content:

- 1) a brief analysis of popular systems of computer mathematics, description of the possibilities of applying Maple to solve the problems of certain fields of mathematics (elementary mathematics, analytical geometry, mathematical analysis, linear algebra, etc.), as well as to use of this virtual environment to create their own mathematical objects or tools for their creation;
- 2) a brief analysis of typical problems of some fields of mathematics and their solution (possibly in several ways) in this environment;
- 3) tasks for individual work, the solution of which provides for the use of the given commands and structures.

Let's describe one of educational situations as an argument. In studying the topic "Integral calculus of functions of several variables" students are often faced with the problem of constructing the body of integration, that is always difficult for reasons of insufficient of stereoscopic imagination, inability to make a good drawing, ignorance of types of surfaces that limit given body.

To solve these problems, we offer the following typical problem.

**Example 1.** Construct the body, limited by surfaces  $z = x^2 + y^2$ ,  $z = 0$ ,  $x = 1$ ,  $y = 2$ ,  $x = 0$ ,  $y = 0$ .

By the moment of study of the course students should know the shapes of given surfaces and be able to draw the body of integration schematically. Unfortunately, by the fourth course knowledge and skills are forgotten, and the problem is solved not by all. Therefore, we offer students to use computer tools.

To construct we will use a graphics package *plots* and commands *implicitplot3d* (construction of implicitly defined surface), *intersectplot* (construction of line of intersection of the surfaces), *polygonplot3d* (construction of broken line), and the command *seq* to create sequences.

Construction of all surfaces that limit the body on the same picture gives only a glimpse of its shape, but not draws it, which confirms the result.

```
> implicitplot3d ([z = x^2+y^2, z = 0, x = 1, y = 2, x = 0, y = 0], x = 0 .. 1, y = 0 .. 2, z = 0 .. 5, color = [red, red, grey, green, grey, green])
```

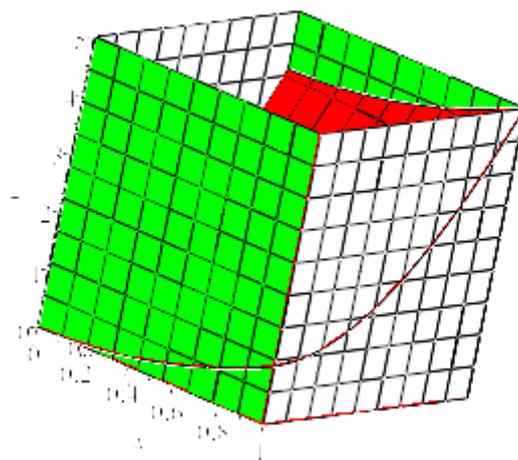


Figure 1. The outlines of the body of integration for example 1

We can guess about the shape of the body of integration by interactively rotation of the resulting object. But since we want to achieve the creation of an ability to create a clear geometric images, we offer more detailed construction of the required geometric body.

To get a clear intersection curves of each pair of surfaces, we use the command *intersectplot* and create geometric objects *p1*, *p2*, *p3*, *p4*, which define the line of intersection of the given surfaces that limit the body (i.e. intersection curves of paraboloid  $z = x^2 + y^2$  with planes). Also we create geometric objects *p5*, *p6*, which determine upper and lower bounds of the body.

```
> p1 := intersectplot(z = x^2+y^2, x = 1, x = 0 .. 1, y = 0 .. 2, z = 0 .. 5, color = black, thickness = 2);
> p2 := intersectplot(z = x^2+y^2, y = 2, x = 0 .. 1, y = 0 .. 2, z = 0 .. 5, color = black, thickness = 2);
> p3 := intersectplot(z = x^2+y^2, y = 0, x = 0 .. 1, y = 0 .. 2, z = 0 .. 5);
> p4 := intersectplot(z = x^2+y^2, x = 0, x = 0 .. 1, y = 0 .. 2, z = 0 .. 5);
> p5 := implicitplot3d(z = x^2+y^2, x = 0 .. 1, y = 0 .. 2, z = 0 .. 5, color = green);
> p6 := implicitplot3d(z = 0, x = 0 .. 1, y = 0 .. 2, z = 0 .. 5, color = grey);
```

