ACTA DIDACTICA NAPOCENSIA

Volume 5, Number 3, 2012

# NON-ROUTINE PROBLEMS IN PRIMARY MATHEMATICS WORKBOOKS FROM ROMANIA

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**Abstarct:** The aim of this paper is to present a research on Hungarian 3th grade primary school textbooks from Romania. These textbooks are analyzed using two classifications. The first classification is based on how much creativity and problem solving skills pupils need to solve a given task. In this classification problems are gouped in three categories: routine problems, gray-area problems and puzzle-like (non-routine) problems. The results show that most of the problems from textbooks are routine-problems. Only about 15% of the problems are more difficult, which can be solved in few steps, but even these problems are not challenging. The second classification divide problems based on how the operation chain they have to solve is given: by numbers, by text or in a word problem. The results show that there are big differences in the percentage of problems from these three categories in different textbooks. In one of the studied textbook half of the problems are word problems, in the other one only one quarter.

**Keywords:** mathematical problem solving; non-routine problems, textbook.

## Introduction

International Mathematics tests focus on problem solving, thus these tests includes non-routine problems too. Romanian pupils have high scores, above international average on routine problems, but they obtain lower scores than the average on non-routine problems.

The aim of this paper is to present a research regarding Hungarian 3th grade primary school textbooks from Romania. The problems from these textbooks are analyzed based on two classifications. The first classification is based on how much creativity and problem solving skills pupils need to solve a given task. In this classification problems are gouped in three categories: routine problems, gray-area problems and puzzle-like (non-routine) problems. The second classification divides problems based on how the operation chain they have to solve is given: by numbers, by text or in a word problem.

## Theoretical background

While learning Mathematics, pupils solve exercises and problems in order to deeper the acquired knowledge and to develop their mathematical skills. Kantowski (1977, p. 163) highlight the differences between exercise and problem: “an individual is faced with a problem when he encounters a question he cannot answer or a situation he is unable to resolve using the knowledge immediately available to him. [.…] A problem differs from an exercise in that the problem solver does not have an algorithm that, when applied, will certainly lead to a solution.” In some literature problems are named “non-routine problems” in order to highlight that when solving a problem “requires a novel idea from the student” (Milgram, 2007, p. 47). In TIMMS 2011 framework “non-routine problems are problems that are very likely to be unfamiliar to students. They make cognitive demands over and above those needed for solution of routine problems, even when the knowledge and skills required for their solution have been learned.” (Mullis et al, 2009, p. 45). So if the student knows what method, algorithm, technique or formula to use for solving a task, then that task is not a problem, it is a routine exercise (Schoenfeld, 1985). Thus it is possible that the same task is a problem for one student and it is an exercise for another one (Zhu & Fan, 2006). Also, a problem is no longer considered a problem for that student, who already solved it (Selden et al, 1999).

*Received 5 November 2012.*

In order to be able to solve non-routine problems, students' problem solving competence has to be developed. According to PISA evaluators "problem solving competency is an individual's capacity to engage in cognitive processing to understand and resolve problem situations where a method of solution is not immediately obvious. It includes the willingness to engage with such situations in order to achieve one's potential as a constructive and reflective citizen." (OECD, 2010) They have extended the cognitive domain underlined in definition used for the PISA 2003 evaluation (OECD, 2003) by the affective domain: the willingness of solving problems. Students' interest in mathematics, their beliefs in the utility of the mathematical knowledge in their future career or in their everyday life determine in a fundamental way their problem-solving behaviour. „Belief systems are one's mathematical world view, the perspective with which one approaches mathematics and mathematical task. One's beliefs about mathematics can determine how one chooses to approach a problem, which techniques will be used or avoided, how long and how hard one will work on it, and so on." (Schoenfeld, 1985, p. 45)

Problem solving competency involves the ability to use the acquired knowledge in a new way, the ability to learn new things which are useful for the problem and to discover new methods for the solution. So the transfer of knowledge and skills to new situation is essential. Creative thinking and critical thinking are important components of problem solving competency (Mayer, 1992).

