



AN INVESTIGATION OF STUDENTS' DIFFICULTIES TO CREATE A BLACKBOARD SKETCH FOR THE LESSON: TABLE OF MULTIPLICATION BY TWO

Ioana Magdaş

Abstract: Specialists in the field of mathematical education consider that the learning of multiplication has an extremely important role not only in the area of mathematics, but also in day to day life. We can distinguish two different stages in the process of teaching-learning multiplication: a first step implies the discovery of the results within the multiplication table and the connections that exist between these results, while a second phase deals with memorizing the results. Our experience has shown that teachers tend to make students learn the table of multiplication by heart and this, on the one hand, overshadows a more logical approach that would aim at the students understanding the way in with the results are obtained and, on the other hand, neglects the subsequent logical way of memorizing the results. The way in with the information is presented is key to both the understanding and the remembering of numbers in the table. Starting from the observation presented previously we have created a pedagogical micro-experiment in which wanted to investigate whether students in their training to become teachers had the ability to work with the content offered in school textbooks and to create a blackboard sketch for *The table of multiplication by 2*.

Key words: mathematics, blackboard sketch, multiplication by two, multiplication table

1. Introduction

Specialists in the field of mathematical education consider that the learning of multiplication has an extremely important role not only in the area of mathematics, but also in day to day life.

Ofsted is the United Kingdom Office for Standards in Education, Children's Services and Skills. "How young learners master maths", an Ofsted report on best practice in early arithmetic, was published in November 2011. "Primary schools which fail to teach times tables by heart are condemning children to a lifetime struggling with numbers", inspectors have warned. Nick Gibb, the schools minister, said: "It is vital that all children can grasp and master arithmetic while they are still at primary school. It is important that pupils are fluent in calculation and have learnt the multiplication tables by heart before they leave primary school".

"We all know that learning multiplication is an essential part of our child's elementary education. Students who have mastered multiplication gain a solid foundation in mathematics that will help them throughout middle school, high school and beyond." (Susan Jarema, 2007).

Learning multiplication tables can be difficult for some students. These difficulties are owed not only to the students themselves, but also to the teachers who, most often, do not find the appropriate ways to make students understand and memorize multiplication charts.

We can distinguish two different stages in the process of teaching-learning multiplication: a first step implies the discovery of the results within the multiplication table and the connections that exist between these results, while a second phase deals with memorizing the results. What is more important understanding or memorization? „It's not one or the other, it's both" (Susan Jarema, 2007). Our experience has shown that teachers tend to make students learn the table of multiplication by heart and this, on the one hand, overshadows a more logical approach that would aim at the students understanding the way in with the results are obtained and, on the other hand, neglects the subsequent

logical way of memorizing the results. The way in which the information is presented is key to both the understanding and the remembering of numbers in the table.

This is precisely why we sought to create a micro-experiment that wanted to investigate whether students in their training to become teachers had the ability to work with the content offered in school textbooks and to create a blackboard sketch for the table of multiplication by 2.

Our goal was to identify the difficulties that students encountered when having to teach multiplication table by 2 and this was directed on several levels:

- Are students aware of the fact that school textbooks do not contain all of the relevant information necessary in the learning of multiplication?
- Are students aware of the fact that the blackboard sketch is essential to the learning of the multiplication table?
- Are students capable to utilize the information they find in school textbooks and to develop a suggestive blackboard sketch for multiplication by 2?

2. Methodological considerations regarding the teaching-learning of multiplication

Considering what was previously discussed, we can postulate that the learning of multiplication is one of the mathematical principles that all pupils have to acquire, although not at the same speed.

Teachers of mathematics and didactics specialists propose a series of modern and amusing methods for memorizing the multiplication table, methods that are complementary: games, songs, manipulatives, stories, short movies, multiplication tricks etc. (Walker, 2000, 2009; Liautaud, J., Rodrigues, D., 1999; Pallotta, J, 2009 etc.). There is even a website dedicated to multiplication: <http://www.multiplication.com>.

