ACTA DIDACTICA NAPOCENSIA
Volume 5 Number 2, 2012

ANALYSIS OF ERRORS AND MISCONCEPTIONS IN THE LEARNING OF CALCULUS BY UNDERGRADUATE STUDENTS

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Abstract. This paper is going to analyse errors and misconceptions in an undergraduate course in Calculus. The study will be based on a group of 10 BEd. Mathematics students at Great Zimbabwe University. Data is gathered through use of two exercises on Calculus 1&2.The analysis of the results from the tests showed that a majority of the errors were due to knowledge gaps in basic algebra. We also noted that errors and misconceptions in calculus were related to learners' lack of advanced mathematical thinking since concepts in calculus are intertwined. Also in this study we highlight some common errors/ mistakes which can be done by lecturers during the teaching process.Students studying calculus often make the same mistakes and similarly lecturers teaching calculus have patterns of mistakes. This paper is derived from practical situations hence it is open to updating and can be adapted by other calculus teachers in different setups.

Key words: errors, misconceptions, calculus

1. Introduction

Calculus is a branch of mathematics that was invented in the 17^{th} century by I. Newton and W. Leibniz. It is a branch of mathematics that deals with infinitesimal quantities of a function i.e. small changes that occur in a function e.g. rates of change, gradient, areas and volumes.

Calculus concepts are widely used in other undergraduate maths courses such as Probability Theory, Optimisation, Ordinary Differential Equations, Analysis, Mechanics and Mathematical Modelling. In Probability Theory, continuous random vectors are handled using Derivatives and Multiple Integrals. The following examples are taken from $\mathbb{R}^2$.

Let (X,Y) be a continuous random vector with the distribution function F(X,Y). Suppose F(X,Y) has finite partial derivatives.

Then the function

$f_{xy}(x,y) = \frac{\partial F(x,y)}{\partial x \partial y}$ gives the joint density function of X and Y and $f_{xy}(x,y)$ satisfies

$\iint_{-\infty}^{\infty} f_{xy}(x,y)dxdy = 1$ and $A \in \mathbb{R}^2$ then $P[(x,y) \in A] = \iint_A f_{xy}(x,y)dxdy$

Other disciplines such as engineering and commerce also use calculus themes. A superficial knowledge of calculus concepts will affect the understanding of a vast number of maths and science disciplines. Thompson (1985) notes that a mathematics curriculum is a collection of activities from which students may construct mathematical knowledge and that it is a sequence of activities, situational context from which students construct a particular way of thinking. The dependence of other mathematical disciplines on Calculus shows that students can construct knowledge from other mathematical disciplines using it. At this point we would like to indicate that a good knowledge of Calculus themes will help students to construct knowledge in other disciplines of mathematics. Other branches of mathematics also seem to consolidate the understanding of Calculus concepts. Mathematical analysis explains the main concepts of Calculus; limits, differentiability, continuity, and integrability while Topology further develops other areas of calculus like sets, limits and continuity.

Received 19 March 2012.

Misconceptions and Errors

According to WWW.Dictionary/Thesaurus (10/10/2011) a misconception happen when a person believes in a concept that is objectively false. Due to the subjective nature of being human it can be assumed that everyone has some kind of misconception. If a concept cannot be proven to be either true or false then it cannot be claimed that disbelievers have a misconception of the concept by believers no matter how much the believers want a concept to be true and vice versa. Misrepresentation of a concept is not a misconception but may produce a misconception. According to Li (2006) student errors are the symptom of misunderstanding. Among many different types of errors, systematic errors occur to many students over a long time period and it is relatively easy and thus possible to research with current knowledge and tools. The cause of systematic errors may relate to student's procedure knowledge, conceptual knowledge, or links between these two types of knowledge. Generally misconceptions manifest through errors. An error can be a mistake, blunder, miscalculation or misjudge and such category falls under unsystematic errors. The challenging issue concerning misconceptions is that many people have difficulty in relinquishing misconceptions because the false concepts may be deeply ingrained in the mental map of an individual. Some people do not like to be proven wrong and will continue clinging to a misconception in the face of evidence to the contrary. This view is consistent with that of Hammer (1996) who thought students' misconceptions:

1. are strongly held, stable cognitive structures;

2. differ from expert understanding;

3. affect in a fundamental sense how students understand natural phenomena and scientific explanations; and

4. must be overcome, avoided, or eliminated for students to achieve expert understanding (p. 99).