For a successful problem solving students need to use various problem solving strategies and to be flexible. Strategy flexibility is "the behaviour of switching strategies during the solution of problem" (Elia, Van den Heuvel- Panhuizen & Kolovou, 2009, p. 607). Also self-regulation is necessary when solving non-routine problems.

Teachers rarely emphasis non-routine problem solving in their classroom (Silver et al, 2005; Leikin & Levav-Waynberg, 2007). In Romania, most of the problems given on national Mathematics tests require to apply formulas or algorithms. These problems has a mathematical formulations, they don't have any connection with real life (Marchis, 2009). Thus teachers are tempted to solve many routine problems that their pupils obtain good results at these tests. But most of the pupils who pass these tests and even they get good marks don't have a good problem solving competence, they have just learnt some techniques, methods or formulas and they know which one to use for a specific problem. Another reason, that teachers don't solve non-routine problems in the classroom is that they are not confident in their problem solving competence and they are not comfortable with handling pedagogical demands required for this type of problem solving activity (Silver et al, 2005).

A study on how primary school teachers in Romania develop their pupils' word problem solving skills shows that three quarters of the teachers guide pupils in order to understand the problem and encourage them for self-control during problem solving; only one third of the respondents encourage their students to solve the problems with more methods. Almost three quarters of the primary school teachers state that they give interesting, real-life problems in class. (Marchis, 2012)

Pupils have to learn the steps of the problem solving. Pólya (1945) has identified four main stages when solving a problem: understanding the problem, making a plan, carrying out the plan, and reviewing the solution. Similar steps are described by other researchers (among others Higgins, 1997; Leader & Middleton, 2004; Ridlon, 2004). The understanding stage includes some text comprehension techniques, for example, to identify the unknown words, to reformulate the problem, to think about a picture or diagram that might help to understand the problem context, and the relations between the given and unknown data (Pólya, 1957). Based on PISA 2012 problem solving has the following four steps: exploring and understanding, representing and formulating, planning and executing, monitoring and reflecting.

Textbooks and other materials are important factors in influencing mathematics teaching (Braslavsky & Halil, 2006; Cueto, Ramírez, & León, 2006; Nicol & Crespo, 2006). Worked examples help pupils to acquire problem solving methods. Research shows that studying worked examples it is an effective and efficient way of learning mathematics (Paas & van Gog, 2006).

## Research design

We have selected two textbooks for 3rd grade (we refer to them by Textbook 1 and Textbook 2) and studied the problems given in three chapters:

- Adding and substracting natural numbers;

- Multiplying natural numbers between 0 and 10 (multiplication table);

- Dividing natural numbers (divisions which can be calculated using the multiplication table).

We classified the problems using two different classifications.

In the first one we used the classification of the tasks given by Kolovou, van den Heuvel-Panhuizen, and Bakker (2009). They have divided textbook problems in three categories:

- *routine exercises*, which require application of a known algorithm.

- non-routine, *puzzle-like* tasks, which are problems that require creative thinking and a higher level of problem solving thinking.

- *gray-area tasks*, which can't be included in any of the above two category. These tasks can't be solved by only applying a known algorithm.

In the second classification we grouped the problems in three categories:

- *solving an operation chain*, where the operations are given by numbers and operation signs, i.e. Calculate 2×6+9.

- *solving an operation chain discribed by text*, where the operations are given by text, i.e. Find the sum and the difference of 56 and 34.

- *word problem*, where pupils have to discover which operation to use, i.e. Ana has 45 glass ball, Peter has 4 less than Ana. How many glass balls they have together?

We also studied if there are tasks in which pupils has to create a word problem. In these task pupils have to formulate a problem:

- *based on an operation chain*;

- *based on a graphical representation*;

- *based on a drawing*.