How should the teacher act in the phase of understanding? One of the specific traits of mathematics is that its learning is done primarily with pen and paper. Taking into consideration the principle that a pupil has to be involved in his or her own learning process, the method of bringing to class a material that was already created and merely giving it to the children is obviously not the best way to learn the table of multiplication. This is why we are of the opinion that the classic method remains the best for the phase of understanding how the table of multiplication works. By "classic" we understand a frontal activity that centers around the teacher writing on the blackboard (flip-chart, smart board, a tablet connected to a projector and other options) and the student writing in his notebook (or on worksheets that have spaces to be filled in). It is in this context that the lesson plan blackboard sketch plays an important role in retaining information. Nevertheless, this technique can be complemented with other didactic devices and methods which are offered to us by new technologies, especially the Internet. We can truly ask ourselves: is the blackboard really a necessity in the age where we have all these other tools that we can use such as standard and auxiliary textbooks, not to mention power-point presentations and worksheets? A first answer to the question could be: "Just because the board represents an old technology doesn't make it outdated one" (Linda B. Nilson, 2010). In any case, there are many modern possibilities today, new versions of the blackboard that we have listed above, that, while preserving the overall advantages of the blackboard, have the additional quality of being interactive.

Consequently, when the lesson is focused on teaching new information and knowledge, the lesson plan has to be completed by a blackboard sketch or draft – especially for pupils in their first years of primary school. We are referring to a plan of the blackboard that is done by the teacher, with or without the help of the pupils, and that is reproduced by each pupil in his or her notebook. Its purpose is to focus children on the relevant information and to guide them towards what they have to learn.

For the understanding stage of multiplication by 2, the subsequent methodological steps will be followed (Magdaş, 2010):

- Multiplication is presented as a repeated addition;
- Students will discover the results of multiplication by 2 and will create a triangular table (see annex 1);

- Students fill in the column and the line corresponding to multiplication by 2 in the multiplication table, taking into consideration also the commutative property of multiplication (see annex 2);

Students notice that results grow by 2 with every new multiplication and understand that every result is obtained by adding 2 to the previous multiplication.

3. The didactic micro-experiment

All of this considered, we developed this experiment to analyze students' ability to create a blackboard plan for multiplication by 2.

The group of students consisted of 39 students in their third year of study, from the Pedagogy of Primary and Kindergarten Education specialty in the Faculty of Psychology and Educational Sciences at the Babeş-Bolyai University in Cluj-Napoca. Out of these 38 were female and one was male, one was a primary school teacher with more than 15 years of work experience, two were kindergarten teachers with 8 years of work experience and another two are students from Spain that had come to study abroad in an Erasmus program; the rest of the students had no previous working experience, but were involved in an internship at a kindergarten.

The experiment took place in November of 2012.

- a) The first part of the course was aimed at gathering information from the students about how they perceived school textbooks and the importance of a lesson plan drafted on the blackboard.

The following questions were asked:

On a scale of 0-100%:

- i. What is the level of trust that you give, from a scientific point of view, to school textbooks?
- ii. How much do you trust the accuracy of the didactic adaptation of the scientific content in school text books?
- iii. To what extent does the presentation of information in a textbook contribute to the learning process?
- iv. To what extent does the lesson sketch contribute to the learning process?

It was noted that approximately 77% of the students trust school textbooks as far as scientific content and didactic potential at above 60%. Interestingly enough, 13 students, which is 33% of the group could not answer question iii. – this means that a significant percentage did not have an opinion on the matter. Also, more than 50% of the students acknowledge the fact that the lesson sketch contributes with more than 50% to the process of learning.

Table 1: A numeric representation of the answers given by the students

Question	Number of answers						
	< 50%	50-59%	60-69%	70-79%	80-89%	90-100%	I don't know / I won't answer
i.	1	1	2	9	6	13	7
ii.	1	0	3	8	13	8	6
iii.	4	9	2	4	2	5	13
iv.	4	7	4	2	5	9	6

- b) A second stage consisted of a frontal presentation in which the students were shown the teaching of multiplication as repeated addition, the notation and the terminology. It was explained that the commutative property is to be introduced as a notion from the beginning and that it will be useful in learning the multiplication table.

Then the students were divided into 19 groups: 18 pairs based on how they were already seated in the lecture hall and a group of 3 with consisted of the two foreign students along with a person that spoke Spanish.

They were asked to utilize the information in the textbook and to create a blackboard draft on a A4 sheet of paper designed to teach multiplication by 2.