One of the main methods used to analyze students' errors is to classify them into certain categorizations based on an analysis of students' behaviours. Through using a cognitive information-processing model and considering the specialties of mathematics, Radatz (1979) classified the errors in terms of (1) language difficulties. Mathematics is like a "foreign language" for students who need to know and understand mathematical concepts, symbols, and vocabulary. Misunderstanding the semantics of mathematics language may cause students' errors at the beginning of problem solving; (2) difficulties in processing iconic and visual representation of mathematical knowledge; (3) deficiency in requisite skills, facts, and concepts; for example, students may forget or be unable to recall related information in solving problems; (4) incorrect associations or rigidity; that is, negative transfer caused by decoding and encoding information; and (5) application of irrelevant rules or strategies.

Orton (1983) classified errors into three categories as follows:

(1) Structural error: is an error which arises from some failure to appreciate the relationship involved in the problem or to grasp some principle essential to solution.
(2) Arbitrary error: is that error in which the subject behaved arbitrarily and failed to take into account the constraints laid down in what was given.
(3) Executive error: is that error where student fails to carry out manipulations, though the principles involved may have been understood.

Mathematical knowledge is interrelated and misconceptions in one branch of mathematics maybe carried into other areas of mathematics. A poor mastery of basic concepts may limit a learner to pursue other areas of study. Over and above all calculus is a core course in any undergraduate mathematics curriculum.

Statement of the research problem

Mathematics knowledge is cumulative in the sense that it builds on knowledge from previous maths lessons which can date back as far as elementary mathematics. Generally on transition from college or high schools to university most students fail to regard the importance of their previous knowledge. A far more valuable aspect of Piaget's theory is the process of transition from one mental state to another. Tall (1991) asserts that during such a transition, unstable behaviour is possible, with the experience of previous ideas conflicting with new elements. He emphasizes on the terms assimilation where students take in new ideas and accommodation when students 'cognitive structure should be modified to live in with the new knowledge. This brings us to another concept of **obstacles** which crops up if new knowledge is not satisfactorily accommodated.

Cornu (1983) defines

an obstacle as a piece of knowledge;it is part of the knowledge of the student. This knowledge was at one time generally satisfactory in solving certain problems.It is precisely this satisfactory aspect which has anchored the concept in the mind and made it an obstacle. The knowledge later proves to be inadequate when faced with new problems and this inadequacy may not be obvious.

The purpose of this study is to investigate error patterns among students taking the course of Calculus for the first time at university. The objective after taking note of these errors and misconceptions would be to improve instructional practices for students taking undergraduate courses in mathematics.

Research question: What are students 'error patterns in solving problems related to single and multivariable Calculus?

2. Theoretical framework and literature review

The learning of mathematics is a constructive process. Dewey and Piaget have researched on this idea. Dewey (1945) pointed out that new objects and events should be related intellectually to those of earlier experiences. Mathematical knowledge therefore should be constructed from related knowledge which the learner already has. It is therefore the role of the maths educator to provide links between existing knowledge of mathematics and new knowledge. Piaget described two aspects of thinking that are different but complementary which are figurative aspect and operative aspect.

The operative thought allows the learner to see what next in relation to what has taken place and this involves intellectual aspect. The figurative aspect involves imitations, perceptions and mental imagery. Piaget made distinction to these two aspects in order to contrast the sensory motor stage and the concrete operational stage. Thompson (1985) found it useful to generalise this to any level of thought as follows: when a person `s actions of thought remain predominantly within schemata associated with a given level then the action is figurative in relation to that level.

When actions of thought move to the level of controlling the schemata then its operative. The thinking of college maths students in an advanced calculus is operative. i.e. learners are given definitions and then asked to apply them. We provide a few examples to illustrate.

(1) Using the definition of a limit prove that $\lim_{x\to 2} 2x+5=9$.