Now we build the sides: calculate the points (sequences *k1*, *k2*, *k3*, *k4*) on intersection curves of paraboloid with vertical planes by command *seq*, which will be connected by closed broken line by the command *polygonplot3d*.

```
> k1 := seq([1, (1/4)*T, 1+(1/16)*T^2], T = 0 .. 8);
```

```

k1 := [1, 0, 1], [1, 1/4, 17/16], [1, 1/2, 5/4], [1, 3/4, 25/16], [1, 1, 2], [1, 5/4,
41/16], [1, 3/2, 13/4], [1, 7/4, 65/16], [1, 2, 5]
> k2 := seq([(1/4)*T, 2, 4+(1/16)*T^2], T = 4 .. 0, -1);
k2 := [1, 2, 5], [3/4, 2, 73/16], [1/2, 2, 17/4], [1/4, 2, 65/16], [0, 2, 4]
> k3 := seq([(1/4)*T, 0, (1/16)*T^2], T = 4 .. 0, -1);
k3 := [1, 0, 1], [3/4, 0, 9/16], [1/2, 0, 1/4], [1/4, 0, 1/16], [0, 0, 0]
> k4 := seq([0, (1/4)*T, (1/16)*T^2], T = 8 .. 0, -1);
k4 := [0, 2, 4], [0, 7/4, 49/16], [0, 3/2, 9/4], [0, 5/4, 25/16], [0, 1, 1], [0, 3/4,
9/16], [0, 1/2, 1/4], [0, 1/4, 1/16], [0, 0, 0]
> p7 := polygonplot3d([[1, 2, 0], [1, 0, 0], k1, k2, [0, 2, 0]]);
> p8 := polygonplot3d([[0, 2, 0], k4]);
> p9 := polygonplot3d([[1, 0, 0], k3]);
All geometric objects are displayed by the command display.
> display(p1, p2, p3, p4, p5, p6, p7, p8, p9, orientation = [-30, 60, 0]);

```

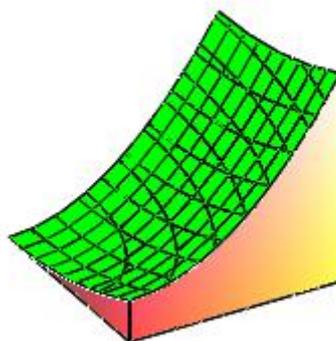


Figure 2. The body of integration for example 1

We offer the typical solutions of the problems in sufficient quantities. Tasks for individual work are identical to solved.

The second part of the practical course envisages the solution of not only typical problems, but also of problems that require a competence approach. For example, we propose to create a tool that allows to animate the approximation of a function by power or trigonometric series. Here the knowledge and skills in the theory of series, programming, proficiency of Maple commands (both standard and narrowly directed), and understanding of mathematical essence of the problem are necessary.

All of the above ideas are implemented (in greater or lesser extent) at the computer workshop (4-th course of preparation of bachelors of mathematics, 54 classroom hours).

Experience of use of computer tools in education, science and professional activity of the future math teachers is accumulated and analyzed during 7 years [1-6].

In particular, studies of students of specialized mathematics packages allows to say about the following typical errors or problems:

- improper or incorrect spelling of the command syntax (students know what the command does, but prescribe the command wrong very often; not adhere the order of the parameters, do not put the additional keys that lead to the absence of the result, or to its incorrect output, or to false or unclear results);
- incorrect understanding or interpretation of the result (for example, in integration);

- the inability to verify the result by additional activities or inverse methods (solution of equations and plotting of graph, finding the highest value analytically and graphically, etc.);
- the inability to improve the quality of presentation of the result (in particular, to change the zoom of the construction, to "smooth" the curve, to construct several images on one drawing, to change the boundaries of output of graphics on the screen, etc.);
- the inability to determine the result of the actions of some command by using examples from the system Help or understand already written command block (lack of knowledge of mathematics, English language, partially programming).

It is sometimes because of insufficient level of classical mathematical background. It makes to search such teaching methods that provide a platform for the evaluation of the correctness of the result and the appropriateness of the choice of computer product by the students. We see the solution of this problem in the active attraction of technologies of problem-based learning and the use of individual research problems, problems with insufficient or excessive amount of data, problems to reveal contradictions, for review, creative tasks of search character, etc.