## Results and discussion

### Dividing tasks in routine, gray-area and puzzle-like categories

Analyzing the problems in the three selected chapters from Textbook 1 and Textbook 2 we can conclude that there is no any puzzle-like problem. There are problems, which can be solved with backward method, which could have been challenging for pupils, but the idea of the solving method is given. It is the same situation with some problems which can be solved by the graphical method. There are some problems, which are more difficult, require more steps in the solution plan, but these problems can't be considered puzzle-like problems. Kolovou, van den Heuvel-Panhuizen, and Bakker (2009) have arrived to similar conclusion in case of two textbooks studied by them. In the other textbooks the maximum percentage of puzzle-like tasks is 2,43%.

It is very difficult to distinguish between routine-problems and gray-area problems, as a problem could be routine problem for somebody already solved something similar and gray-area problem for somebody first time sees that type of problem. Thus problems, which can be solved in more steps, could be considered gray-area problems for some pupils and routine-problems for other pupils. In this study we considered multiple step problems as gray-area problems. Problem 1 is an example of gray-area problem from Textbook 2 and Problem 2 is such an example from Textbook 1. We have counted the number of routine-problems and gray-area problems for each operation in Textbook 1 and Textbook 2 (see Table 1). We could observe that the number of gray-area problem is quite similar for the two textbooks, 16.8% for Textbook 1 and 14.8% for Textbook 2. In Textbook 2 the percentage of gray-area problems for addition and substraction is much higher than in Textbook 1 (30.0% in Textbook 2 and 17.5% in Textbook 1). In case of division in Textbook 1 there are more gray-area problems than in Textbook 2 (26.5% in Textbook 1 and 16.7% in Textbook 2).

| | |
|---|---|
| A housewife bought 798 kg of vegetables: potato, onion and cabbage. She bought 251 kg of cabbage and 118 kg less onion, than cabbage. How many kg of onion and potato did she buy? | If we increase the triple of a number with the fivefold of it, we get 64. Find the number! |
| Problem 1. | Problem 2. |

**Table 1.** Number of routine and gray-area problems

| Operation | Textbook 1 | | | Textbook 2 | | |
|---|---|---|---|---|---|---|
| | **Routine problem** | **Gray-area problem** | **Total** | **Routine problem** | **Gray-area problem** | **Total** |
| **+, -** | 33 (82.5%) | 7 (17.5%) | 40 | 14 (70%) | 6 (30.0%) | 20 |
| **×** | 85 (88.5%) | 11 (11.5%) | 96 | 73 (90.1%) | 8 (9.9%) | 81 |
| **:** | 36 (23.5%) | 13 (26.5%) | 49 | 45 (83.3%) | 9 (16.7%) | 54 |
| **Total** | 154 (83.2%) | 31 (16.8%) | 185 | 132 (85.2%) | 23 (14.8%) | 155 |

These results are similar with that obtained by Kolovou, van den Heuvel-Panhuizen, and Bakker (2009). They the textbooks analyzed by them the percentage of gray-area and puzzle-like tasks together were between 5% and 13%. They also mentioned the difficulty on deciding if a task is gray-area or routine.

### Dividing tasks by how the operations needed to carry out are given

The most easier is when the operations to be performed are given by an operation chain with numbers and operation signs (i.e. 3×4+24:6).

A bit more difficult is when the operations to be carried out are hidden in a text (i.e. calculate the sum of 345 and 234). In this case pupils have to know specific mathematical terms, as sum, difference, product, etc.

