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A METHOD TO DETERMINE OF ALL NON-ISOMORPHIC GROUPS OF ORDER 16

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Abstract. Many students or teachers ask themselves: Being given a natural number n, how many non-isomorphic groups of order n exists? The answer, generally, is not yet given. But, for certain values of the number n have answered this question. The present work gives a method to determine of all non-isomorphic groups of order 16 and gives descriptions of all these groups. The results obtained here can be used by students in the senior year, students in courses or seminars, or other Mathematics teachers are in different stages of their professional development.

Key words: non-isometric groups of order n

The problem of determining all non-isomorphic groups of a given order is not new. She appeared in the latter part of the nineteenth century and part was solved by Cayley and Frobenius - abelian case. The problem is still open. In 1905, Dickson presents some general solutions, and in 1992, Jungnickel solve the problem of uniqueness of the groups of order n.

In [6] Purdea has determined all non-isomorphic groups of order $n \leq 10$, and I have determined in [9] all these groups of order $n \in \{p, q, pq, p^2, p^3\}$, where p and q are two distinct prime numbers.

In this work we will present a method to determine of all non-isomorphic groups of order 16. In this context, throughout this paper by group we mean a group (denoted by G) of order 16 in multiplicative notation and we will denote: by 1 the identity (the "neutral") element of G, by $\operatorname{ord}(g)$ the order of the element $g \in G$ and by $|A|$ the cardinal of the set A. If A is a subgroup of G, then $|A|$ is (also) the order of A.

Following the same reasoning as in [9], respectively [10], we will prove the main result of this paper:

Theorem: There are 14 non-isomorphic groups of order 16.

Proof: So, let G be a group of order 16 and $Z(G)$ his center. According to Lagrange's theorem and to Burnside's theorem we have the following cases:

Case I: $|Z(G)| = 16$. In this case G is commutative and by [3,8.4] and [5,2.2 (p. 86) and 6.1 (p. 97)] it follows that we have the following possibilities:

1) $G = G_1 \cong \mathbf{Z}_{16}$; so, there is an element $x \in G_1$ such that $\operatorname{ord}(x) = 16$ and

$$G_1 = \{1, x, x^2, x^3, x^4, x^5, x^6, x^7, x^8, x^9, x^{10}, x^{11}, x^{12}, x^{13}, x^{14}, x^{15}\}.$$

For this group see Table 1;

2) $G = G_2 \cong \mathbf{Z}_2 \times \mathbf{Z}_8$; so, there are $x, y \in G_2$ such that $\operatorname{ord}(x) = 8$, $\operatorname{ord}(y) = 2$, $xy = yx$ and

$$G_2 = \{1, x, x^2, x^3, x^4, x^5, x^6, x^7, y, xy, x^2y, x^3y, x^4y, x^5y, x^6y, x^7y\}.$$

For this group see Table 2;

3) $G = G_3 \cong \mathbf{Z}_4 \times \mathbf{Z}_4$; so, there are $x, y \in G_3$ such that $\operatorname{ord}(x) = 4$, $\operatorname{ord}(y) = 4$, $xy = yx$ and

$$G_3 = \{1, x, x^2, x^3, y, y^2, y^3, xy, x^2y, x^3y, xy^2, xy^3, x^2y^2, x^2y^3, x^3y^2, x^3y^3\}.$$

For this group see Table 3;

4) $G = G_4 \cong \mathbf{Z}_2 \times \mathbf{Z}_2 \times \mathbf{Z}_4$; so, there are $x,y,z \in G_4$ such that $\mathrm{ord}(x) = 4$, $\mathrm{ord}(y) = \mathrm{ord}(z) = 2$, $xy = yx$, $xz = zx$, $yz = yz$ and

$$G_4 = \{1,x,x^2,x^3,y,xy,x^2y,x^3y,z,xz,x^2z,x^3z,yz,xyz,x^2yz,x^3yz,x^3y^3\}.$$

For this group see Table 4;

5) $G = G_5 \cong \mathbf{Z}_2 \times \mathbf{Z}_2 \times \mathbf{Z}_2 \times \mathbf{Z}_2$; so, there are $x,y,z,t \in G_5$ such that $\mathrm{ord}(x) = \mathrm{ord}(y) = \mathrm{ord}(z) = \mathrm{ord}(t) = 2$, $xy = yx$, $xz = zx$, $xt = tx$, $yz = zy$, $yt = ty$, $zt = tz$ and

$$G_5 = \{1,x,y,z,t,xy,xz,xt,yz,yt,zt,xyz,xyt,xzt,yzt,xyzt\}.$$

For this group see Table 5.

Case II: $|Z(G)| = 8$. Then $|G/Z(G)| = 2$ and since the group $G/Z(G)$ is cyclic, by [5,2.2 (p. 143)] it follows that G is commutative-contradiction to the hypothesis. So, there is no group G of order 16 with $|Z(G)| = 8$.

Case III: $|Z(G)| = 4$. In this case $|G/Z(G)| = 4$ and according to [8,5.5] the group $G/Z(G)$ is commutative. Again by [3,8.4] and [5,2.2 (p. 86) and 6.1 (p. 97)] it follows that either $G/Z(G) \cong \mathbf{Z}_4$ or $G/Z(G) \cong \mathbf{Z}_2 \times \mathbf{Z}_2$. Since the group $\mathbf{Z}_4$ is cyclic, it follows that only the second possibility holds.

Therefore

$$G/Z(G) = \{Z(G), xZ(G), yZ(G), xyZ(G)\}$$

, where $x^2 \in Z(G)$, $y^2 \in Z(G)$ and $xyZ(G) = yxZ(G)$. If $xy = yx$ then G is commutative, which is impossible. So, $xy \neq yx$ and there is an element $b \in Z(G)$ such that $yx = xyb$. Then

$$y^2x = y(yx) = y(xyb) = (yx)yb = xybyb = xy^2b^2.$$

Since $y^2 \in Z(G)$ it follows that $b^2 = 1$. On the other hand, $Z(G)$ being a (sub)group of order 4 (of G) we distinguish the following subcases:

Subcase 1: Assume $Z(G) \cong \mathbf{Z}_4$, so $Z(G) = \{1,a,a^2,a^3\}$, with $a \in G$. From the above proved facts it follows that $yx = xya^2$. Then $xy = yxa^2$ and we distinguish the following possibilities:

a) $x^2 = y^2 = 1$. Then there is a group $G_6 = \langle x,y,a \rangle$, with x, y and a satisfying the above conditions; see Table 6.

b) $x^2 = 1$ and $y^2 = c \in \{a,a^2,a^3\}$.

i) If $y^2 = a$ then there is a group $G_7 = \langle x,y,a \rangle = \langle x,y \rangle$, with x, y and a satisfying the above conditions; see Table 7.

ii) If $y^2 = a^2$ then there is a group $G_8 = \langle x,y,a \rangle$, with x, y and a satisfying the above conditions; see Table 8.

iii) If $y^2 = a^3$ then there is a group $G_8 = \langle x,y,a \rangle$, with x, y and a satisfying the above conditions; see Table 9.

c) $x^2 = c$ and $y^2 = d$, where c, $d \in \{a,a^2,a^3\}$.

i) If $x^2 = y^2 = a$ then there is a group $G_{10} = \langle x,y,a \rangle$, with x, y and a satisfying the above conditions; see Table 10.

ii) If $x^2 = a$ and $y^2 = a^2$ then there is a group $G_{11} = \langle x,y,a \rangle$, with x, y and a satisfying the above conditions; see Table 11.

iii) If $x^2 = a$ and $y^2 = a^3$ then there is a group $G_{12} = \langle x,y,a \rangle$, with x, y and a satisfying the above conditions; see Table 12.

iv) If $x^2 = y^2 = a^2$ then there is a group $G_{13} = \langle x,y,a \rangle$, with x, y and a satisfying the above conditions; see Table 13.

v) If $x^2 = a^2$ and $y^2 = a^3$ then there is a group $G_{14} = \langle x, y, a\rangle$, with x, y and a satisfying the above conditions; see Table 14.

vi) If $x^2 = y^2 = a^3$ then there is a group $G_{15} = \langle x, y, a\rangle$, with x, y and a satisfying the above conditions; see Table 15.

Subcase 2: Assume $Z(G) \cong \mathbf{Z}_2 \times \mathbf{Z}_2$, so $Z(G) = \{1, a, b, ab\}$, with a, b and $ab = ba$, $a^2 = b^2 = 1$. Since $xyZ(G) = yxZ(G)$, $xy \neq yx$ and x^2, $y^2 \in Z(G)$ it follows that (say) $xy = yxa$, $G = \langle x, y, a, b\rangle$ and we distinguish the following possibilities:

a) If $x^2 = y^2 = 1$ then there is a group $G_{16} = \langle x, y, a, b\rangle$, with x, y, a and b satisfying the above conditions; see Table 16.

b) $x^2 = 1$ and $y^2 = c \in \{a, b, ab\}$.

i) If $y^2 = a$ then there is a group $G_{17} = \langle x, y, a, b\rangle$, with x, y, a and b satisfying the above conditions; see Table 17.

ii) If $y^2 = b$ then there is a group $G_{18} = \langle x, y, a, b\rangle$, with x, y, a and b satisfying the above conditions; see Table 18.

iii) If $y^2 = ab$ then there is a group $G_{19} = \langle x, y, a, b\rangle$, with x, y, a and b satisfying the above conditions; see Table 19.

c) $x^2 = d$ and $y^2 = e$, where d, $e \in \{a, b, ab\}$.

i) If $x^2 = y^2 = a$ then there is a group $G_{20} = \langle x, y, a, b\rangle$, with x, y, a and b satisfying the above conditions; see Table 20.

ii) If $x^2 = a$ and $y^2 = b$ then there is a group $G_{21} = \langle x, y, a, b\rangle$, with x, y, a and b satisfying the above conditions; see Table 21.

iii) If $x^2 = a$ and $y^2 = ab$ then there is a group $G_{22} = \langle x, y, a, b\rangle$, with x, y, a and b satisfying the above conditions; see Table 22.

iv) If $x^2 = y^2 = b$ then there is a group $G_{23} = \langle x, y, a, b\rangle$, with x, y, a and b satisfying the above conditions; see Table 23.

v) If $x^2 = b$ and $y^2 = ab$ then there is a group $G_{24} = \langle x, y, a, b\rangle$, with x, y, a and b satisfying the above conditions; see Table 24.

vi) If $x^2 = ab$ and $y^2 = a$ then there is a group $G_{25} = \langle x, y, a, b\rangle$, with x, y, a and b satisfying the above conditions; see Table 25.

vii) If $x^2 = ab$ and $y^2 = b$ then there is a group $G_{26} = \langle x, y, a, b\rangle$, with x, y, a and b satisfying the above conditions; see Table 26.

viii) If $x^2 = y^2 = ab$ then there is a group $G_{27} = \langle x, y, a, b\rangle$, with x, y, a and b satisfying the above conditions; see Table 27.

Case IV: $|Z(G)| = 2$. Then $|G/Z(G)| = 8$ and according to [6,p.141-142] and [5,2.2 (p. 143)] either $G/Z(G) \cong \mathbf{Z}_4 \times \mathbf{Z}_2$ or $G/Z(G) \cong \mathbf{Z}_2 \times \mathbf{Z}_2 \times \mathbf{Z}_2$, or $G/Z(G) \cong D_4$, where D_4 is the dihedral group of order 4, or $G/Z(G) \cong Q$, where Q is the quaternions group.

Therefore $Z(G) = \langle a\rangle = \{1, a\}$, with $a \in G$ and $a^2 = 1$.

Subcase 1: $G/Z(G) \cong \mathbf{Z}_4 \times \mathbf{Z}_2$. It follows that

$$G/Z(G) = \{Z(G), xZ(G), x^2Z(G), x^3Z(G), yZ(G), xyZ(G), x^2yZ(G), x^3yZ(G)\}$$

, where x, $y \in G$, x^4, $y^2 \in Z(G)$ and $xyZ(G) = yxZ(G)$. Thus $G =< x, y, a >$ and $xy = yxa$. But in these conditions $yx^2 = x^2y$ and $x^2 \in Z(G)$-which is impossible; so, there is no exist group G with the above properties.

Subcase 2: $G/Z(G) \cong \mathbf{Z}_2 \times \mathbf{Z}_2 \times \mathbf{Z}_2$. It follows that

$$G/Z(G) = \{Z(G), xZ(G), yZ(G), zZ(G), xyZ(G), xzZ(G), yzZ(G), xyzZ(G)\}$$

, where x, y, $z \in G$, x^2, y^2, $z^2 \in Z(G)$ and $xyZ(G) = yxZ(G)$, $xzZ(G) = zxZ(G)$, $yzZ(G) = zyZ(G)$. Thus we have the following possibilities:

a) $yx = xy$, $zx = xza$ and $yz = zya$. In these conditions

$$z(xy) = (xza)y = x(zya) = (xy)z$$

and $xy \in Z(G)$-which is impossible; so, it doesn't exist group G with the above properties.

b) $yx = xya$, $zx = xz$ and $yz = zya$. In these conditions

$$(xz)y = x(yza) = (yx)z = y(xz)$$

and $xz \in Z(G)$-which is impossible; so, it doesn't exist group G with the above properties.

c) $yx = xya$, $zx = xza$ and $yz = zy$. In these conditions

$$(yz)x = y(xza) = x(yz)$$

and $yz \in Z(G)$-which is impossible; so, it doesn't exist group G with the above properties.

d) $yx = xya$, $zx = xza$ and $yz = zya$. Then we obtain the following equalities:

$$x(xyz) = yz = (xyz)x$$

,

$$y(xyz) = xza = (xyz)y$$

and

$$z(xyz) = xy = (xyz)z$$

. Therefore $xyz \in Z(G)$-which is impossible. So, also in these conditions there is no group G of order 16.