During the first 5-10 minutes we noticed that the students asked for support because their tendency was to develop a lesson plan, not a blackboard draft. However, they subsequently focused on the task at hand, the professor monitoring activities and providing help where needed.

Although we initially noted that the students had difficulties, we later observed that they developed the blackboard draft or sketch reasonably well. In order to make a clearer classification of the results, we established the following grading system:

Grade 4: the students did not understand the task/did not develop the draft

Grade 5: only the results of multiplication by 2 appear, without the justification of the results

Grade 6: the results of the multiplication appear and some justification (though drawings and/or calculations), but the justifications are incomplete and the blackboard is too crowded to be understood by pupils

Grade 7: the results of the multiplication appear, along with full justification (though drawings and/or calculations), but the blackboard is too crowded to be understood by pupils

Grade 8: the results of the multiplication appear with justification based on calculation, but the sketch requires improvement

Grade 9: the results of the multiplication appear with justification based on calculation, but the growth by 2 with each step is not explained or does not appear on the sketch

Grade 10: the results of the multiplication appear with justification based on calculation and the growth by 2 with each step is explained on the sketch.

The results can be observed in Table 2 and in the Figures 3 and 4.

Table 2. The grades obtained by students on the task

Grade	4	5	6	7	8	9	10
Number of sketches	1	5	3	1	4	3	2

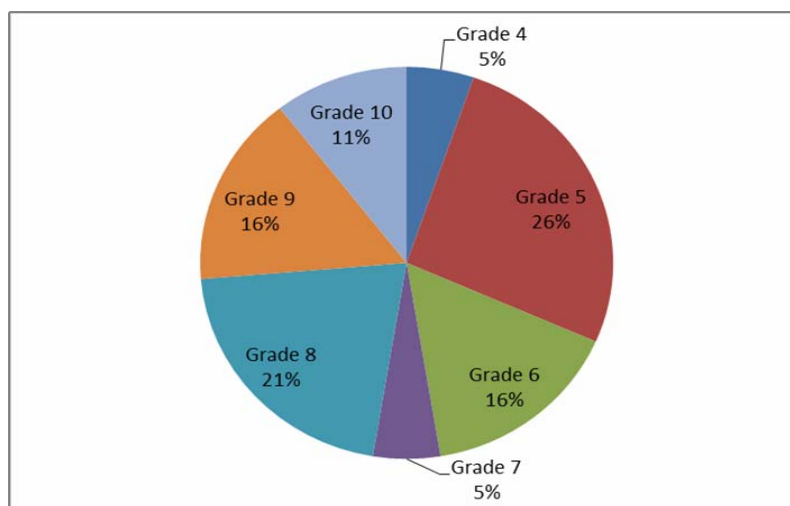


Figure 3. The grades percent distribution of students' blackboard sketches

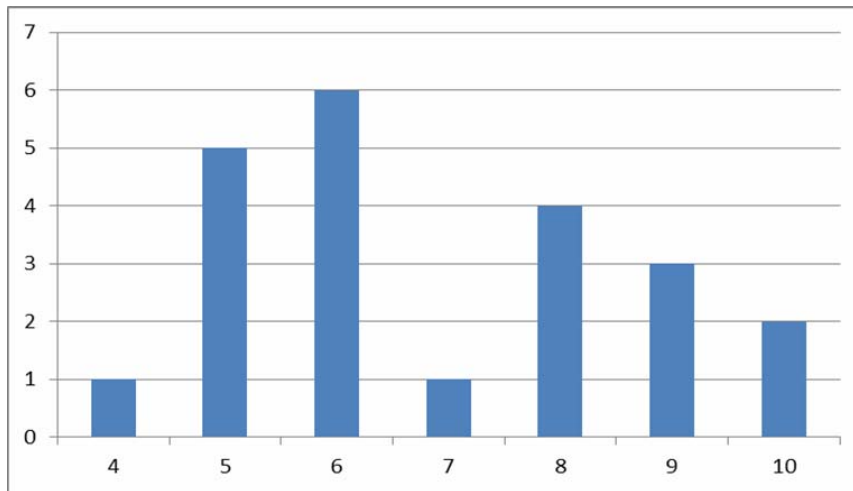


Figure 4. Chart of the grades obtained by students on the blackboard sketch

Apart from the analysis of the grades, we were also able to note the following: none of the groups changed the title of the lesson from the one that was given in the textbook: *Multiplication when one of the factors is 2* with a more suggestive title such as *Multiplication table by 2*. This title would have been more appropriate if we think about how, at the end, the results of the multiplication are put in a square table with two entrances and where every little square is filled in as the results are discovered. More importantly, none of the groups thought about creating a square table of representation. However, although this aspect was not covered, we still gave two maximum grades of 10 because we took into consideration the lack of experience of the groups. A sketch in this style is presented in annex number 3. Out of the 19 sketches, 9 of them – approximately 47% – which obtained grades above 8 could, be used in classrooms. We noticed that three pairs transformed the information in the textbook and created a triangular multiplication table, this making the lesson more accessible and easier to understand. One of the groups obtained the grade 9 and the other two 8. Even though these sketches were not perfect, they highlighted the students' ability to utilize information offered and to filter it through their own thought process. Annex 4 presents one of the sketches that obtained grade 9. The experiment showed that a large number of students need help when it comes to not only developing a lesson plan, but also when they have to produce the blackboard sketch. Annex 5 shows a sketch that was graded 6, representative for this category of students.