The most difficult is when a text problem formulation is given because solving these problems requires text comprehension, problem representation, selection of the adequate operations, solving these operations (Kintsch & Greeno, 1985; Swanson, 2004). When solving these problems the most important difficulties are related with forming an operation based on the text of the problem (Carey, 1991; English, 1998). Usually pupils try to use the operations which they last learnt (i.e. if they learn multiplication they tend to use multiplication when solving text problems). Thus it is important that in a chapter related with some operation to include also text problems which have to be solved using other operations. In Textbook 1 we have found few problems hidden in other chapters than the operation needed in those problems. A sequence of problems included in the division chapter highlights the differences between „3 more", „3 times more" and „3 times less" (see Problem 3, 4 and 5). In Textbook 2 there are also problems which need other operations than discussed in the current chapter. Problem 6 is in multiplication chapter, but it highlighs the difference between increasing by 2 (so adding 2 to the given number) and inceasing by 2 times (so multiplying by 2 the give number). Problem 7 is included in division chapter, but it contains also addition not only to calculate the total, but also to calculate the numbers of green balls, as the problem states that „András has 3 more green balls, than red balls".

| | | |
|---|---|---|
| On the meadow there are 8 sheeps and 3 more horses than sheeps. How many animals are on the meadow? | In the courtyard there are 6 girls and 3 times more boys than girsl. How many childran are in the courtyard? | In a fruitbowl there are 9 apples and 3 times less pears. How many fruits are in the fruitbowl? |
| Problem 3 | Problem 4 | Problem 5 |

| | |
|---|---|
| Increase the given numbers by 2 then increase by 2 times: 2, 6, 9, 7, 10, 5, 8, 1, 3, 4. | András has 15 red, 3 times more blue balls, and 3 more green balls, than red balls. How many balls does András have? |
| Problem 6 | Problem 7 |

To see the difference between the problems where the operations are given in text and word problems, see Problem 8 and 9 from Textbook 1). In Problem 8 the terms *difference* and *sum* indicates the operations needed. In Problem 9 the operation is also indicated by the word *total*, but it is not so obvious than in case of the term *sum*.

| | |
|---|---|
| Calculate the difference between the sum of 378 and 266 respectively the sum of 445 and 87. | In a greenhouse there are 165 red and 128 yellow tulips. How many tulips are in total? |
| Problem 8 | Problem 9 |

It is important that in different word problems the operation which pupils should use to be expressed differently. For example, addition could be suggested by the words *total*, *together*, *more*, etc. Multiplication also could be expressed in different ways. Problems 10 and 11 from Textbook 1 show two different problems in which multiplication should be used.

| | |
|---|---|
| Pali caught 5 fish Peti 4 times more. How many fish did they cacth together? | On the window still there are 3 flowerpots. How many flowerpots are on 5 window stills? |
| Problem 10 | Problem 11 |

We have counted the numbers of each type of problems in above described three categories in each chapter. The results are included in Table 2.

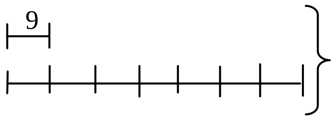
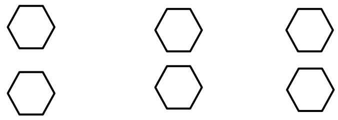
In the studied three chapters we have analyzed 185 problems from Texbook 1 and 155 problems from Textbook 2. We can observe, that in Textbook 1 the percetage of word problems is double than in Textbook 2 (44.9% in Textbook 1 and 21.9% in Textbook 2). In Textbook 2 almost half of the problems (46.5%) are operation chains which have to be calculated, while in Textbook 1 only one third (30.3%) of the problems are of this type.

**Table 2.** Number of problems in which the operation is given by operation chain, operation described by text, and word problems

| Operation | Textbook 1 | | | | Textbook 2 | | | |
|---|---|---|---|---|---|---|---|---|
| | Operation chain | Operation described by text | Word problem | Total | Operation chain | Operation described by text | Word problem | Total |
| +, - | 3 (7.5%) | 18 (45%) | 19 (47.5%) | 40 | 10 (50%) | 3 (15%) | 7 (35%) | 20 |
| × | 36 (37.5%) | 21 (21.9%) | 39 (40.6%) | 96 | 37 (45.7%) | 26 (32.1%) | 18 (22.2) | 81 |
| : | 17 (34.7%) | 7 (14.3%) | 25 (51.0%) | 49 | 25 (46.3%) | 20 (37.0%) | 9 (16.7%) | 54 |
| **Total** | 56 (30.3%) | 46 (24.9%) | 83 (44.9%) | 185 | 72 (46.5%) | 49 (31.6) | 34 (21.9) | 155 |