Subcase 3: $G/Z(G) \cong D_4$. It follows that

$$G/Z(G) = \{Z(G), xZ(G), x^2Z(G), x^3Z(G), yZ(G), xyZ(G), x^2yZ(G), x^3yZ(G)\}$$

, where x, $y \in G$, x^4, $y^2 \in Z(G)$ and $yxZ(G) = x^3yZ(G)$. Thus in this subcase we have the following possibilities:

a) $yx = x^3y$.

i) If $x^4 = 1$ then

$$yx^2 = (x^3y)x = x^3(x^3y) = x^2y$$

and $x^2 \in Z(G)$-which is impossible. So, in these conditions there is no group G of order 16.

ii) If $x^4 = a$ and $y^2 = 1$ then

$$yx^2 = (x^3y)x = x^3(x^3y) = x^2ya$$

and there is a group $G_{28} = \langle x, y, a \rangle$, with x, y and a satisfying the above conditions; see Table 28.

iii) If $x^4 = y^2 = a$ then again we obtain that

$$yx^2 = (x^3y)x = x^3(x^3y) = x^2ya$$

and there is a group $G_{29} = \langle x, y, a \rangle$, with x, y and a satisfying the above conditions; see Table 29.

b) $yx = x^3ya$.

i) If $x^4 = 1$ then again we obtain that

$$yx^2 = (x^3y)x = x^3(x^3y) = x^2y$$

and $x^2 \in Z(G)$-which is impossible. So, in these conditions there is no group G of order 16.

ii) If $x^4 = a$ and $y^2 = 1$ then

$$yx^2 = (x^3ya)x = x^3(x^3ya)a = x^2ya$$

and there is a group $G_{30} = \langle x, y, a \rangle$, with x, y and a satisfying the above conditions; see Table 30.

iii) If $x^4 = y^2 = a$ then again we obtain that

$$yx^2 = (x^3ya)x = x^3(x^3y)a = x^2y$$

and $x^2 \in Z(G)$-which is impossible. So, in these conditions there is no group G of order 16.

Subcase 4: $G/Z(G) \cong Q$. By [8, p.93] It follows that again

$$G/Z(G) = \{Z(G), xZ(G), x^2Z(G), x^3Z(G), yZ(G), xyZ(G), x^2yZ(G), x^3yZ(G)\}$$

, where x, $y \in G$, x^4, $y^4 \in Z(G)$, $x^2Z(G) = y^2Z(G)$ and $yxZ(G) = x^3yZ(G)$. Thus in this subcase we have the following possibilities:

a) $yx = x^3y$ and $x^2 = y^2$.

i) If $x^4 = 1$ then

$$yx^2 = (x^3y)x = x^3(x^3y) = x^2y$$

and $x^2 \in Z(G)$-which is impossible. So, in these conditions there is no group G of order 16.

ii) If $x^4 = a$ then

$$yx^2 = (yx)x = x^3(yx) = x^2ya$$

and

$$y^3 = yy^2 = yx^2 = x^2ya.$$

On the other hand,

$$y^3 = y^2y = x^2y$$

and $a = 1$-which is impossible. So, in these conditions there is no group G of order 16.

b) $yx = x^3y$ and $x^2 = y^2a$. If $x^4 = a$ then

$$yx^2 = (x^3y)x = x^3(x^3y) = x^2ya$$

and

$$y^3 = yy^2 = yx^2a = x^2y$$

. On the other hand,

$$y^3 = y^2y = x^2ya$$

and $a = 1$-which is impossible. So, in these conditions there is no group G of order 16.

c) $yx = x^3ya$ and $x^2 = y^2$.

i) If $x^4 = 1$ then

$$yx^2 = (x^3ya)x = x^3(x^3ya)a = x^2y$$

and $x^2 \in Z(G)$-which is impossible. So, in these conditions there is no group G of order 16.

ii) If $x^4 = a$ then
$$yx^2 = (yx)x = x^3(yx)a = x^2ya$$
and
$$y^3 = yy^2 = yx^2 = x^2ya.$$
On the other hand,
$$y^3 = y^2y = x^2y$$
and $a = 1$-which is impossible. So, in these conditions there is no group G of order 16.

d) $yx = x^3ya$ and $x^2 = y^2a$.

i) If $x^4 = 1$ then
$$yx^2 = (x^3ya)x = x^3(x^3ya)a = x^2y$$
and $x^2 \in Z(G)$-which is impossible. So, in these conditions there is no group G of order 16.

ii) If $x^4 = a$ then
$$yx^2 = (yx)x = x^3(yx)a = x^2ya$$
and
$$y^3 = yy^2 = yx^2a = x^2y.$$
On the other hand,
$$y^3 = y^2y = x^2ya$$
and $a = 1$-which is impossible. So, in these conditions there is no group G of order 16.

It follows that in the conditions from this subcase doesn't exist group G of order 16.

Therefore we have determined 30 groups of order 16. Afterwards we are going to show the following isomorphisms:
$$G_6 \cong G_8 \cong G_{13},$$
$$G_9 \cong G_{10} \cong G_{11} \cong G_{12} \cong G_{14} \cong G_{15},$$
$$G_{16} \cong G_{17}$$
and
$$G_{18} \cong G_{19} \cong G_{24} \cong G_{26},$$
$$G_{20} \cong G_{21} \cong G_{22} \cong G_{23} \cong G_{25} \cong G_{27}.$$

By the tables of group it follows the Table 0, which presents all the elements of order 2, 4 and 8 in each group determined above.

Indeed, using the Table 0, it is straightforward to verify that:

α) the map $f : G_6 \to G_8$ defined by $f(x) = x$, $f(y) = xy$ and $f(a) = a$ is an isomorphism of groups and $G_6 \cong G_8$;

β) the map $g : G_8 \to G_{13}$ defined by $g(x) = xa$, $g(y) = y$ and $g(a) = a$ is an isomorphism of groups and $G_8 \cong G_{13}$;

γ) so, the map $g \circ f : G_6 \to G_{13}$ defined by $(g \circ f)(x) = xa$, $g(\circ f)(y) = xya$ and $(g \circ f)(a) = a$ is an isomorphism of groups and $G_6 \cong G_{13}$.

Similarly, check the other isomorphisms above.

Therefore only 14 groups from these above presented are not isomorphic, namely: G_1, G_2, G_3, G_4, G_5, G_6, G_7, G_9, G_{16}, G_{18}, G_{20}, G_{28}, G_{29}, G_{33} - the first five are abelian and last nine non-abelian. Now the theorem is completely proved.

At the end of this paper we present the Table 0 - above mentioned and the (multiplication) tables of these 30 groups which have been determined.

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TABLE 0: with the elements of order 2, 4 and 8 in each of the 30 groups determined in the paper.

The group	The elements of order 2	The elements of order 4	The elements of order 8
G_1	x^8	x^4, x^{12}	x^2, x^6, x^{10}, x^{14}
G_2	x^4, y, x^4y	x^2, x^6, x^2y, x^6y	$x, x^3, x^5, x^7\ xy, x^3y, x^5y, x^7y$
G_3	x^2, y^2, x^2y^2	$x, x^3, y, y^3, xy, x^2y, x^3y, xy^2, xy^3, x^2y^3, x^3y^2, x^3y^3$	-
G_4	$x^2, y, x^2y, z, x^2z, yz, x^2yz$	$x, x^3, xy, x^3y, xz, x^3z, xyz, x^3yz$	-
G_5	all elements	-	-
G_6	$x, y, a^2, xa^2, ya^2, xya, xya^3$	$a, a^3, xa, xa^3, ya, ya^3, xy, xya^3$	-
G_7	x, a^2, xa^2	a, a^3	$y, xy, ya, ya^2, ya^3, xya, xya^2, xya^3$
G_8	$x, xy, a^2, xa^2, ya, ya^3, xya^2$	$y, a, a^3, xa, xa^3, ya^2, xya, xya^3$	-
G_9	x, a^2, xa^2	a, a^3, xa, xa^3	$y, xy, ya, ya^2, ya^3, xya, xya^2, xya^3$
G_{10}	xy, a^2, xya^2	a, a^3, xya, xya^3	$x, y, xa, xa^2, xa^3, ya, ya^2, ya^3$
G_{11}	a^2, ya, ya^3	y, a, a^3, ya^2	$x, xy, xa^2, xa^3, xya, xya^2, xya^3$
G_{12}	a^2, xya, xya^3	xy, a, a^3, xya^2	$x, xy, xa, xa^2, xa^3, ya, ya^2, ya^3$
G_{13}	$a^2, xa, xa^3, ya, ya^3, xya, xya^3$	$x, y, xy, a, a^3, xa^2, ya^2, xya^2$	-
G_{14}	a^2, xa, xa^3	x, a, a^3, xa^2	$y, xy, ya, ya^2, ya^3, xya, xya^2, xya^3$
G_{15}	xy, a^2, xya^2	a, a^3, xya, xya^3	$x, y, xa, xa^2, xa^3, ya, ya^2, ya^3$
G_{16}	x, y, a, b, ab, xa, xb, xab, ya, yb, yab	xy, xya, xyb, xyab	-
G_{17}	x, xy, a, b, ab, xa, xb, xab, xya, xyb, xyab	y, ya, yb, yab	-
G_{18}	x, a, b, ab, xa, xb, xab	y, xy, ya, yb, yab, xya, xyb, xyab	-
G_{19}	x, a, b, ab, xa, xb, xab	y, xy, ya, yb, yab, xya, xyb, xyab	-
G_{20}	a, b, ab	x, y, xy, xa, xb, xab, ya, yb, yab, xya, xyb, xyab	-
G_{21}	a, b, ab	x, y, xy, xa, xb, xab, ya, yb, yab, xya, xyb, xyab	-
G_{22}	a, b, ab	x, y, xy, xa, xb, xab, ya, yb, yab, xya, xyb, xyab	-
G_{23}	a, b, ab	x, y, xy, xa, xb, xab, ya, yb, yab, xya, xyb, xyab	-
G_{24}	xy, a, b, ab, xya, xyb, xyab	x, y, xa, xb, xab, ya, yb, yab	-
G_{25}	a, b, ab	x, y, xy, xa, xb, xab, ya, yb, yab, xyab, xya, xyb, xyab	-
G_{26}	xy, a, b, ab, xya, xyb, xyab	x, y, xa, xb, xab, ya, yb, yab	-
G_{27}	a, b, ab	x, y, xy, xa, xb, xab, ya, yb, yab, xya, xyb, xyab	-
G_{28}	y, x^2y, a, ya, x^2ya	$x^2, xy, x^3y, x^2a, xya, x^3ya$	x, x^3, xa, x^3a
G_{29}	xy, x^3y, a, xya, x^3ya	$x^2, y, x^2y, x^2a, ya, x^2ya$	x, x^3, xa, x^3a
G_{30}	$y, xy, x^2y, x^3y, a, ya, xya, x^2ya, x^3ya$	x^2, x^2a	x, x^3, xa, x^3a

OPERATIONS GROUP TABLES

TABLE 1: $G_1 \cong \mathbf{Z}_{16} = \langle x \rangle$, $x^{16}=1$; $Z(G_1)=G_1$.