In particular, with the aim of focusing on the critical analysis of the results, which were obtained by IT, the students were proposed to test the results of their own individual works at different mathematical courses, that were written and defended by them during the whole training. We note that among mathematics computer environments that were familiar to students (future math teachers) were SCM *Maple*, dynamic mathematics packages "*Live mathematics*", "*MathKit*", *Gran1*, *Gran2d*, *Gran3d*, *GeoGebra*, various plotters, spreadsheet software.

The implementation of the task has shown that very often students search ready solutions of problems in the Internet (about 80% of the respondents), actively use online communication on the forums (about 40%), even pay money (!) for computer solution of problem instead of individual studying of possibilities of SCM or writing code (about 10%). Also we fixed the tendency to use a limited range of commands without additional analysis of available subpackages (about 15 %) or even to write their own procedures instead of using the existent command (about 5%).

Also, we have registered local student "dissatisfaction" about the lack of result or incorrect result, which appear on the monitor screen. Not always students critically evaluated the command spelling (syntax) (about 70%), the input parameters (about 30%), and also the presentation of the result. It was especially felt in transformations, in particular trigonometric expressions (about 30%). And only hints to students, the need for self-analysis of each stage of solving the problem by SCM contributed to obtaining the correct result, and the solution of subsequent problems contributed to the formation of the skills of their critical assessment.

The solution results of problems also showed special activity in the use of typical computer tools, in particular, integration, plotting of graphics, etc. (about 90%), and some passivity in the independent searches inside mathematics package that is due to not only inadequate knowledge of the English language, but also natural laziness to spend forces for the development of a new tool (about 80%).

It should also be noted that successful students are more actively and in a balanced manner solve problems by professional software than are looking for the answer in paper-and-pencil way (80% vs. 20%) unlike students who study mediocre. However unfamiliar mathematics problem, even for advanced students, does not become a "motive force" in their own research. They refuse to do it more often, because they understand that it is necessary not only to study the theoretical basis of the problem, but also to spend time on finding the right computer product. Also, experience shows that often the intermediate results (although they receive in virtual environments) in most cases are calculated manually by students without using IT.

Powerful mathematics packages such as *Maple* causes "fear" in the average student because of English interface, a large number of unclear commands, the inability to understand the system Help. All of this becomes an insurmountable obstacle in the use of this environment.

According to surveys of students, future math teachers, they will use virtual shells for the modeling of the problem, for the research of solutions or for the demonstration of the properties of geometric objects in their own careers. Among software they mention, as a rule, the dynamic mathematics software with intuitive interface (mostly Ukrainian software, recommended by Ministry of Education and Science of Ukraine (*Gran1*, *Gran2d*, *Gran3d*, *DG*), less the Ukrainian version of *GeoGebra* and software *Live mathematics*, *Live geometry*, *MathKit* with Russian

interface). Some students said about the use of more powerful SCM at the mathematics clubs or at lessons of computer science during the study of algorithms (in particular, SCM Maple).

### **Conclusion**

Summarizing our experience, we want to say that today it is not a question of necessity of studying the virtual computer tools to support the teaching of mathematics; it is today understood by all. The inevitability of studying of computer mathematics in the modern system of training of math teachers is due both to the development of IT in the mathematics, and to the fact that today's youth are in the majority refuses to work in paper-and-pencil style. E-readers, pads, smartphones are an integral attribute of a modern student, and if a simple calculation can be performed in the mind, more complex mathematics operations, as a rule, already performed by gadgets.

And no matter how long we talked that skills of orally performing of calculations, of drawing any geometric object, of calculations tabulated integral and derivative are basic in the system of preparation of mathematician, time shows the necessity and the inevitability to equip the youth with computer tools with the formation of skills to use them correctly at solving any mathematics problem.

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**Компьютерные математические инструменты:  
практический опыт обучения их использовать**

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**Аннотация.** В статье приведена обобщенная информация об использовании специализированных компьютерных программ математического направления при подготовке учителей математики. Проанализирован опыт преподавания компьютерного практикума по изучению математических компьютерных инструментов студентам педагогического университета.

**Ключевые слова:** математика; компьютерная математика; математические компьютерные инструменты; спецкурсы по компьютерной математике.