In this case the learner should know the definition of a limit i.e. $\lim_{x\to a} f(x) = L \quad iff$ $\forall\epsilon > 0 \ \exists\, \delta > 0 : |x-a| < \delta \quad implies \ |f(x)-L| < \epsilon$

This is the figurative aspect. The operative stage now requires the learner to move to the next stage i.e. to apply the definition.

i.e. Let $\epsilon > 0 \ be$ given, we want to find $\delta > 0$ such that $|x-2| < \delta \Rightarrow |f(x)-L| < \epsilon$

Now $|f(x)-L| = |2x+5-9| = 2\,|x-2|$, so choose $2\delta = \epsilon$ or $\delta = \epsilon/2$

2.Using L`Hospital` rule prove that $\lim_{x\to 0}\frac{\sin x}{x} = 1$.

In this case the learner should know L`Hospital` rule (figurative)

Then apply the rule $\lim_{x\to 0}\frac{\sin x}{x}$ [0/0] $=\lim_{x\to 0}\frac{\cos x}{1} = 1$.

3. Prove that the composition of injective functions is injective. The learner is required to know the definition of an injective function (one –to –one function.)

Suppose f o $g(x_1)$ =f o $g(x_2)$ then $g(x_1) = g(x_2)$ since f is injective and $x_1=x_2$ since g is injective. Therefore f o g is injective.

The cognitivists seem to point out that the learning of mathematical concepts is from the known to the unknown and when the application of a theorem or definition is needed then we are being required to recall a known result and then apply it to a new situation. The basic themes in a calculus course are functions and limits of functions. Most key concepts may require the learner to represent a function or a relation by a graph (Kyvatinsky & Even, 2004). Problems on finding the area below a curve of solids of revolution require a correct sketch of a graph.

Problems on double integrals that require changing the order of integration are also typical i.e. $\iint_D f(x,y)dxdy = \iint_D f(x,y)dydx.$ Transformations of multiple integrals also require the learner to give a sketch of the region of integration.

Most of calculus concepts require a good knowledge of limits of functions. Morn et al (2005) observed that students encounter epistemological problems with the limit concept and that these problems emanated from language and symbolism used. Robert & Speer (2001), Cornu (1991), Eisenberg (1990), Orton (1983) all have shown that learners have difficulties in understanding limits and derivatives. The following are examples of concepts that require knowledge of limits.

1. Improper Integrals e.g. $\int_1^\infty \frac{1}{x^2}dx = \lim_{m\to\infty} \int_1^m \frac{1}{x^2}dx$
2. Differentiation from first principles where the derivative of f(x) is defined as: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

Definitions of key concepts play a crucial role in most undergraduate maths courses since they assist learners to understand basics. Vinner (1991) suggests that definitions do not only help in concept formation but they also help learners to handle tasks involving mathematics concepts.

Vinner (1991) also gave two modes of the use of definitions namely the everyday use and the technical mode. Most branches of mathematics like Analysis and Topology also rely on definitions. At this point we would like to conjecture that a good knowledge of definitions will lead to a good understanding of calculus. Alternative definitions also help learners to have an idea of a concept. The following example on continuity illustrate this.

Definition 1. A function f(x) is continuous at x_0 iff
$\forall\epsilon > 0 \ \exists\,\delta > 0 : |x - x_0| < \delta \quad implies \ |f(x) - f(x_0)| < \epsilon$

This is considered as an analytic definition. Tsvigu et al (2005) have shown that most learners have difficulties with such analytic definitions. Alternative definitions are:

1) A function f(x) is continuous at x_0 if $\lim_{x\to x_0} f(x) = f(x_0)$
2) A function f(x) is continuous on a set A if its graph has no jumps.

Most calculus concepts are based on limits of functions and their representation as graphs. Nyikahadzoyi (2005) observed that certain "A" level teachers had difficulties with definitions of functions, and the same authority also noted that some "A" level textbooks called relations like $x^2 + y^2 = 1$ functions which is misleading. Breidenback (1992) suggested different terms for the function concept such as pre-function concept, action process and object conception. A learner is said to have developed the pre-function concept if he has little or no idea of a function with elements of one set fall for this category. An action is a way of dealing with some objects. Learners who view a function as a formula that is used to obtain values of x are said to have attained the action concept. In this case a function is taken to be a rule for obtaining a dependent variable from an independent variable.Dubnisky and Harel (1992)argue that another group of learners may view a function as a process i.e. a function is taken to be causing change to elements of a set. A typical example is a transfer function in the branch of mathematics called Control Theory where there is an input and an output result. Nyikahadzoyi (2005) defined learners who can use functions correctly without the concern of correct definition as a structural approach to the function concept.