**Composing word problems**

Composing their own word problems also helps students in changing their attitudes regarding these problems and becoming familiar with the mathematical terminology (Edwards et al., 2002). In Textbook 1 there are problems in which pupils have to formulate the question(s) of the problem or to formulate the problem based on given arithmetic operations (see Problem 12) or graphical representation (see Problem 13). In Textbook 2 there are problems where pupils have to formulate a word problem based on a picture (see Problem 14, in the original problem three pairs of snowmen where represented, we have simplified the drawing using hexagons instead of snowmen).

| | | |
|---|---|---|
| Formulate a problem which can be solved by the following operations:<br>(7 × 5) – 18 = | Formulate a problem based on the following graphical representation:<br> | Formulate a problem based on the picture:<br><br>$\square\square : \square = \square$ , because<br>$\square \times \square = \square\square$ |
| Problem 12 | Problem 13 | Problem 14 |

The number of those task were pupils have to formulate a word problem is very low, 2 in Textbook 1 and 4 in Textbook 2.

## Conclusion

As regarding the first classification, dividing problems in routine, gray-area and puzzle-like problems, the results show that most of the problems from textbooks are routine-problems. Only about 15% of the problems are more difficult, which can be solved in few steps, but even these problems are not challenging, pupils don't need to develop new solving methods. None of the problems can't be considered puzzle-like problem.

As regarding the second classification, dividing problems based on how the operation chain they have to solve is given (by numbers, by text or in a word problem), the result show that there are big differences in the percentage of problems from these three categories in different textbooks. In one of the studied textbook half of the problems are word problems, in the other one only one quarter. Thus teacher can choose the most appropriate textbook for their classroom.

The number of tasks where pupils have to formulate word problems is very limited.

## References

[1] Braslavsky, C., & Halil, K. (Eds.) (2006). Textbooks and Quality Learning for all: Some Lessons Learned form International Experiences. Genève: International Bureau of Education, UNESCO.

[2] Carey, D. (1991). Number Sentences: Linking Addition and Subtraction Word Problems and Symbols. Journal for Research in Mathematics Education, 22(4), 266-280

[3] Cueto, S., Ramírez, C. & León, J. (2006). Opportunities to learn and achievement in mathematics in a sample of sixth grade students in Lima, Peru. Educational Studies in Mathematics, 62(1), 25-55.

[4] Edwards, S. A., Maloy, R. W., & Verock-O'Loughlin, R. (2002). *Ways of writing with young kids: Teaching creativity and conventions unconventionally*. Boston: Allyn & Bacon.

[5] Elia, I.; Van den Heuvel-Panhuizen, M. & Kolovou, A. (2009). Exploring strategy use and strategy flexibility in non-routine problem solving by primary school high achievers in mathematics. ZDM The International Journal on Mathematics Education, 41, 605–618.

[6] English, L. (1998). Children’s Problem Posing within Formal and Informal Context. Journal of Research in Mathematics Education, 29(1), 83-106.

[7] Higgins, K. M. (1997). The effect of year-long instruction in mathematical problem solving on middle-school students' attitudes, beliefs, and abilities. Journal of Experimental Education, 66(1), 5-29.

[8] Kantowski, M. G. (1977). Processes Involved in Mathematical Problem Solving. Journal for Research in Mathematics Education, 8(3), 163-180.

[9] Kintsch, W., & Greeno, J. G. (1985). Understanding and solving word arithmetic problems. Psychological Review, *92*, 109-129.