At the end of the activity one of the pairs presented their sketch (this can be seen in annex 3). Students then read the corresponding material from their course readers and the professor added explanations and alternative approaches based on the steps outlined previously in paragraph 2 (see annexes 1 and 2). The professor encouraged the students to participate in a debate starting from what was discussed and they showed enthusiasm in expressing their opinions.

Conclusion

The experiment showed that:

- Approximately 55% of students are aware of the fact that the content presented in textbooks is not clear enough for an efficient learning of multiplication, but approximately 30% do not have an opinion on the matter.
- Students acknowledge the importance of the blackboard sketch in overall learning and particularly in the learning of the multiplication table.
- More than 50% of students need guidance not only for creating the lesson plan, but also for drafting the blackboard sketch.
- Only a small percentage of students are able to transform the content and information presented in textbooks in order to make it more accessible for pupils.

- Students are not able to change the title of the lesson or to add certain details (such as multiplication table in this case)

This micro-experiment has shown that, for a lesson designed to present new information in primary school, future teachers need to be taught to create, as a complement to the lesson plan, an adequate blackboard sketch. This is an essential exercise for students that want to develop their ability to teach mathematics, more specifically; it is an indispensable trait for future teachers in primary school education.

References

- [1] Jarema, Susan, 2007, *The Importance of Memorizing the Times Tables*, published in <http://thephantomwriters.com>
- [2] Liautaud, J., Rodrigues, D., 1999, *Times tables the fun way*, City Creek PressInc
- [3] Magdaş, I., 2010, *Didactica matematicii pentru învățământul primar și preșcolar (Didactics of Mathematics for Primary Schools and Kindergarten)*, Presa Universitară Clujeană
- [4] Nilson, Linda B., 2010, *Teaching and Its Best*, John Wiley & Sons
- [5] Pallotta, J., 2009, *Hershey's Milk Chocolate Multiplication Book*, Scholastic
- [6] Russell, Deb, 2007, *Tricks to Learn the Multiplication Facts*, <http://math.about.com/od/multiplication/a/Multiplication-Tricks.htm>
- [7] Walker, A., 2009, *Multiplication in a flash*, Krimsten Publishing, USA
- [8] Walker, A., 2000, *Memorize in minutes: the times tables*, Krimsten Publishing, USA
- [9] OFSTED report (<http://www.ofsted.gov.uk>), 2011, *How young learners master maths*

Authors

Magdaş Ioana, Babes-Bolyai University, Cluj-Napoca, Romania, e-mail: magdas_ioana@yahoo.com

Annex 1

MULTIPLICATION BY TWO

$$\begin{aligned}
 2 \times 1 &= \boxed{2} \xrightarrow{+2} \\
 2 \times 2 &= 2 + 2 = \boxed{4} \xrightarrow{+2} \\
 2 \times 3 &= 2 + 2 + 2 = \boxed{6} \xrightarrow{+2} \\
 2 \times 4 &= 2 + 2 + 2 + 2 = \boxed{8} \xrightarrow{+2} \\
 2 \times 5 &= 2 + 2 + 2 + 2 + 2 = \boxed{10} \xrightarrow{+2} \\
 2 \times 6 &= 2 + 2 + 2 + 2 + 2 + 2 = \boxed{12} \xrightarrow{+2} \\
 2 \times 7 &= 2 + 2 + 2 + 2 + 2 + 2 + 2 = \boxed{14} \xrightarrow{+2} \\
 2 \times 8 &= 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = \boxed{16} \xrightarrow{+2} \\
 2 \times 9 &= 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = \boxed{18} \xrightarrow{+2} \\
 2 \times 10 &= 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = \boxed{20}
 \end{aligned}$$