Some of the concepts in Calculus may require learners to represent a function or a relation in a table or graph. Kyvatinsky and Even (2004) noted that the ability to represent a function in various forms allows the learner to have a deeper understanding of the concept. Certain calculus problems require the learner to sketch the graph of the functions or relations.Typical examples are problems on finding area enclosed by curves, find volumes of solids of revolution and most problems on multiple integrals.

3. Research

Research methods

In this research we explore errors, misconceptions and their causes in a Calculus course offered to B.Ed. Mathematics (Secondary) in-service students at Great Zimbabwe University. Tests will be used to collect data from learners. Students had covered Calculus 1 in semester 1.A pre-test was given at the beginning of the course to asses the level of the learners at the beginning of the course and this will be done to check whether causes of certain misconceptions are due to the background of the learners. A second post-test will be administered at the end of lectures (60hrs). The test will cover all the main themes of Calculus2 which are limits, continuity, functions of several variables, partial differentiation, multiple integrals and application. In this research a group of 10 B.Ed. students was used for the study. Half of the group have an “A” level pass in mathematics and the other half has done mathematics up to teachers` colleges level equivalent to at least “A” level. For the purpose of this research a pre-calculus test was administered. The test was given after a few revision lectures on introduction topics which were functions, limits, continuity, induction, basic differentiation and integration. The test was as follows:

Table 1. Pre-Test : Errors and Misconceptions in learning Calculus

	Topic	Question	Option1	Option2
1	Definition of function	What is the domain and range of the given function? f(x)= $\frac{\lvert x\rvert}{x}$	Domain: x is defined for all real numbers. Range: y=1	Domain: x is defined for all real numbers. Range: y=1 and -1
2	Graphs of functions	Draw/Sketch the graph of the function ? f(x)= $\frac{\lvert x\rvert}{x}$		
3	Types of functions	Classify the following functions as (a) even (b) odd (c) periodic : $y=x^2$, $y=x^3$,y=sinx, y=cosx		
4	Basic Trigonometry	What is the correct expanded form of cos(x+y) ?	cos(x+y)=cosx+cosy	cos(x+y)=cosxcosy-sinxsiny
5	Method of induction	Prove by induction that $1+2+\cdots+n=\frac{n}{2}(n+1)$		
6	Limits of functions	Find $\lim_{x\to 0}\frac{\sin 5x}{x}$		
7	Derivatives	What is the derivative of $y=x^x$	xx^{x-1}	$x^x(1+\ln x)$
8	Rules of derivatives	What is the derivative of $\sin x^2$	$2x\cos x^2$	$\mathrm{Cos}x^2$
9	Integration by parts	Identify u and dv on $\int \ln x dx$	(a) u=lnx, dv=dx	(b) u=1,dv=lnxdx
10	Improper Integrals	Evaluate $\int_{-1}^{1}\frac{1}{x}$dx		

The test is composed 10 items with 5 multiple choice questions(1,4,7,8,9) and 5 show working questions (2,3,5,6,10).Credit is given to a totally correct response for each item there are no partial marks.

Table 2. Results of pre-test

Subjects

Item	A	B	C	D	E	F	G	H	I	J	Total	p	q	pq
1	1	1	1	1	1	1	1	1	1	1	10	1	0	0
2	0	0	0	0	0	0	0	0	0	1	1	0.1	0.9	0.09
3	0	0	0	0	0	0	0	0	0	0	0	0	1	0
4	1	0	1	0	0	1	0	0	0	1	4	0.4	0.6	0.24
5	1	1	0	1	1	1	0	0	0	1	6	0.6	0.4	0.24
6	0	0	0	0	0	0	0	0	0	0	0	0	1	0
7	0	0	0	0	0	0	0	0	0	0	0	0	1	0
8	0	0	0	0	1	1	1	1	0	0	4	0.4	0.6	0.24
9	1	1	1	1	1	1	1	1	1	1	10	1	0	0
10	0	0	0	0	0	0	0	1	0	1	2	0.2	0.8	0.16
Total	4	3	3	3	4	5	3	4	2	6	37			0.97
Mean											3.7			
$x=X-\bar{x}$	0.3	-0.7	-0.7	-0.7	0.3	1.3	-0.7	0.3	-1.7	2.3				
x^2	0.09	0.49	0.49	0.49	0.09	1.69	0.49	0.09	2.89	5.29	12.1			