	1	**x**	**x^2**	**x^3**	**x^4**	**x^5**	**x^6**	**x^7**	**x^8**	**x^9**	**x^{10}**	**x^{11}**	**x^{12}**	**x^{13}**	**x^{14}**	**x^{15}**
1	1	x	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	x^{10}	x^{11}	x^{12}	x^{13}	x^{14}	x^{15}
x	x	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	x^{10}	x^{11}	x^{12}	x^{13}	x^{14}	x^{15}	1
x^2	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	x^{10}	x^{11}	x^{12}	x^{13}	x^{14}	x^{15}	1	x
x^3	x^3	x^4	x^5	x^6	x^7	x^8	x^9	x^{10}	x^{11}	x^{12}	x^{13}	x^{14}	x^{15}	1	x	x^2
x^4	x^4	x^5	x^6	x^7	x^8	x^9	x^{10}	x^{11}	x^{12}	x^{13}	x^{14}	x^{15}	1	x	x^2	x^3
x^5	x^5	x^6	x^7	x^8	x^9	x^{10}	x^{11}	x^{12}	x^{13}	x^{14}	x^{15}	1	x	x^2	x^3	x^4
x^6	x^6	x^7	x^8	x^9	x^{10}	x^{11}	x^{12}	x^{13}	x^{14}	x^{15}	1	x	x^2	x^3	x^4	x^5
x^7	x^7	x^8	x^9	x^{10}	x^{11}	x^{12}	x^{13}	x^{14}	x^{15}	1	x	x^2	x^3	x^4	x^5	x^6
x^8	x^8	x^9	x^{10}	x^{11}	x^{12}	x^{13}	x^{14}	x^{15}	1	x	x^2	x^3	x^4	x^5	x^6	x^7
x^9	x^9	x^{10}	x^{11}	x^{12}	x^{13}	x^{14}	x^{15}	1	x	x^2	x^3	x^4	x^5	x^6	x^7	x^8
x^{10}	x^{10}	x^{11}	x^{12}	x^{13}	x^{14}	x^{15}	1	x	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
x^{11}	x^{11}	x^{12}	x^{13}	x^{14}	x^{15}	1	x	X^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	x^{10}
x^{12}	x^{12}	x^{13}	x^{14}	x^{15}	1	X	x^2	X^3	x^4	x^5	x^6	x^7	x^8	x^9	x^{10}	x^{11}
x^{13}	x^{13}	x^{14}	x^{15}	1	x	x^2	x^3	X^4	x^5	x^6	x^7	x^8	x^9	x^{10}	x^{11}	x^{12}
x^{14}	x^{14}	x^{15}	1	x	x^2	x^3	x^4	X^5	x^6	x^7	x^8	x^9	x^{10}	x^{11}	x^{12}	x^{13}
x^{15}	x^{15}	1	x	x^2	x^3	x^4	x^5	X^6	x^7	x^8	x^9	x^{10}	x^{11}	x^{12}	x^{13}	x^{14}

TABLE 2: $G_2 \cong \mathbf{Z}_8 \times \mathbf{Z}_2 = \langle x,y \rangle$, $x^8 = y^2 = 1$, $xy = yx$; $Z(G_2) = G_2$.

	1	**x**	$\mathbf{x^2}$	$\mathbf{x^3}$	$\mathbf{x^4}$	$\mathbf{x^5}$	$\mathbf{x^6}$	$\mathbf{X^7}$	**y**	**xy**	$\mathbf{x^2y}$	$\mathbf{x^3y}$	$\mathbf{x^4y}$	$\mathbf{x^5y}$	$\mathbf{x^6y}$	$\mathbf{x^7y}$
1	1	x	x^2	x^3	x^4	x^5	x^6	X^7	y	xy	x^2y	x^3y	x^4y	x^5y	x^6y	x^7y
x	x	x^2	x^3	x^4	x^5	x^6	x^7	1	xy	x^2y	x^3y	x^4y	x^5y	x^6y	x^7y	y
$\mathbf{x^2}$	x^2	x^3	x^4	x^5	x^6	x^7	1	x	x^2y	x^3y	x^4y	x^5y	x^6y	x^7y	y	xy
$\mathbf{x^3}$	x^3	x^4	x^5	x^6	x^7	1	x	X^2	x^3y	x^4y	x^5y	x^6y	x^7y	y	xy	x^2y
$\mathbf{x^4}$	x^4	x^5	x^6	x^7	1	x	x^2	X^3	x^4y	x^5y	x^6y	x^7y	y	xy	x^2y	x^3y
$\mathbf{x^5}$	x^5	x^6	x^7	1	x	x^2	x^3	X^4	x^5y	x^6y	x^7y	y	xy	x^2y	x^3y	x^4y
$\mathbf{x^6}$	x^6	x^7	1	x	x^2	x^3	x^4	X^5	x^6y	x^7y	y	xy	x^2y	x^3y	x^4y	x^5y
$\mathbf{x^7}$	x^7	1	x	x^2	x^3	x^4	x^5	X^6	x^7y	y	xy	x^2y	x^3y	x^4y	x^5y	x^6y
y	y	xy	x^2y	x^3y	x^4y	x^5y	x^6y	X^7y	1	x	x^2	x^3	x^4	x^5	x^6	x^7
xy	xy	x^2y	x^3y	x^4y	x^5y	x^6y	x^7y	y	x	x^2	x^3	x^4	x^5	x^6	x^7	1
$\mathbf{x^2y}$	x^2y	x^3y	x^4y	x^5y	x^6y	x^7y	y	xy	x^2	x^3	x^4	x^5	x^6	x^7	1	x
$\mathbf{x^3y}$	x^3y	x^4y	x^5y	x^6y	x^7y	y	xy	X^2y	x^3	x^4	x^5	x^6	x^7	1	x	x^2
$\mathbf{x^4y}$	x^4y	x^5y	x^6y	x^7y	y	xy	x^2y	X^3y	x^4	x^5	x^6	x^7	1	x	x^2	x^3
$\mathbf{x^5y}$	x^5y	x^6y	x^7y	y	xy	x^2y	x^3y	X^4y	x^5	x^6	x^7	1	x	x^2	x^3	x^4
$\mathbf{x^6y}$	x^6y	x^7y	y	xy	x^2y	x^3y	x^4y	X^5y	x^6	x^7	1	x	x^2	x^3	x^4	x^5
$\mathbf{x^7y}$	x^7y	y	xy	x^2y	x^3y	x^4y	x^5y	X^6y	x^7	1	x	x^2	x^3	x^4	x^5	x^6

TABLE 3: $G_3 \cong \mathbf{Z}_4 \times \mathbf{Z}_4 = \langle x,y \rangle$, $x^4=y^4=1$, $xy=yx$; $Z(G_3)=G_3$.

	1	**x**	**x^2**	**x^3**	**y**	**y^2**	**y^3**	**xy**	**x^2y**	**x^3y**	**xy^2**	**xy^3**	**x^2y^2**	**x^2y^3**	**x^3y^2**	**x^3y^3**
1	1	x	x^2	x^3	y	y^2	y^3	xy	x^2y	x^3y	xy^2	xy^3	x^2y^2	x^2y^3	x^3y^2	x^3y^3
x	x	x^2	x^3	1	xy	xy^2	xy^3	X^2y	x^3y	y	x^2y^2	x^2y^3	x^3y^2	x^3y^3	y^2	y^3
x^2	x^2	x^3	1	x	x^2y	x^2y^2	x^2y^3	X^3y	y	xy	x^3y^2	x^3y^3	y^2	y^3	xy^2	xy^3
x^3	x^3	1	x	x^2	x^3y	x^3y^2	x^3y^3	y	xy	x^2y	y^2	y^3	xy^2	xy^3	x^2y^2	x^2y^3
y	y	xy	x^2y	x^3y	y^2	y^3	1	xy^2	x^2y^2	x^3y^2	xy^3	x	x^2y^3	x^2	x^3y^3	x^3
y^2	y^2	xy^2	x^2y^2	x^3y^2	y^3	1	y	xy^3	x^2y^3	x^3y^3	x	xy	x^2	x^2y	x^3	x^3y
y^3	y^3	xy^3	x^2y^3	x^3y^3	1	y	y^2	x	x^2	x^3	xy	xy^2	x^2y	x^2y^2	x^3y	x^3y^2
xy	xy	x^2y	x^3y	y	xy^2	xy^3	x	x^2y^2	x^3y^2	y^2	x^2y^3	x^2	x^3y^3	x^3	y^3	1
x^2y	x^2y	x^3y	y	xy	x^2y^2	x^2y^3	x^2	x^3y^2	y^2	xy^2	x^3y^3	x^3	y^3	1	xy^3	x
x^3y	x^3y	y	xy	x^2y	x^3y^2	x^3y^3	x^3	Y^2	xy^2	x^2y^2	y^3	1	xy^3	x	x^2y^3	x^2
xy^2	xy^2	x^2y^2	x^3y^2	y^2	xy^3	x	xy	x^2y^3	x^3y^3	y^3	x^2	x^2y	x^3	x^3y	1	y
xy^3	xy^3	x^2y^3	x^3y^3	y^3	x	xy	xy^2	X^2	x^3	1	x^2y	x^2y^2	x^3y	x^3y^2	y	y^2
x^2y^2	x^2y^2	x^3y^2	y^2	xy^2	x^2y^3	x^2	x^2y	x^3y^3	y^3	xy^3	x^3	x^3y	1	y	x	xy
x^2y^3	x^2y^3	x^3y^3	y^3	xy^3	x^2	x^2y	x^2y^2	X^3	1	x	x^3y	x^3y^2	y	y^2	xy	xy^2
x^3y^2	x^3y^2	y^2	xy^2	x^2y^2	x^3y^3	x^3	x^3y	Y^3	xy^3	x^2y^3	1	y	x	xy	x^2	x^2y
x^3y^3	x^3y^3	y^3	xy^3	x^2y^3	x^3	x^3y	x^3y^2	1	x	x^2	y	y^2	xy	xy^2	x^2y	x^3y

TABLE 4: $G_4 \cong \mathbf{Z}_4 \times \mathbf{Z}_2 \times \mathbf{Z}_2 = <x,y,z>$, $x^4=y^2=z^2=1$, $xy=yx$, $yz=zy$, $xz=zx$; $Z(G_4)=G_4$.

	1	**x**	**x^2**	**x^3**	**y**	**xy**	**x^2y**	**X^3y**	**z**	**xz**	**x^2z**	**x^3z**	**yz**	**xyz**	**x^2yz**	**x^3yz**
1	1	x	x^2	x^3	y	xy	x^2y	X^3y	z	xz	x^2z	x^3z	yz	xyz	x^2yz	x^3yz
x	x	x^2	x^3	1	xy	x^2y	x^3y	y	xz	x^2z	x^3z	z	xyz	x^2yz	x^3yz	yz
x^2	x^2	x^3	1	x	x^2y	x^3y	y	xy	x^2z	x^3z	z	xz	x^2yz	x^3yz	yz	xyz
x^3	x^3	1	x	x^2	x^3y	y	xy	X^2y	x^3y	z	xz	x^2z	x^3yz	yz	xyz	x^2yz
y	y	xy	x^2y	x^3y	1	x	x^2	X^3	yz	xyz	x^2yz	x^3yz	z	xz	x^2z	x^3z
xy	xy	x^2y	x^3y	y	x	x^2	x^3	1	xyz	x^2yz	x^3yz	yz	xz	x^2z	x^3z	z
x^2y	x^2y	x^3y	y	xy	x^2	x^3	1	x	x^2yz	x^3yz	yz	xyz	x^2z	x^3z	z	xz
x^3y	x^3y	y	xy	x^2y	x^3y	1	x	X^2	x^3yz	yz	xyz	x^2yz	x^3z	z	xz	x^2z
z	z	xz	x^2z	x^3z	yz	xyz	x^2yz	x^3yz	1	x	x^2	x^3	y	xy	x^2y	x^3y
xz	xz	x^2z	x^3z	z	xyz	x^2yz	x^3yz	yz	x	x^2	x^3	1	xy	x^2y	x^3y	y
x^2z	x^2z	x^3z	z	xz	x^2yz	x^3yz	yz	xyz	x^2	x^3	1	x	x^2y	x^3y	y	xy
x^3z	x^3z	z	xz	x^2z	x^3yz	yz	xyz	x^2yz	x^3	1	x	x^2	x^3y	y	xy	x^2y
yz	yz	xyz	x^2yz	x^3yz	z	xz	x^2z	X^3z	y	xy	x^2y	x^3y	1	x	x^2	x^3
xyz	xyz	x^2yz	x^3yz	yz	xz	x^2z	x^3z	z	xy	x^2y	x^3y	y	x	x^2	x^3	1
x^2yz	x^2yz	x^3yz	yz	xyz	x^2z	x^3z	z	xz	x^2y	x^3y	y	xy	x^2	x^3	1	x
x^3yz	x^3yz	yz	xyz	x^2yz	x^3z	z	xz	X^2z	x^3y	y	xy	x^2y	x^3	1	x	x^2

TABLE 5: $G_5 \cong \mathbf{Z}_2 \times \mathbf{Z}_2 \times \mathbf{Z}_2 \times \mathbf{Z}_2 = \langle x,y,z,t \rangle$, $x^2=y^2=z^2=t^2=1$, $xy=yx$, $xz=zx$, $xt=tx$, $yz=zy$, $yt=ty$, $zt=tz$; $Z(G_5)=G_5$.