Annex 2

MULTIPLICATION TABLE

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6								
4	4	8								
5	5	10								
6	6	12								
7	7	14								
8	8	16								
9	9	18								
10	10	20								

Annex 3

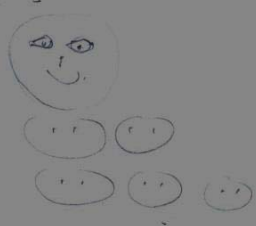
Sketch of the lesson graded 10: the results of the multiplication appear with justification based on calculation and the growth by 2 with each step is explained on the sketch.

5. XI, 2012

Înmulțirea când unul din factori este 2

1. ... 25 persoane ... câte 2 bețe
? bețe în total
 $25 \times 2 = 2 \times 25 = 25 + 25 = 50$ (bețe)

2. $1 \times 2 = 2 \times 1 = 2$ } +2
 $2 \times 2 = 2 + 2 = 4$ } +2
 $3 \times 2 = 2 \times 3 = 3 + 3 = 6$ } +2
 $4 \times 2 = 2 \times 4 = 4 + 4 = 8$ } +2
 $5 \times 2 = 2 \times 5 = 5 + 5 = 10$ } +2
 $6 \times 2 = 2 \times 6 = 6 + 6 = 12$ } +2
 $7 \times 2 = 2 \times 7 = 7 + 7 = 14$ } +2
 $8 \times 2 = 2 \times 8 = 8 + 8 = 16$ } +2
 $9 \times 2 = 2 \times 9 = 9 + 9 = 18$ } +2
 $10 \times 2 = 2 \times 10 = 10 + 10 = 20$ } +2



⚠️ Gradul de înmulțire cu 2 crește din 2 în 2.

Aplicații:

a) ... I 5 mere ... A cu 2 mai multe ? mere are A.
 $5 + 2 = 7$ (mere)