[10] Kolovou, A.; van den Heuvel-Panhuizen, M. & Bakker, A. (2009). Non-Routine Problem Solving Tasks in Primary School Mathematics Textbooks – A Needle in a Haystack, Mediterranean Journal for Research in Mathematics Education, 8(2), 31-68.

[11] Leader, L. F. & Middleton, J. A. (2004). Promoting critical-thinking dispositions by using problem solving in middle school mathematics. Research in Middle Level Education, 28(1), 55-71.

[12] Leikin, R. & Levav-Waynberg, A. (2007). Exploring mathematics teacher knowledge to explain the gap between theory-based recommendations and school practice in the use of connecting tasks. Educational Studies in Mathematics, 66, 349-371.

[13] Mayer, R. E. (1992). Thinking Problem Solving, Cognition. New York: Freemann.

[14] Marchis, I. (2009). Comparative analysis of the mathematics problems given at international tests and at the Romanian national tests, Acta Didactica Napocensia, 2(2) 141-148

[15] Marchis**,** I. (2012)**.** How primary school teachers develop their pupils' mathematical word problem solving skills, Studia Universitatis Babes-Bolyai Psychologia-Paedagogia, LVII/1, 97-107

[16] Milgram, R. J. (2007). What is mathematical proficiency? In A. H. Schoenfeld (Ed.), Assessing mathematical proficiency (pp. 31-58). New York: Cambridge University Press.

[17] Mullis, I. V. S., Martin, M. O.; Ruddock, G. J.; O'Sullivan, C. Y. & Preuschoff, C. (2009). TIMSS 2011 Assesment Frameworks. http://timssandpirls.bc.edu/timss2011/downloads/TIMSS2011_Frameworks-Chapter1.pdf

[18] Nicol, C.C., & Crespo, S.M. (2006). Learning to teach with mathematics textbooks: How preservice teachers interpret and use curriculum materials. Educational Studies in Mathematics, 62(3), 331-355.

[19] OECD (2003). The PISA 2003. Assessment framework- mathematics, reading, science and problem solving knowlegde and skills. http://www.oecd.org/dataoecd/46/14/33694881.pdf [October 2012]

[20] OECD (2010). PISA 2012 field trial: problem solving framework, http://www.oecd.org/pisa/pisaproducts/46962005.pdf [November 2012]

[21] Paas, F., & Van Gog, T. (2006). Optimising worked example instruction: Different ways to increase germane cognitive load. Learning and Instruction, 16, 87-91

[22] Pólya, G. (1945). How to Solve It. Princeton University Press

[23] Pólya, G. (1957). How to Solve It. Doubleday

[24] Ridlon, C. L. (2004). The effect of a problem centered approach on low achieving six graders. Focus on learning problems in mathematics, 28(4), 6-29.

[25] Schoenfeld, A. H. (1985). Mathematical problem solving. San Diego: Academic Press, Inc.

[26] Selden, A., Selden, J., Hauk, S., & Mason, A. (1999). Do calculus students eventually learn to solve non-routine problems? Tennessee Technological University Technical Report No. 1999-5. http://math.tntech.edu/techreports/TR_1999_5.pdf [November 2012]

[27] Silver, E. A., Ghousseini, H., Gosen, D., Charalambous, C. & Strawhun, B.T.F. (2005) Moving from rhetoric to praxis: Issues faced by teachers in having students consider multiple solutions for problems in the mathematics classroom. Journal of Mathematical Behavior, 24, 287-301.

[28] Swanson, H. L. (2004). Working memory and phonological processing as predictors of children's mathematical problem solving at different ages. Memory & Cognition, 32(4), 648-661.

[29] Zhu, Y. & Fan, L. (2006). Focus on the Representation of Problem Types in Intended Curriculum: A Comparison of Selected Mathematics Textbooks from Mainland China and the United States. International Journal of Science and Mathematics Education, 4(4), 609-626.

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## Acknowledgment

This research was founded by Domus Hungarica research grant, contract number DSZ/34/2012.