	1	**x**	**y**	**z**	**t**	**xy**	**xz**	**xt**	**yz**	**yt**	**zt**	**xyz**	**xyt**	**xzt**	**yzt**	**xyzt**
1	1	x	y	z	t	xy	xz	xt	yz	yt	zt	xyz	xyt	xzt	yzt	xyzt
x	x	1	xy	xz	xt	y	z	t	xyz	xyt	xzt	yz	yt	zt	xyzt	yzt
y	y	xy	1	yz	yt	x	xyz	xyt	z	t	yzt	xz	xt	xyzt	zt	xzt
z	z	xz	yz	1	zt	xyz	x	xzt	y	yzt	t	xy	xyzt	xt	yt	xyt
t	t	xt	yt	zt	1	xyt	xzt	x	yzt	y	z	xyzt	xy	xz	yz	xyz
xy	xy	y	x	xyz	xyt	1	yz	yt	xz	xt	xyzt	z	t	yzt	xzt	zt
xz	xz	z	xyz	x	xzt	yz	1	zt	xy	xyzt	xt	y	yzt	t	xyt	yt
xt	xt	t	xyt	xzt	x	yt	zt	1	xyzt	xy	xz	yzt	y	z	xyz	yz
yz	yz	xyz	z	y	yzt	xz	xy	xyzt	1	zt	yt	x	xzt	xyt	t	xt
yt	yt	xyt	t	yzt	y	xt	xyzt	xy	zt	1	yz	xzt	x	xyz	z	xz
zt	zt	xzt	yzt	t	z	xyzt	xt	xz	yt	yz	1	xyt	xyz	x	y	xy
xyz	xyz	yz	xz	xy	xyzt	z	y	yzt	x	xzt	xyt	1	zt	yt	xt	t
xyt	xyt	yt	xt	xyzt	xy	t	yzt	y	xzt	x	xyz	zt	1	yz	xz	z
xzt	xzt	zt	xyzt	xt	xz	yzt	t	z	xyt	xyz	x	yt	yz	1	xy	y
yzt	yzt	xyzt	zt	yt	yz	xzt	xyt	xyz	t	z	y	xt	xz	xy	1	x
xyzt	xyzt	yzt	xzt	xyt	xyz	zt	yt	yz	xt	xz	xy	t	z	y	x	1

TABLE 6: $G_6=\langle x,y,a\rangle$, $x^2=y^2=a^4=1$, $yx=xya^2$, $Z(G_6)=\langle a\rangle$.

	1	**x**	**y**	**xy**	**a**	**a^2**	**a^3**	**xa**	**xa^2**	**xa^3**	**ya**	**ya^2**	**ya^3**	**xya**	**xya^2**	**xya^3**
1	1	x	y	xy	a	a^2	a^3	xa	xa^2	xa^3	ya	ya^2	ya^3	xya	xya^2	xya^3
x	x	1	xy	y	xa	xa^2	xa^3	a	a^2	a^3	xya	xya^2	xya^3	ya	ya^2	ya^3
y	y	xya^2	1	xa^2	ya	ya^2	ya^3	xya^3	xy	xya	a	a^2	a^3	xa^3	x	xa
xy	xy	ya^2	x	a^2	xya	xya^2	xya^3	ya^2	y	ya	xa	xa^2	xa^3	a^3	1	a
a	a	xa	ya	xya	a^2	a^3	1	xa^2	xa^3	x	ya^2	ya^3	y	xya^2	xya^3	xy
a^2	a^2	xa^2	ya^2	xya^2	a^3	1	a	xa^3	x	xa	ya^3	y	ya	xya^3	xy	xya
a^3	a^3	xa^3	ya^3	xya^3	1	a	a^2	x	xa	xa^2	y	ya	ya^2	xy	xya	xya^2
xa	xa	a	xya	ya	xa^2	xa^3	x	A^2	a^3	1	xya^2	xya^3	xy	ya^2	ya^3	y
xa^2	xa^2	a^2	xya^2	ya^2	xa^3	x	xa	A^3	1	a	xya^3	xy	xya	ya^3	y	ya
xa^3	xa^3	a^3	xya^3	ya^3	x	xa	xa^2	1	a	a^2	xy	xya	xya^2	y	ya	ya^2
ya	ya	xya^3	a	xa^3	ya^2	ya^3	y	xy	xya	xya^2	a^2	a^3	1	x	xa	xa^2
ya^2	ya^2	xy	a^2	x	ya^3	y	ya	xya	xya^2	xya^3	a^3	1	a	xa	xa^2	xa^3
ya^3	ya^3	xya	a^3	xa	y	ya	ya^2	xya^2	xya^3	xy	1	a	a^2	xa^2	xa^3	x
xya	xya	ya^3	xa	a^2	xya^2	xya^3	xy	ya^3	ya	ya^2	xa^2	xa^3	x	1	a	a^2
xya^2	xya^2	y	xa^2	a^3	xya^3	xy	xya	y	ya^2	ya^3	xa^3	x	xa	a	a^2	a^3
xya^3	xya^3	ya	xa^3	1	xy	xya	xya^2	ya	ya^3	y	x	xa	xa^2	a^2	a^3	1

TABLE 7: $G_7=\langle x,y,a\rangle$, $x^2=a^4=1$, $y^2=a$, $yx=xya^2$, $Z(G_7)=\langle a\rangle$.

	1	**x**	**y**	**xy**	**a**	**a^2**	**a^3**	**xa**	**xa^2**	**xa^3**	**ya**	**ya^2**	**ya^3**	**xya**	**xya^2**	**xya^3**
1	1	x	y	xy	a	a^2	a^3	xa	xa^2	xa^3	ya	ya^2	ya^3	xya	xya^2	xya^3
x	x	1	xy	y	xa	xa^2	xa^3	a	a^2	a^3	xya	xya^2	xya^3	ya	ya^2	ya^3
y	y	xya^2	a	xa^3	ya	ya^2	ya^3	xya^3	xy	xya	a^2	a^3	1	x	xa	xa^2
xy	xy	ya^2	xa	a^3	xya	xya^2	xya^3	ya^3	y	ya	xa^2	xa^3	x	1	a	a^2
a	a	xa	ya	xya	a^2	a^3	1	xa^2	xa^3	x	ya^2	ya^3	y	xya^2	xya^3	xy
a^2	a^2	xa^2	ya^2	xya^2	a^3	1	a	xa^3	x	xa	ya^3	y	ya	xya^3	xy	xya
a^3	a^3	xa^3	ya^3	xya^3	1	a	a^2	x	xa	xa^2	y	ya	ya^2	xy	xya	xya^2
xa	xa	a	xya	ya	xa^2	xa^3	x	A^2	a^3	1	xya^2	xya^3	xy	ya^2	ya^3	y
xa^2	xa^2	a^2	xya^2	ya^2	xa^3	x	xa	A^3	1	a	xya^3	xy	xya	ya^3	y	ya
xa^3	xa^3	a^3	xya^3	ya^3	x	xa	xa^2	1	a	a^2	xy	xya	xya^2	y	ya	ya^2
ya	ya	xya^3	a^2	x	ya^2	ya^3	y	xy	xya	xya^2	a^3	1	a	xa	xa^2	xa^3
ya^2	ya^2	xy	a^3	xa	ya^3	y	ya	xya	xya^2	xya^3	1	a	a^2	xa^2	xa^3	x
ya^3	ya^3	xya	1	xa^2	y	ya	ya^2	xya^2	xya^3	xy	a	a^2	a^3	xa^3	x	xa
xya	xya	ya^3	xa^2	1	xya^2	xya^3	xy	y	ya	ya^2	xa^3	x	xa	a	a^2	a^3
xya^2	xya^2	y	xa^3	a	xya^3	xy	xya	ya	ya^2	ya^3	x	xa	xa^2	a^2	a^3	1
xya^3	xya^3	ya	x	a^2	xy	xya	xya^2	ya^2	ya^3	y	xa	xa^2	xa^3	a^3	1	a

TABLE 8: G_8=<x,y,a>, $x^2=a^4=1$, $y^2=a^2$, $yx=xya^2$, $Z(G_8)$=<a>.

	1	**x**	**y**	**xy**	**a**	**a^2**	**a^3**	**xa**	**xa^2**	**xa^3**	**ya**	**ya^2**	**ya^3**	**xya**	**xya^2**	**xya^3**
1	1	x	y	xy	a	a^2	a^3	xa	xa^2	xa^3	ya	ya^2	ya^3	xya	xya^2	xya^3
x	x	1	xy	y	xa	xa^2	xa^3	a	a^2	a^3	xya	xya^2	xya^3	ya	ya^2	ya^3
y	y	xya^2	a^2	x	ya	ya^2	ya^3	xya^3	xy	xya	a^3	1	a	xa	xa^2	xa^3
xy	xy	ya^2	xa^2	1	xya	xya^2	xya^3	ya^3	y	ya	xa^3	x	xa	a	a^2	a^3
a	a	xa	ya	xya	a^2	a^3	1	xa^2	xa^3	x	ya^2	ya^3	y	xya^2	xya^3	xy
a^2	a^2	xa^2	ya^2	xya^2	a^3	1	a	xa^3	x	xa	ya^3	y	ya	xya^3	xy	xya
a^3	a^3	xa^3	ya^3	xya^3	1	a	a^2	x	xa	xa^2	y	ya	ya^2	xy	xya	xya^2
xa	xa	a	xya	ya	xa^2	xa^3	x	A^2	a^3	1	xya^2	xya^3	xy	ya^2	ya^3	y
xa^2	xa^2	a^2	xya^2	ya^2	xa^3	x	xa	A^3	1	a	xya^3	xy	xya	ya^3	y	ya
xa^3	xa^3	a^3	xya^3	ya^3	x	xa	xa^2	1	a	a^2	xy	xya	xya^2	y	ya	ya^2
ya	ya	xya^3	a^3	xa	ya^2	ya^3	y	xy	xya	xya^2	1	a	a^2	xa^2	xa^3	x
ya^2	ya^2	xy	1	xa^2	ya^3	y	ya	xya	xya^2	xya^3	a	a^2	a^3	xa^3	x	xa
ya^3	ya^3	xya	a	xa^3	y	ya	ya^2	xya^2	xya^3	xy	a^2	a^3	1	x	xa	xa^2
xya	xya	ya^3	xa^3	a	xya^2	xya^3	xy	y	ya	ya^2	x	xa	xa^2	a^2	a^3	1
xya^2	xya^2	y	x	a^2	xya^3	xy	xya	ya	ya^2	ya^3	xa	xa^2	xa^3	a^3	1	a
xya^3	xya^3	ya	xa	a^3	xy	xya	xya^2	ya^2	ya^3	y	xa^2	xa^3	x	1	a	a^2

TABLE 9: $G_9=\langle x,y,a\rangle$, $x^2=a^4=1$, $y^2=a^3$, $yx=xya^2$, $Z(G_9)=\langle a\rangle$.

	1	**x**	**y**	**xy**	**a**	**a^2**	**a^3**	**xa**	**xa^2**	**xa^3**	**ya**	**ya^2**	**ya^3**	**xya**	**xya^2**	**xya^3**
1	1	x	y	xy	a	a^2	a^3	xa	xa^2	xa^3	ya	ya^2	ya^3	xya	xya^2	xya^3
x	x	1	xy	y	xa	xa^2	xa^3	a	a^2	a^3	xya	xya^2	xya^3	ya	ya^2	ya^3
y	y	xya^2	a^3	xa	ya	ya^2	ya^3	xya^3	xy	xya	1	a	a^2	xa^2	xa^3	x
xy	xy	ya^2	xa^3	a	xya	xya^2	xya^3	ya^3	y	ya	x	xa	xa^2	a^2	a^3	1
a	a	xa	ya	xya	a^2	a^3	1	xa^2	xa^3	x	ya^2	ya^3	y	xya^2	xya^3	xy
a^2	a^2	xa^2	ya^2	xya^2	a^3	1	a	xa^3	x	xa	ya^3	y	ya	xya^3	xy	xya
a^3	a^3	xa^3	ya^3	xya^3	1	a	a^2	x	xa	xa^2	y	ya	ya^2	xy	xya	xya^2
xa	xa	a	xya	ya	xa^2	xa^3	x	A^2	a^3	1	xya^2	xya^3	xy	ya^2	ya^3	y
xa^2	xa^2	a^2	xya^2	ya^2	xa^3	x	xa	A^3	1	a	xya^3	xy	xya	ya^3	y	ya
xa^3	xa^3	a^3	xya^3	ya^3	x	xa	xa^2	1	a	a^2	xy	xya	xya^2	y	ya	ya^2
ya	ya	xya^3	1	xa^2	ya^2	ya^3	y	xy	xya	xya^2	a	a^2	a^3	xa^3	x	xa
ya^2	ya^2	xy	a	xa^3	ya^3	y	ya	xya	xya^2	xya^3	a^2	a^3	1	x	xa	xa^2
ya^3	ya^3	xya	a^2	x	y	ya	ya^2	xya^2	xya^3	xy	a^3	1	a	xa	xa^2	xa^3
xya	xya	ya^3	x	a^2	xya^2	xya^3	xy	y	ya	ya^2	xa	xa^2	xa^3	a^3	1	a
xya^2	xya^2	y	xa	a^3	xya^3	xy	xya	ya	ya^2	ya^3	xa^2	xa^3	x	1	a	a^2
xya^3	xya^3	ya	xa^2	1	xy	xya	xya^2	ya^2	ya^3	y	xa^3	x	xa	a	a^2	a^3

TABLE 10: $G_{10}=\langle x,y,a\rangle$, $x^2=y^2=a$, $a^4=1$, $yx=xya^2$, $Z(G_{10})=\langle a\rangle$.