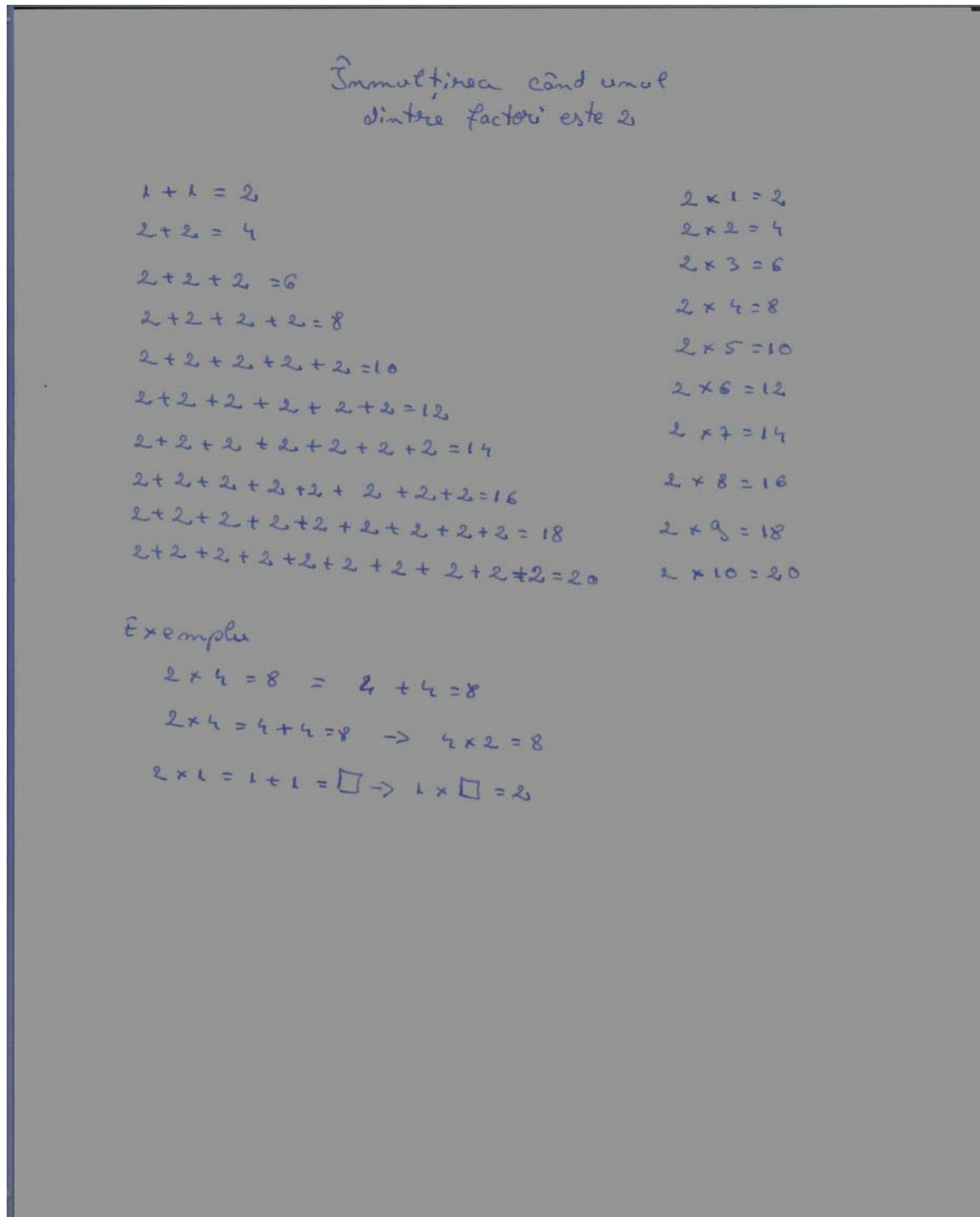
b) ... I 5 mere ... A de 2 ori mai multe ? mere are A.
 $2 \times 5 = 10$ (mere)

⚠️!! Dublul unui nr. se obține prin înmulțirea acestuia cu 2.
 $2 \times 18 = 18 + 18 = 36$ (dublul lui 18)

Teme: ...

Annex 4

Sketch of the lesson graded 9: the results of the multiplication appear with justification based on calculation, but the growth by 2 with each step does not appear on the sketch



Annex 5

Sketch of the lesson graded 7: the results of the multiplication appear and justification through drawings and calculations, but some of them are unclear and the blackboard is too crowded

5. XI. 2012.

Inmultirea când unul dintre factori este 2

Exerciții:

1) $|+| \Rightarrow 2$ $\triangle \triangle \Rightarrow 4$ $\circ \circ \Rightarrow 6$ $* * * \Rightarrow 8$ $\square \square \square \square \Rightarrow 10$

$1+1=2$ $2+2=4$ $3+3=6$ $4+4=8$ $5+5=10$

$2 \times 1 = 2$ $2 \times 2 = 4$ $2 \times 3 = 6$ $2 \times 4 = 8$ $2 \times 5 = 10$

$\textcircled{6} \textcircled{6} \textcircled{6} \textcircled{6} \textcircled{6} \textcircled{6} \Rightarrow 12$ $\heartsuit \heartsuit \heartsuit \heartsuit \heartsuit \heartsuit \heartsuit \Rightarrow 14$ $\downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \Rightarrow 16$

$6+6=12$ $7+7=14$ $8+8=16$

$2 \times 6 = 12$ $2 \times 7 = 14$ $2 \times 8 = 16$

$\textcircled{0} \textcircled{0} \textcircled{0} \textcircled{0} \textcircled{0} \textcircled{0} \textcircled{0} \textcircled{0} \textcircled{0} \textcircled{0} \textcircled{0} \Rightarrow 18$ $\textcircled{10} \textcircled{10} \textcircled{10} \textcircled{10} \textcircled{10} \textcircled{10} \textcircled{10} \textcircled{10} \Rightarrow 20$

$9+9=18$ $10+10=20$

$2 \times 9 = 18$ $2 \times 10 = 20$

-1-