	1	**x**	**y**	**xy**	**a**	**a^2**	**a^3**	**xa**	**xa^2**	**xa^3**	**ya**	**ya^2**	**ya^3**	**xya**	**xya^2**	**xya^3**
1	1	x	y	xy	a	a^2	a^3	xa	xa^2	xa^3	ya	ya^2	ya^3	xya	xya^2	xya^3
x	x	a	xy	ya	xa	xa^2	xa^3	A^2	a^3	1	xya	xya^2	xya^3	ya^2	ya^3	y
y	y	xya^2	a	xa^3	ya	ya^2	ya^3	xya^3	xy	xya	a^2	a^3	1	x	xa	xa^2
xy	xy	ya^3	xa	1	xya	xya^2	xya^3	y	ya	ya^2	xa^2	xa^3	x	a	a^2	a^3
a	a	xa	ya	xya	a^2	a^3	1	xa^2	xa^3	x	ya^2	ya^3	y	xya^2	xya^3	xy
a^2	a^2	xa^2	ya^2	xya^2	a^3	1	a	xa^3	x	xa	ya^3	y	ya	xya^3	xy	xya
a^3	a^3	xa^3	ya^3	xya^3	1	a	a^2	x	xa	xa^2	y	ya	ya^2	xy	xya	xya^2
xa	xa	a^2	xya	ya^2	xa^2	xa^3	x	a^3	1	a	xya^2	xya^3	xy	ya^3	y	ya
xa^2	xa^2	a^3	xya^2	ya^3	xa^3	x	xa	1	a	a^2	xya^3	xy	xya	y	ya	ya^2
xa^3	xa^3	1	xya^3	y	x	xa	xa^2	A	a^2	a^3	xy	xya	xya^2	ya	ya^2	ya^3
ya	ya	xya^3	a^2	x	ya^2	ya^3	y	xy	xya	xya^2	a^3	1	a	xa	xa^2	xa^3
ya^2	ya^2	xy	a^3	xa	ya^3	y	ya	xya	xya^2	xya^3	1	a	a^2	xa^2	xa^3	x
ya^3	ya^3	xya	1	xa^2	y	ya	ya^2	xya^2	xya^3	xy	a	a^2	a^3	xa^3	x	xa
xya	xya	y	xa^2	a	xya^2	xya^3	xy	ya	ya^2	ya^3	xa^3	x	xa	a^2	a^3	1
xya^2	xya^2	ya	xa^3	a^2	xya^3	xy	xya	ya^2	ya^3	y	x	xa	xa^2	a^3	1	a
xya^3	xya^3	ya^2	x	a^3	xy	xya	xya^2	ya^3	y	ya	xa	xa^2	xa^3	1	a	a^2

TABLE 11: $G_{11}=\langle x,y,a\rangle$, $x^2=a$, $y^2=a^2$, $a^4=1$, $yx=xya^2$, $Z(G_{11})=\langle a\rangle$.

	1	**x**	**y**	**xy**	**a**	**a^2**	**a^3**	**xa**	**xa^2**	**xa^3**	**ya**	**ya^2**	**ya^3**	**xya**	**xya^2**	**xya^3**
1	1	x	y	xy	a	a^2	a^3	xa	xa^2	xa^3	ya	ya^2	ya^3	xya	xya^2	xya^3
x	x	a	xy	ya	xa	xa^2	xa^3	a^2	a^3	1	xya	xya^2	xya^3	ya^2	ya^3	y
y	y	xya^2	a^2	x	ya	ya^2	ya^3	xya^3	xy	xya	a^3	1	a	xa	xa^2	xa^3
xy	xy	ya^3	xa^2	a	xya	xya^2	xya^3	Y	ya	ya^2	xa^3	x	xa	a^2	a^3	1
a	a	xa	ya	xya	a^2	a^3	1	xa^2	xa^3	x	ya^2	ya^3	y	xya^2	xya^3	xy
a^2	a^2	xa^2	ya^2	xya^2	a^3	1	a	xa^3	x	xa	ya^3	y	ya	xya^3	xy	xya
a^3	a^3	xa^3	ya^3	xya^3	1	a	a^2	X	xa	xa^2	y	ya	ya^2	xy	xya	xya^2
xa	xa	a^2	xya	ya^2	xa^2	xa^3	x	a^3	1	a	xya^2	xya^3	xy	ya^3	y	ya
xa^2	xa^2	a^3	xya^2	ya^3	xa^3	x	xa	1	a	a^2	xya^3	xy	xya	y	ya	ya^2
xa^3	xa^3	1	xya^3	y	x	xa	xa^2	a	a^2	a^3	xy	xya	xya^2	ya	ya^2	ya^3
ya	ya	xya^3	a^3	xa	ya^2	ya^3	y	xy	xya	xya^2	1	a	a^2	xa^2	xa^3	x
ya^2	ya^2	xy	1	xa^2	ya^3	y	ya	xya	xya^2	xya^3	a	a^2	a^3	xa^3	x	xa
ya^3	ya^3	xya	a	xa^3	y	ya	ya^2	xya^2	xya^3	xy	a^2	a^3	1	x	xa	xa^2
xya	xya	y	xa^3	a^2	xya^2	xya^3	xy	ya	ya^2	ya^3	x	xa	xa^2	a^3	1	a
xya^2	xya^2	ya	x	a^3	xya^3	xy	xya	Ya^2	ya^3	y	xa	xa^2	xa^3	1	a	a^2
xya^3	xya^3	ya^2	xa	1	xy	xya	xya^2	Ya^3	y	ya	xa^2	xa^3	x	a	a^2	a^3

TABLE 12: G_{12}=<x,y,a>, x^2=a, $y^2=a^3$, a^4=1, yx=xya^2, $Z(G_{12})$=<a>.

	1	**x**	**y**	**xy**	**a**	**a^2**	**a^3**	**xa**	**xa^2**	**xa^3**	**ya**	**ya^2**	**ya^3**	**xya**	**xya^2**	**xya^3**
1	1	x	y	xy	a	a^2	a^3	xa	xa^2	xa^3	ya	ya^2	ya^3	xya	xya^2	xya^3
x	x	a	xy	ya	xa	xa^2	xa^3	a^2	a^3	1	xya	xya^2	xya^3	ya^2	ya^3	y
y	y	xya^2	a^3	xa	ya	ya^2	ya^3	xya^3	xy	xya	1	a	a^2	xa^2	xa^3	x
xy	xy	ya^3	xa^3	a^2	xya	xya^2	xya^3	Y	ya	ya^2	x	xa	xa^2	a^3	1	a
a	a	xa	ya	xya	a^2	a^3	1	xa^2	xa^3	x	ya^2	ya^3	y	xya^2	xya^3	xy
a^2	a^2	xa^2	ya^2	xya^2	a^3	1	a	xa^3	x	xa	ya^3	y	ya	xya^3	xy	xya
a^3	a^3	xa^3	ya^3	xya^3	1	a	a^2	X	xa	xa^2	y	ya	ya^2	xy	xya	xya^2
xa	xa	a^2	xya	ya^2	xa^2	xa^3	x	a^3	1	a	xya^2	xya^3	xy	ya^3	y	ya
xa^2	xa^2	a^3	xya^2	ya^3	xa^3	x	xa	1	a	a^2	xya^3	xy	xya	y	ya	ya^2
xa^3	xa^3	1	xya^3	y	x	xa	xa^2	a	a^2	a^3	xy	xya	xya^2	ya	ya^2	ya^3
ya	ya	xya^3	1	xa^2	ya^2	ya^3	y	xy	xya	xya^2	a	a^2	a^3	xa^3	x	xa
ya^2	ya^2	xy	a	xa^3	ya^3	y	ya	xya	xya^2	xya^3	a^2	a^3	1	x	xa	xa^2
ya^3	ya^3	xya	a^2	x	y	ya	ya^2	xya^2	xya^3	xy	a^3	1	a	xa	xa^2	xa^3
xya	xya	y	x	a^3	xya^2	xya^3	xy	ya	ya^2	ya^3	xa	xa^2	xa^3	1	a	a^2
xya^2	xya^2	ya	xa	1	xya^3	xy	xya	Ya^2	ya^3	y	xa^2	xa^3	x	a	a^2	a^3
xya^3	xya^3	ya^2	xa^2	a	xy	xya	xya^2	Ya^3	y	ya	xa^3	x	xa	a^2	a^3	1

TABLE 13: $G_{13}=\langle x,y,a\rangle$, $x^2=y^2=a^2$, $a^4=1$, $yx=xya^2$, $Z(G_{13})=\langle a\rangle$.

	1	**x**	**y**	**xy**	**a**	**a^2**	**a^3**	**xa**	**xa^2**	**xa^3**	**ya**	**ya^2**	**ya^3**	**xya**	**xya^2**	**xya^3**
1	1	x	y	xy	a	a^2	a^3	xa	xa^2	xa^3	ya	ya^2	ya^3	xya	xya^2	xya^3
x	x	a^2	xy	ya^2	xa	xa^2	xa^3	A^3	1	a	xya	xya^2	xya^3	ya^3	y	ya
y	y	xya^2	a^2	x	ya	ya^2	ya^3	xya^3	xy	xya	a^3	1	a	xa	xa^2	xa^3
xy	xy	y	xa^2	a^2	xya	xya^2	xya^3	ya	ya^2	ya^3	xa^3	x	xa	a^3	1	a
a	a	xa	ya	xya	a^2	a^3	1	xa^2	xa^3	x	ya^2	ya^3	y	xya^2	xya^3	xy
a^2	a^2	xa^2	ya^2	xya^2	a^3	1	a	xa^3	x	xa	ya^3	y	ya	xya^3	xy	xya
a^3	a^3	xa^3	ya^3	xya^3	1	a	a^2	x	xa	xa^2	y	ya	ya^2	xy	xya	xya^2
xa	xa	a^3	xya	ya^3	xa^2	xa^3	x	1	a	a^2	xya^2	xya^3	xy	y	ya	ya^2
xa^2	xa^2	1	xya^2	y	xa^3	x	xa	a	a^2	a^3	xya^3	xy	xya	ya	ya^2	ya^3
xa^3	xa^3	a	xya^3	ya	x	xa	xa^2	A^2	a^3	1	xy	xya	xya^2	ya^2	ya^3	y
ya	ya	xya^3	a^3	xa	ya^2	ya^3	y	xy	xya	xya^2	1	a	a^2	xa^2	xa^3	x
ya^2	ya^2	xy	1	xa^2	ya^3	y	ya	xya	xya^2	xya^3	a	a^2	a^3	xa^3	x	xa
ya^3	ya^3	xya	a	xa^3	y	ya	ya^2	xya^2	xya^3	xy	a^2	a^3	1	x	xa	xa^2
xya	xya	ya	xa^3	a^3	xya^2	xya^3	xy	ya^2	ya^3	y	x	xa	xa^2	1	a	a^2
xya^2	xya^2	ya^2	x	1	xya^3	xy	xya	ya^3	y	ya	xa	xa^2	xa^3	a	a^2	a^3
xya^3	xya^3	ya^3	xa	a	xy	xya	xya^2	y	ya	ya^2	xa^2	xa^3	x	a^2	a^3	1

TABLE 14: $G_{14}=<x,y,a>$, $x^2=a^2$, $y^2=a^3$, $a^4=1$, $yx=xya^2$, $Z(G_{14})=<a>$.

	1	**x**	**y**	**xy**	**a**	**a^2**	**a^3**	**xa**	**xa^2**	**xa^3**	**ya**	**ya^2**	**ya^3**	**xya**	**xya^2**	**xya^3**
1	1	x	y	xy	a	a^2	a^3	xa	xa^2	xa^3	ya	ya^2	ya^3	xya	xya^2	xya^3
x	x	a^2	xy	ya^2	xa	xa^2	xa^3	A^3	1	a	xya	xya^2	xya^3	ya^3	y	ya
y	y	xya^2	a^3	xa	ya	ya^2	ya^3	xya^3	xy	xya	1	a	a^2	xa^2	xa^3	x
xy	xy	y	xa^3	a^3	xya	xya^2	xya^3	ya	ya^2	ya^3	x	xa	xa^2	1	a	a^2
a	a	xa	ya	xya	a^2	a^3	1	xa^2	xa^3	x	ya^2	ya^3	y	xya^2	xya^3	xy
a^2	a^2	xa^2	ya^2	xya^2	a^3	1	a	xa^3	x	xa	ya^3	y	ya	xya^3	xy	xya
a^3	a^3	xa^3	ya^3	xya^3	1	a	a^2	x	xa	xa^2	y	ya	ya^2	xy	xya	xya^2
xa	xa	a^3	xya	ya^3	xa^2	xa^3	x	1	a	a^2	xya^2	xya^3	xy	y	ya	ya^2
xa^2	xa^2	1	xya^2	y	xa^3	x	xa	a	a^2	a^3	xya^3	xy	xya	ya	ya^2	ya^3
xa^3	xa^3	a	xya^3	ya	x	xa	xa^2	A^2	a^3	1	xy	xya	xya^2	ya^2	ya^2	ya^3
ya	ya	xya^3	1	xa^2	ya^2	ya^3	y	xy	xya	xya^2	a	a^2	a^3	xa^3	x	xa
ya^2	ya^2	xy	a	xa^3	ya^3	y	ya	xya	xya^2	xya^3	a^2	a^3	1	x	xa	xa^2
ya^3	ya^3	xya	a^2	x	y	ya	ya^2	xya^2	xya^3	xy	a^3	1	a	xa	xa^2	xa^3
xya	xya	ya	x	1	xya^2	xya^3	xy	ya^2	ya^3	y	xa	xa^2	xa^3	a	a^2	a^3
xya^2	xya^2	ya^2	xa	a	xya^3	xy	xya	ya^3	y	ya	xa^2	xa^3	x	a^2	a^3	1
xya^3	xya^3	ya^3	xa^2	a^2	xy	xya	xya^2	y	ya	ya^2	xa^3	x	xa	a^3	1	a

TABLE 15: $G_{14}=\langle x,y,a\rangle$, $x^2=y^2=a^3$, $a^4=1$, $yx=xya^2$, $Z(G_{14})=\langle a\rangle$.

	1	**x**	**y**	**xy**	**a**	**a^2**	**a^3**	**xa**	**xa^2**	**xa^3**	**ya**	**ya^2**	**ya^3**	**xya**	**xya^2**	**xya^3**
1	1	x	y	xy	a	a^2	a^3	xa	xa^2	xa^3	ya	ya^2	ya^3	xya	xya^2	xya^3
x	x	a^3	xy	ya^3	xa	xa^2	xa^3	1	a	a^2	xya	xya^2	xya^3	y	ya	ya^2
y	y	xya^2	a^3	xa	ya	ya^2	ya^3	xya^3	xy	xya	1	a	a^2	xa^2	xa^3	x
xy	xy	ya	xa^3	1	xya	xya^2	xya^3	ya^2	ya^3	y	x	xa	xa^2	a	a^2	a^3
a	a	xa	ya	xya	a^2	a^3	1	xa^2	xa^3	x	ya^2	ya^3	y	xya^2	xya^3	xy
a^2	a^2	xa^2	ya^2	xya^2	a^3	1	a	xa^3	x	xa	ya^3	y	ya	xya^3	xy	xya
a^3	a^3	xa^3	ya^3	xya^3	1	a	a^2	x	xa	xa^2	y	ya	ya^2	xy	xya	xya^2
xa	xa	1	xya	y	xa^2	xa^3	x	a	a^2	a^3	xya^2	xya^3	xy	ya	ya^2	ya^3
xa^2	xa^2	a	xya^2	ya	xa^3	x	xa	A^2	a^3	1	xya^3	xy	xya	ya^2	ya^3	y
xa^3	xa^3	a^2	xya^3	ya^2	x	xa	xa^2	A^3	1	a	xy	xya	xya^2	ya^3	y	ya
ya	ya	xya^3	1	xa^2	ya^2	ya^3	y	xy	xya	xya^2	a	a^2	a^3	xa^3	x	xa
ya^2	ya^2	xy	a	xa^3	ya^3	y	ya	xya	xya^2	xya^3	a^2	a^3	1	x	xa	xa^2
ya^3	ya^3	xya	a^2	x	y	ya	ya^2	xya^2	xya^3	xy	a^3	1	a	xa	xa^2	xa^3
xya	xya	ya^2	x	a	xya^2	xya^3	xy	ya^3	y	ya	xa	xa^2	xa^3	a^2	a^3	1
xya^2	xya^2	ya^3	xa	a^2	xya^3	xy	xya	y	ya	ya^2	xa^2	xa^3	x	a^3	1	a
xya^3	xya^3	y	xa^2	a^3	xy	xya	xya^2	ya	ya^2	ya^3	xa^3	x	xa	1	a	a^2

TABLE 16: $G_{16}=\langle x,y,a,b\rangle$, $x^2=y^2=a^2=b^2=1$, $xy=yxa$, $ab=ba$, $Z(G_{16})=\langle a,b\rangle$.

	1	**x**	**y**	**xy**	**a**	**b**	**ab**	**xa**	**xb**	**xab**	**ya**	**yb**	**yab**	**xya**	**xyb**	**xyab**
1	1	x	y	xy	a	b	ab	xa	xb	xab	ya	yb	yab	xya	xyb	xyab
x	x	1	xy	y	xa	xb	xab	a	b	ab	xya	xyb	xyab	ya	yb	yab
y	y	xya	1	xa	ya	yb	yab	xy	xyab	xyb	a	b	ab	x	xab	xb
xy	xy	ya	x	a	xya	xyb	xyab	y	yab	yb	xa	xb	xab	1	ab	b
a	a	xa	ya	xya	1	ab	b	x	xab	xb	y	yab	yb	xy	xyab	xyb
b	b	xb	yb	xyb	ab	1	a	xab	x	xa	yab	y	ya	xyab	xy	xya
ab	ab	xab	yab	xyab	b	a	1	xb	xa	x	yb	ya	y	xyb	xya	xy
xa	xa	a	xya	ya	x	xab	xb	1	ab	b	xy	xyab	xyb	y	yab	yb
xb	xb	b	xyb	yb	xab	x	xa	ab	1	a	xyab	xy	xya	yab	y	ya
xab	xab	ab	xyab	yab	xb	xa	x	b	a	1	xyb	xya	xy	yb	ya	y
ya	ya	xy	a	x	y	yab	yb	xya	xyb	xyab	1	ab	b	xa	xb	xab
yb	yb	xyab	b	xab	yab	y	ya	xyb	xya	xy	ab	1	a	xb	xa	x
yab	yab	xyb	ab	xb	yb	ya	y	xyab	xy	xya	b	a	1	xab	x	xa
xya	xya	y	xa	1	xy	xyab	xyb	ya	yb	yab	x	xab	xb	a	b	ab
xyb	xyb	yab	xb	ab	xyab	xy	xya	yb	ya	y	xab	x	xa	b	a	1
xyab	xyab	yb	xab	b	xyb	xya	xy	yab	y	ya	xb	xa	x	ab	1	a

TABLE 17: G_{17}=<x,y,a,b>, $x^2=a^2=b^2=1$, $y^2=a$, xy=yxa, $Z(G_{17})$=<a,b>.

	1	**x**	**y**	**xy**	**a**	**b**	**ab**	**xa**	**xb**	**xab**	**ya**	**yb**	**yab**	**xya**	**xyb**	**xyab**
1	1	x	y	xy	a	b	ab	xa	xb	xab	ya	yb	yab	xya	xyb	xyab
x	x	1	xy	y	xa	xb	xab	a	b	ab	xya	xyb	xyab	ya	yb	yab
y	y	xya	a	x	ya	yb	yab	xy	xyab	xyb	1	ab	b	xa	xb	xab
xy	xy	ya	xa	1	xya	xyb	xyab	y	yab	yb	x	xab	xb	a	b	ab
a	a	xa	ya	xya	1	ab	b	x	xab	xb	y	yab	yb	xy	xyab	xyb
b	b	xb	yb	xyb	ab	1	a	xab	x	xa	yab	y	ya	xyab	xy	xya
ab	ab	xab	yab	xyab	b	a	1	xb	xa	x	yb	ya	y	xyb	xya	xy
xa	xa	a	xya	ya	x	xab	xb	1	ab	b	xy	xyab	xyb	y	yab	yb
xb	xb	b	xyb	yb	xab	x	xa	ab	1	a	xyab	xy	xya	yab	y	ya
xab	xab	ab	xyab	yab	xb	xa	x	b	a	1	xyb	xya	xy	yb	ya	y
ya	ya	xy	1	xa	y	yab	yb	xya	xyb	xyab	a	b	ab	x	xab	xb
yb	yb	xyab	ab	xb	yab	y	ya	xyb	xya	xy	b	a	1	xab	x	xa
yab	yab	xyb	b	xab	yb	ya	y	xyab	xy	xya	ab	1	a	xb	xa	x
xya	xya	y	x	a	xy	xyab	xyb	ya	yb	yab	xa	xb	xab	1	ab	b
xyb	xyb	yab	xab	b	xyab	xy	xya	yb	ya	y	xb	xa	x	ab	1	a
xyab	xyab	yb	xb	ab	xyb	xya	xy	yab	y	ya	xab	x	xa	b	a	1

TABLE 18: G_{18}=<x,y,a,b>, $x^2=a^2=b^2=1$, $y^2=b$, xy=yxa, $Z(G_{18})$=<a,b>.

	1	**x**	**y**	**xy**	**a**	**b**	**ab**	**xa**	**xb**	**xab**	**ya**	**yb**	**yab**	**xya**	**xyb**	**xyab**
1	1	x	y	xy	a	b	ab	xa	xb	xab	ya	yb	yab	xya	xyb	xyab
x	x	1	xy	y	xa	xb	xab	a	b	ab	xya	xyb	xyab	ya	yb	yab
y	y	xya	b	xab	ya	yb	yab	xy	xyab	xyb	ab	1	a	xb	xa	x
xy	xy	ya	xb	ab	xya	xyb	xyab	y	yab	yb	xab	x	xa	b	a	1
a	a	xa	ya	xya	1	ab	b	x	xab	xb	y	yab	yb	xy	xyab	xyb
b	b	xb	yb	xyb	ab	1	a	xab	x	xa	yab	y	ya	xyab	xy	xya
ab	ab	xab	yab	xyab	b	a	1	xb	xa	x	yb	ya	y	xyb	xya	xy
xa	xa	a	xya	ya	x	xab	xb	1	ab	b	xy	xyab	xyb	y	yab	yb
xb	xb	b	xyb	yb	xab	x	xa	ab	1	a	xyab	xy	xya	yab	y	ya
xab	xab	ab	xyab	yab	xb	xa	x	b	a	1	xyb	xya	xy	yb	ya	y
ya	ya	xy	ab	xb	y	yab	yb	xya	xyb	xyab	b	a	1	xab	x	xa
yb	yb	xyab	1	xa	yab	y	ya	xyb	xya	xy	a	b	ab	x	xab	xb
yab	yab	xyb	a	x	yb	ya	y	xyab	xy	xya	1	ab	b	xa	xb	xab
xya	xya	y	xab	b	xy	xyab	xyb	ya	yb	yab	xb	xa	x	ab	1	a
xyb	xyb	yab	x	a	xyab	xy	xya	yb	ya	y	xa	xb	xab	1	ab	b
xyab	xyab	yb	xa	1	xyb	xya	xy	yab	y	ya	x	xab	xb	a	b	ab

TABLE 19: G_{19}=<x,y,a,b>, $x^2=a^2=b^2=1$, $y^2=ab$, xy=yxa, $Z(G_{19})$=<a,b>.

	1	**x**	**y**	**xy**	**a**	**b**	**ab**	**xa**	**xb**	**xab**	**ya**	**yb**	**yab**	**xya**	**xyb**	**xyab**
1	1	x	y	xy	a	b	ab	xa	xb	xab	ya	yb	yab	xya	xyb	xyab
x	x	1	xy	y	xa	xb	xab	a	b	ab	xya	xyb	xyab	ya	yb	yab
y	y	xya	ab	xb	ya	yb	yab	xy	xyab	xyb	b	a	1	xab	x	xa
xy	xy	ya	xab	b	xya	xyb	xyab	y	yab	yb	xb	xa	x	ab	1	a
a	a	xa	ya	xya	1	ab	b	x	xab	xb	y	yab	yb	xy	xyab	xyb
b	b	xb	yb	xyb	ab	1	a	xab	x	xa	yab	y	ya	xyab	xy	xya
ab	ab	xab	yab	xyab	b	a	1	xb	xa	x	yb	ya	y	xyb	xya	xy
xa	xa	a	xya	ya	x	xab	xb	1	ab	b	xy	xyab	xyb	y	yab	yb
xb	xb	b	xyb	yb	xab	x	xa	ab	1	a	xyab	xy	xya	yab	y	ya
xab	xab	ab	xyab	yab	xb	xa	x	b	a	1	xyb	xya	xy	yb	ya	y
ya	ya	xy	b	xab	y	yab	yb	xya	xyb	xyab	ab	1	a	xb	xa	x
yb	yb	xyab	a	x	yab	y	ya	xyb	xya	xy	1	ab	b	xa	xb	xab
yab	yab	xyb	1	xa	yb	ya	y	xyab	xy	xya	a	b	ab	x	xab	xb
xya	xya	y	xb	ab	xy	xyab	xyb	ya	yb	yab	xab	x	xa	b	a	1
xyb	xyb	yab	xa	1	xyab	xy	xya	yb	ya	y	x	xab	xb	a	b	ab
xyab	xyab	yb	x	a	xyb	xya	xy	yab	y	ya	xa	xb	xab	1	ab	b

TABLE 20: $G_{20}=<x,y,a,b>$, $x^2=a=y^2$, $a^2=b^2=1$, $xy=yxa$, $Z(G_{20})=<a,b>$.

	1	**x**	**y**	**xy**	**a**	**b**	**ab**	**xa**	**xb**	**xab**	**ya**	**yb**	**yab**	**xya**	**xyb**	**xyab**
1	1	x	y	xy	a	b	ab	xa	xb	xab	ya	yb	yab	xya	xyb	xyab
x	x	a	xy	ya	xa	xb	xab	1	ab	b	xya	xyb	xyab	y	yab	yb
y	y	xya	a	x	ya	yb	yab	xy	xyab	xyb	1	ab	b	xa	xb	xab
xy	xy	y	xa	a	xya	xyb	xyab	ya	yb	yab	x	xab	xb	1	ab	b
a	a	xa	ya	xya	1	ab	b	x	xab	xb	y	yab	yb	xy	xyab	xyb
b	b	xb	yb	xyb	ab	1	a	xab	x	xa	yab	y	ya	xyab	xy	xya
ab	ab	xab	yab	xyab	b	a	1	xb	xa	x	yb	ya	y	xyb	xya	xy
xa	xa	1	xya	y	x	xab	xb	a	b	ab	xy	xyab	xyb	ya	yb	yab
xb	xb	ab	xyb	yab	xab	x	xa	b	a	1	xyab	xy	xya	yb	ya	y
xab	xab	b	xyab	yb	xb	xa	x	ab	1	a	xyb	xya	xy	yab	y	ya
ya	ya	xy	1	xa	y	yab	yb	xya	xyb	xyab	a	b	ab	x	xab	xb
yb	yb	xyab	ab	xb	yab	y	ya	xyb	xya	xy	b	a	1	xab	x	xa
yab	yab	xyb	b	xab	yb	ya	y	xyab	xy	xya	ab	1	a	xb	xa	x
xya	xya	ya	x	1	xy	xyab	xyb	y	yab	yb	xa	xb	xab	a	b	ab
xyb	xyb	yb	xab	ab	xyab	xy	xya	yab	y	ya	xb	xa	x	b	a	1
xyab	xyab	yab	xb	b	xyb	xya	xy	yb	ya	y	xab	x	xa	ab	1	a

TABLE 21: G_{21}=<x,y,a,b>, x^2=a, y^2=b, a^2=b^2=1, xy=yxa, $Z(G_{21})$=<a,b>.

	1	**x**	**y**	**xy**	**a**	**b**	**ab**	**xa**	**xb**	**xab**	**ya**	**yb**	**yab**	**xya**	**xyb**	**xyab**
1	1	x	y	xy	a	b	ab	xa	xb	xab	ya	yb	yab	xya	xyb	xyab
x	x	a	xy	ya	xa	xb	xab	1	ab	b	xya	xyb	xyab	y	yab	yb
y	y	xya	b	xab	ya	yb	yab	xy	xyab	xyb	ab	1	a	xb	xa	x
xy	xy	y	xb	b	xya	xyb	xyab	ya	yb	yab	xab	x	xa	ab	1	a
a	a	xa	ya	xya	1	ab	b	x	xab	xb	y	yab	yb	xy	xyab	xyb
b	b	xb	yb	xyb	ab	1	a	xab	x	xa	yab	y	ya	xyab	xy	xya
ab	ab	xab	yab	xyab	b	a	1	xb	xa	x	yb	ya	y	xyb	xya	xy
xa	xa	1	xya	y	x	xab	xb	a	b	ab	xy	xyab	xyb	ya	yb	yab
xb	xb	ab	xyb	yab	xab	x	xa	b	a	1	xyab	xy	xya	yb	ya	y
xab	xab	b	xyab	yb	xb	xa	x	ab	1	a	xyb	xya	xy	yab	y	ya
ya	ya	xy	ab	xb	y	yab	yb	xya	xyb	xyab	b	a	1	xab	x	xa
yb	yb	xyab	1	xa	yab	y	ya	xyb	xya	xy	a	b	ab	x	xab	xb
yab	yab	xyb	a	x	yb	ya	y	xyab	xy	xya	1	ab	b	xa	xb	xab
xya	xya	ya	xab	ab	xy	xyab	xyb	y	yab	yb	xb	xa	x	b	a	1
xyb	xyb	yb	x	1	xyab	xy	xya	yab	y	ya	xa	xb	xab	a	b	ab
xyab	xyab	yab	xa	a	xyb	xya	xy	yb	ya	y	x	xab	xb	1	ab	b

TABLE 22: $G_{22}=\langle x,y,a,b\rangle$, $x^2=a$, $y^2=ab$, $a^2=b^2=1$, $xy=yxa$, $Z(G_{22})=\langle a,b\rangle$.

	1	**x**	**y**	**xy**	**a**	**b**	**ab**	**xa**	**xb**	**xab**	**ya**	**yb**	**yab**	**xya**	**xyb**	**xyab**
1	1	x	y	xy	a	b	ab	xa	xb	xab	ya	yb	yab	xya	xyb	xyab
x	x	a	xy	ya	xa	xb	xab	1	ab	b	xya	xyb	xyab	y	yab	yb
y	y	xya	ab	xb	ya	yb	yab	xy	xyab	xyb	b	a	1	xab	x	xa
xy	xy	y	xab	ab	xya	xyb	xyab	ya	yb	yab	xb	xa	x	b	a	1
a	a	xa	ya	xya	1	ab	b	x	xab	xb	y	yab	yb	xy	xyab	xyb
b	b	xb	yb	xyb	ab	1	a	xab	x	xa	yab	y	ya	xyab	xy	xya
ab	ab	xab	yab	xyab	b	a	1	xb	xa	x	yb	ya	y	xyb	xya	xy
xa	xa	1	xya	y	x	xab	xb	a	b	ab	xy	xyab	xyb	ya	yb	yab
xb	xb	ab	xyb	yab	xab	x	xa	b	a	1	xyab	xy	xya	yb	ya	y
xab	xab	b	xyab	yb	xb	xa	x	ab	1	a	xyb	xya	xy	yab	y	ya
ya	ya	xy	b	xab	y	yab	yb	xya	xyb	xyab	ab	1	a	xb	xa	x
yb	yb	xyab	a	x	yab	y	ya	xyb	xya	xy	1	ab	b	xa	xb	xab
yab	yab	xyb	1	xa	yb	ya	y	xyab	xy	xya	a	b	ab	x	xab	xb
xya	xya	ya	xb	b	xy	xyab	xyb	y	yab	yb	xab	x	xa	ab	1	a
xyb	xyb	yb	xa	a	xyab	xy	xya	yab	y	ya	x	xab	xb	1	ab	b
xyab	xyab	yab	x	1	xyb	xya	xy	yb	ya	y	xa	xb	xab	a	b	ab

TABLE 23: $G_{23}=\langle x,y,a,b\rangle$, $x^2=y^2=b$, $a^2=b^2=1$, $xy=yxa$, $Z(G_{23})=\langle a,b\rangle$.

	1	**x**	**y**	**xy**	**a**	**b**	**ab**	**xa**	**xb**	**xab**	**ya**	**yb**	**yab**	**xya**	**xyb**	**xyab**
1	1	x	y	xy	a	b	ab	xa	xb	xab	ya	yb	yab	xya	xyb	xyab
x	x	b	xy	yb	xa	xb	xab	ab	1	a	xya	xyb	xyab	yab	y	ya
y	y	xya	b	xab	ya	yb	yab	xy	xyab	xyb	ab	1	a	xb	xa	x
xy	xy	yab	xb	a	xya	xyb	xyab	yb	ya	y	xab	x	xa	1	ab	b
a	a	xa	ya	xya	1	ab	b	x	xab	xb	y	yab	yb	xy	xyab	xyb
b	b	xb	yb	xyb	ab	1	a	xab	x	xa	yab	y	ya	xyab	xy	xya
ab	ab	xab	yab	xyab	b	a	1	xb	xa	x	yb	ya	y	xyb	xya	xy
xa	xa	ab	xya	yab	x	xab	xb	b	a	1	xy	xyab	xyb	yb	ya	y
xb	xb	1	xyb	y	xab	x	xa	a	b	ab	xyab	xy	xya	ya	yb	yab
xab	xab	a	xyab	ya	xb	xa	x	1	ab	b	xyb	xya	xy	y	yab	yb
ya	ya	xy	ab	xb	y	yab	yb	xya	xyb	xyab	b	a	1	xab	x	xa
yb	yb	xyab	1	xa	yab	y	ya	xyb	xya	xy	a	b	ab	x	xab	xb
yab	yab	xyb	a	x	yb	ya	y	xyab	xy	xya	1	ab	b	xa	xb	xab
xya	xya	yb	xab	1	xy	xyab	xyb	yab	y	ya	xb	xa	x	a	b	ab
xyb	xyb	ya	x	ab	xyab	xy	xya	y	yab	yb	xa	xb	xab	b	a	1
xyab	xyab	y	xa	b	xyb	xya	xy	ya	yb	yab	x	xab	xb	ab	1	a

TABLE 24: $G_{24}=\langle x,y,a,b\rangle$, $x^2=b$, $y^2=ab$, $a^2=b^2=1$, $xy=yxa$, $Z(G_{24})=\langle a,b\rangle$.

	1	**x**	**y**	**xy**	**a**	**b**	**ab**	**xa**	**xb**	**xab**	**ya**	**yb**	**yab**	**xya**	**xyb**	**xyab**
1	1	x	y	xy	a	b	ab	xa	xb	xab	ya	yb	yab	xya	xyb	xyab
x	x	b	xy	yb	xa	xb	xab	ab	1	a	xya	xyb	xyab	yab	y	ya
y	y	xya	ab	xb	ya	yb	yab	xy	xyab	xyb	b	a	1	xab	x	xa
xy	xy	yab	xab	1	xya	xyb	xyab	yb	ya	y	xb	xa	x	a	b	ab
a	a	xa	ya	xya	1	ab	b	x	xab	xb	y	yab	yb	xy	xyab	xyb
b	b	xb	yb	xyb	ab	1	a	xab	x	xa	yab	y	ya	xyab	xy	xya
ab	ab	xab	yab	xyab	b	a	1	xb	xa	x	yb	ya	y	xyb	xya	xy
xa	xa	ab	xya	yab	x	xab	xb	b	a	1	xy	xyab	xyb	yb	ya	y
xb	xb	1	xyb	y	xab	x	xa	a	b	ab	xyab	xy	xya	ya	yb	yab
xab	xab	a	xyab	ya	xb	xa	x	1	ab	b	xyb	xya	xy	y	yab	yb
ya	ya	xy	b	xab	y	yab	yb	xya	xyb	xyab	ab	1	a	xb	xa	x
yb	yb	xyab	a	x	yab	y	ya	xyb	xya	xy	1	ab	b	xa	xb	xab
yab	yab	xyb	1	xa	yb	ya	y	xyab	xy	xya	a	b	ab	x	xab	xb
xya	xya	yb	xb	a	xy	xyab	xyb	yab	y	ya	xab	x	xa	1	ab	b
xyb	xyb	ya	xa	b	xyab	xy	xya	y	yab	yb	x	xab	xb	ab	1	a
xyab	xyab	y	x	ab	xyb	xya	xy	ya	yb	yab	xa	xb	xab	b	a	1

TABLE 25: G_{25}=<x,y,a,b>, x^2=ab, y^2=a, a^2=b^2=1, xy=yxa, Z(G_{25})=<a,b>.

	1	**x**	**y**	**xy**	**a**	**b**	**ab**	**xa**	**xb**	**xab**	**ya**	**yb**	**yab**	**xya**	**xyb**	**xyab**
1	1	x	y	xy	a	b	ab	xa	xb	xab	ya	yb	yab	xya	xyb	xyab
x	x	ab	xy	yab	xa	xb	xab	b	a	1	xya	xyb	xyab	yb	ya	y
y	y	xya	a	x	ya	yb	yab	xy	xyab	xyb	1	ab	b	xa	xb	xab
xy	xy	yb	xa	ab	xya	xyb	xyab	yab	y	ya	x	xab	xb	b	a	1
a	a	xa	ya	xya	1	ab	b	x	xab	xb	y	yab	yb	xy	xyab	xyb
b	b	xb	yb	xyb	ab	1	a	xab	x	xa	yab	y	ya	xyab	xy	xya
ab	ab	xab	yab	xyab	b	a	1	xb	xa	x	yb	ya	y	xyb	xya	xy
xa	xa	b	xya	yb	x	xab	xb	ab	1	a	xy	xyab	xyb	yab	y	ya
xb	xb	a	xyb	ya	xab	x	xa	1	ab	b	xyab	xy	xya	y	yab	yb
xab	xab	1	xyab	y	xb	xa	x	a	b	ab	xyb	xya	xy	ya	yb	yab
ya	ya	xy	1	xa	y	yab	yb	xya	xyb	xyab	a	b	ab	x	xab	xb
yb	yb	xyab	ab	xb	yab	y	ya	xyb	xya	xy	b	a	1	xab	x	xa
yab	yab	xyb	b	xab	yb	ya	y	xyab	xy	xya	ab	1	a	xb	xa	x
xya	xya	yab	x	b	xy	xyab	xyb	yb	ya	y	xa	xb	xab	ab	1	a
xyb	xyb	y	xab	a	xyab	xy	xya	ya	yb	yab	xb	xa	x	1	ab	b
xyab	xyab	ya	xb	1	xyb	xya	xy	y	yab	yb	xab	x	xa	a	b	ab

TABLE 26: G_{26}=<x,y,a,b>, x^2=ab, y^2=b, $a^2=b^2$=1, xy=yxa, Z(G_{26})=<a,b>.

	1	**x**	**y**	**xy**	**a**	**b**	**ab**	**xa**	**xb**	**xab**	**ya**	**yb**	**yab**	**xya**	**xyb**	**xyab**
1	1	x	y	xy	a	b	ab	xa	xb	xab	ya	yb	yab	xya	xyb	xyab
x	x	ab	xy	yab	xa	xb	xab	b	a	1	xya	xyb	xyab	yb	ya	y
y	y	xya	b	xab	ya	yb	yab	xy	xyab	xyb	ab	1	a	xb	xa	x
xy	xy	yb	xa	1	xya	xyb	xyab	yab	y	ya	xab	x	xa	a	b	ab
a	a	xa	ya	xya	1	ab	b	x	xab	xb	y	yab	yb	xy	xyab	xyb
b	b	xb	yb	xyb	ab	1	a	xab	x	xa	yab	y	ya	xyab	xy	xya
ab	ab	xab	yab	xyab	b	a	1	xb	xa	x	yb	ya	y	xyb	xya	xy
xa	xa	b	xya	yb	x	xab	xb	ab	1	a	xy	xyab	xyb	yab	y	ya
xb	xb	a	xyb	ya	xab	x	xa	1	ab	b	xyab	xy	xya	y	yab	yb
xab	xab	1	xyab	y	xb	xa	x	a	b	ab	xyb	xya	xy	ya	yb	yab
ya	ya	xy	ab	xb	y	yab	yb	xya	xyb	xyab	b	a	1	xab	x	xa
yb	yb	xyab	1	xa	yab	y	ya	xyb	xya	xy	a	b	ab	x	xab	xb
yab	yab	xyb	a	x	yb	ya	y	xyab	xy	xya	1	ab	b	xa	xb	xab
xya	xya	yab	x	a	xy	xyab	xyb	yb	ya	y	xb	xa	x	1	ab	b
xyb	xyb	y	xab	b	xyab	xy	xya	ya	yb	yab	xa	xb	xab	ab	1	a
xyab	xyab	ya	xb	ab	xyb	xya	xy	y	yab	yb	x	xab	xb	b	a	1

TABLE 27: $G_{27}=\langle x,y,a,b\rangle$, $x^2=y^2=ab$, $a^2=b^2=1$, $xy=yxa$, $Z(G_{27})=\langle a,b\rangle$.

	1	**x**	**y**	**xy**	**a**	**b**	**ab**	**xa**	**xb**	**xab**	**ya**	**yb**	**yab**	**xya**	**xyb**	**xyab**
1	1	x	y	xy	a	b	ab	xa	xb	xab	ya	yb	yab	xya	xyb	xyab
x	x	ab	xy	yab	xa	xb	xab	b	a	1	xya	xyb	xyab	yb	ya	y
y	y	xya	ab	xb	ya	yb	yab	xy	xyab	xyb	b	a	1	xab	x	xa
xy	xy	yb	xab	a	xya	xyb	xyab	yab	y	ya	xb	xa	x	1	ab	b
a	a	xa	ya	xya	1	ab	b	x	xab	xb	y	yab	yb	xy	xyab	xyb
b	b	xb	yb	xyb	ab	1	a	xab	x	xa	yab	y	ya	xyab	xy	xya
ab	ab	xab	yab	xyab	b	a	1	xb	xa	x	yb	ya	y	xyb	xya	xy
xa	xa	b	xya	yb	x	xab	xb	ab	1	a	xy	xyab	xyb	yab	y	ya
xb	xb	a	xyb	ya	xab	x	xa	1	ab	b	xyab	xy	xya	y	yab	yb
xab	xab	1	xyab	y	xb	xa	x	a	b	ab	xyb	xya	xy	ya	yb	yab
ya	ya	xy	b	xab	y	yab	yb	xya	xyb	xyab	ab	1	a	xb	xa	x
yb	yb	xyab	a	x	yab	y	ya	xyb	xya	xy	1	ab	b	xa	xb	xab
yab	yab	xyb	1	xa	yb	ya	y	xyab	xy	xya	a	b	ab	x	xab	xb
xya	xya	yab	xb	1	xy	xyab	xyb	yb	ya	y	xab	x	xa	a	b	ab
xyb	xyb	y	xa	ab	xyab	xy	xya	ya	yb	yab	x	xab	xb	b	a	1
xyab	xyab	ya	x	b	xyb	xya	xy	y	yab	yb	xa	xb	xab	ab	1	a

TABLE 28: $G_{28}=<x, y, a>, x^4=a, y^2=a^2=1, yx=x^3y, Z(G_{28})=<a>$.

	1	**x**	**x^2**	**x^3**	**y**	**xy**	**x^2y**	**X^3y**	**a**	**xa**	**x^2a**	**x^3a**	**ya**	**xya**	**x^2ya**	**x^3ya**
1	1	x	x^2	x^3	y	xy	x^2y	X^3y	a	xa	x^2a	x^3a	ya	xya	x^2ya	x^3ya
x	x	x^2	x^3	a	xy	x^2y	x^3y	ya	xa	x^2a	x^3a	1	xya	x^2ya	x^3ya	y
x^2	x^2	x^3	a	xa	x^2y	x^3y	ya	xya	x^2a	x^3a	1	x	x^2ya	x^3ya	y	xy
x^3	x^3	a	xa	x^2a	x^3y	ya	xya	x^2ya	x^3a	1	x	x^2	x^3ya	y	xy	x^2y
y	y	x^3y	x^2ya	xy	1	x^3	x^2a	x	ya	x^3ya	x^2y	xya	a	x^3a	x^2	xa
xy	xy	ya	x^3ya	x^2y	x	a	x^3a	X^2	xya	y	x^3y	x^2ya	xa	1	x^3	x^2a
x^2y	x^2y	xya	y	x^3y	x^2	xa	1	X^3	x^2ya	xy	ya	x^3ya	x^2a	x	a	x^3a
x^3y	x^3y	x^2ya	xy	ya	x^3	x^2a	x	a	x^3ya	x^2y	xya	y	x^3a	x^2	xa	1
a	a	xa	x^2a	x^3a	ya	xya	x^2ya	x^3ya	1	x	x^2	x^3	y	xy	x^2y	x^3y
xa	xa	x^2a	x^3a	1	xya	x^2ya	x^3ya	y	x	x^2	x^3	a	xy	x^2y	x^3y	ya
x^2a	x^2a	x^3a	1	x	x^2ya	x^3ya	y	xy	x^2	x^3	a	xa	x^2y	x^3y	ya	xya
x^3a	x^3a	1	x	x^2	x^3ya	y	xy	X^2y	x^3	a	xa	x^2a	x^3y	ya	xya	x^2ya
ya	ya	x^3ya	x^2y	xya	a	x^3a	x^2	xa	y	x^3y	x^2ya	xy	1	x^3	x^2a	x
xya	xya	y	x^3y	x^2ya	xa	1	x^3	X^2a	xy	ya	x^3ya	x^2y	x	a	x^3a	x^2
x^2ya	x^2ya	xy	ya	x^3ya	x^2a	x	a	X^3a	x^2y	xya	y	x^3y	x^2	xa	1	x^3
x^3ya	x^3ya	x^2y	xya	y	x^3a	x^2	xa	1	x^3y	x^2ya	xy	ya	x^3	x^2a	x	a

TABLE 29: G_{29}=<x,y,a>, $x^4=y^2=a$, $a^2=1$, $yx=x^3y$, $Z(G_{29})$=<a>.

	1	**x**	**x^2**	**x^3**	**y**	**xy**	**x^2y**	**X^3y**	**a**	**xa**	**x^2a**	**x^3a**	**ya**	**xya**	**x^2ya**	**x^3ya**
1	1	x	x^2	x^3	y	xy	x^2y	X^3y	a	xa	x^2a	x^3a	ya	xya	x^2ya	x^3ya
x	x	x^2	x^3	a	xy	x^2y	x^3y	ya	xa	x^2a	x^3a	1	xya	x^2ya	x^3ya	y
x^2	x^2	x^3	a	xa	x^2y	x^3y	ya	xya	x^2a	x^3a	1	x	x^2ya	x^3ya	y	xy
x^3	x^3	a	xa	x^2a	x^3y	ya	xya	x^2ya	x^3a	1	x	x^2	x^3ya	y	xy	x^2y
y	y	x^3y	x^2ya	xy	a	x^3a	x^2	xa	ya	x^3ya	x^2y	xya	1	x^3	x^2a	x
xy	xy	ya	x^3ya	x^2y	xa	1	x^3	X^2a	xya	y	x^3y	x^2ya	x	a	x^3a	x^2
x^2y	x^2y	xya	y	x^3y	x^2a	x	a	X^3a	x^2ya	xy	ya	x^3ya	x^2	xa	1	x^3
x^3y	x^3y	x^2ya	xy	ya	x^3a	x^2	xa	1	x^3ya	x^2y	xya	y	x^3	x^2a	x	a
a	a	xa	x^2a	x^3a	ya	xya	x^2ya	x^3ya	1	x	x^2	x^3	y	xy	x^2y	x^3y
xa	xa	x^2a	x^3a	1	xya	x^2ya	x^3ya	y	x	x^2	x^3	a	xy	x^2y	x^3y	ya
x^2a	x^2a	x^3a	1	x	x^2ya	x^3ya	y	xy	x^2	x^3	a	xa	x^2y	x^3y	ya	xya
x^3a	x^3a	1	x	x^2	x^3ya	y	xy	X^2y	x^3	a	xa	x^2a	x^3y	ya	xya	x^2ya
ya	ya	x^3ya	x^2y	xya	1	x^3	x^2a	x	y	x^3y	x^2ya	xy	a	x^3a	x^2	xa
xya	xya	y	x^3y	x^2ya	x	a	x^3a	X^2	xy	ya	x^3ya	x^2y	xa	1	x^3	x^2a
x^2ya	x^2ya	xy	ya	x^3ya	x^2	xa	1	X^3	x^2y	xya	y	x^3y	x^2a	x	a	x^3a
x^3ya	x^3ya	x^2y	xya	y	x^3	x^2a	x	a	x^3y	x^2ya	xy	ya	x^3a	x^2	xa	1

TABLE 30: $G_{30}=\langle x,y,a\rangle$, $x^4=a$, $y^2=a^2=1$, $yx=x^3ya$, $Z(G_{30})=\langle a\rangle$.

	1	**x**	**x^2**	**x^3**	**y**	**xy**	**x^2y**	**X^3y**	**a**	**xa**	**x^2a**	**x^3a**	**ya**	**xya**	**x^2ya**	**x^3ya**
1	1	x	x^2	x^3	y	xy	x^2y	X^3y	a	xa	x^2a	x^3a	ya	xya	x^2ya	x^3ya
x	x	x^2	x^3	a	xy	x^2y	x^3y	ya	xa	x^2a	x^3a	1	xya	x^2ya	x^3ya	y
x^2	x^2	x^3	a	xa	x^2y	x^3y	ya	xya	x^2a	x^3a	1	x	x^2ya	x^3ya	y	xy
x^3	x^3	a	xa	x^2a	x^3y	ya	xya	x^2ya	x^3a	1	x	x^2	x^3ya	y	xy	x^2y
y	y	x^3ya	x^2ya	xya	1	x^3a	x^2a	xa	ya	x^3y	x^2y	xy	a	x^3	x^2	x
xy	xy	y	x^3ya	x^2ya	x	1	x^3a	X^2a	xya	ya	x^3y	x^2y	xa	a	x^3	x^2
x^2y	x^2y	xy	y	x^3ya	x^2	x	1	X^3a	x^2ya	xya	ya	x^3y	x^2a	xa	a	x^3
x^3y	x^3y	x^2y	xy	y	x^3	x^2	x	1	x^3ya	x^2ya	xya	ya	x^3a	x^2a	xa	a
a	a	xa	x^2a	x^3a	ya	xya	x^2ya	x^3ya	1	x	x^2	x^3	y	xy	x^2y	x^3y
xa	xa	x^2a	x^3a	1	xya	x^2ya	x^3ya	y	x	x^2	x^3	a	xy	x^2y	x^3y	ya
x^2a	x^2a	x^3a	1	x	x^2ya	x^3ya	y	xy	x^2	x^3	a	xa	x^2y	x^3y	ya	xya
x^3a	x^3a	1	x	x^2	x^3ya	y	xy	X^2y	x^3	a	xa	x^2a	x^3y	ya	xya	x^2ya
ya	ya	x^3y	x^2y	xy	a	x^3	x^2	x	y	x^3ya	x^2ya	xya	1	x^3a	x^2a	xa
xya	xya	ya	x^3y	x^2y	xa	a	x^3	X^2	xy	y	x^3ya	x^2ya	x	1	x^3a	x^2a
x^2ya	x^2ya	xya	ya	x^3y	x^2a	xa	a	X^3	x^2y	xy	y	x^3ya	x^2	x	1	x^3a
x^3ya	x^3ya	x^2ya	xya	ya	x^3a	x^2a	xa	a	x^3y	x^2y	xy	y	x^3	x^2	x	1