Key: 0=incorrect response; 1= correct response

Estimating Reliability using the *Kuder-Richardson Formula 20*

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{10}(x-\bar{x})^2 = \frac{12.1}{9} = 1.34$$, the sample variance.

$$KR_{20} = \frac{k}{k-1}\left(1-\frac{\sum pq}{\sigma^2}\right)$$ where : k=number of test items (10)

$KR_{20} = \frac{10}{9}\left(1-\frac{0.97}{1.34}\right) = 0.31$

which signifies a low estimate of reliability. Definitely the test items are questions which are not obvious to the calibre of students and also the most tricky questions to most students taking Calculus.

Analysis of errors and misconceptions

From the table the most difficulty items were 2, 3,6,7and 10. On item 2 some students sketched the graph of y=|x| while others the graph of y=1/x.They did not realize that item2 was a follow up of item1. Vinner (1983) notes that students do not necessarily use definitions to identify maths concepts and this suggests that definitions especially analytic ones may complicate certain learning situations. On item3 most students were able to classify easily the x^2 and x^3 but failed to identify that cosx and sinx are even and odd respectively, however they were able to classify periodic functions. All students were not able to solve item 6.The solution was as follows : $\lim_{x\to 0}\frac{5\sin 5x}{5x}$ =5. Some came up with strange answers like sin5 which came as a result cancelling by x. Again all students wrongly answered item 7 on the derivative of x^x. All students chose Option1, a misconception of applying the rule for x^n,where n is a real number. Instructors should emphasize the difference between those two terms. Only two students got item 10 correctly. The rest could not realize the area of integration where the integrand had a point of discontinuity at x=0.

Table3. Analysis of Pre-test question by question

	Question	Option1	Option2	Errors	Category
1	What is the domain and range of the given function? $f(x)=\frac{\lvert x\rvert}{x}$	Domain: x is defined for all real numbers. Range: y=1	Domain: x is defined for all real numbers. Range: y=1 and -1	none	N/A
2	Draw/Sketch the graph of the function ? $f(x)=\frac{\lvert x\rvert}{x}$			3 sketch y=1/x 3 sketch y=x 3 sketch y=|x|	Executive
3	Classify the following functions as (a) even (b) odd (c) periodic : $y=x^2$, $y=x^3$,y=sinx, y=cosx			All failed to identify that cosx is even and sinx is odd.	Structural
4	What is the correct expanded form of cos(x+y) ?	cos(x+y)= cosx+cosy	cos(x+y)=cosx cosy-sinxsiny	6 chose Option 1	Misconception of distributive law
5	Prove by induction that $1+2+\ldots\ldots+n=\frac{n}{2}(n+1)$			4 failed to show all the 3steps of induction	Executive
6	Find $\lim_{x\to 0}\frac{\sin 5x}{x}$			A common answer was sin5 found after dividing by x throughout	Misconception by relating to limits of rational functions.
7	What is the derivative of $y=x^x$	xx^{x-1}	$x^x(1+\ln x)$	All students chose Option1 and failed	Misconception of x^n
8	What is the derivative of $\sin x^2$	$2x\cos x^2$	$\cos x^2$	6 chose Option 2	Structural& Executive
9	Identify u and dv on $\int \ln x dx$	(a) u=lnx, dv=dx	(b) u=1,dv=lnxdx	None	N/A
10	Evaluate $\int_{-1}^{1}\frac{1}{x}dx$			8 did not realize the point of discontinuity at x=0 and proceeded to integrate	Arbitrary &Structural

A post calculus test was given to learners at the end of the semester after 60 hrs of lectures. The main themes of calculus of several variables were considered. The results for the topics of functions of several variables, functions & limits, basic differentiation and integration and applications were summarised.

Table 4: Results of the post-test on Calculus of several variables

Items	1	2	3	4	5	6	Total
Mark/Subject	3	1	2	2	7	5	20
A	0	1	1	2	4	0	8
B	0	1	0	2	4	0	7
C	0	0	0	2	4	0	6
D	0	1	1	2	4	0	8
E	0	1	0	0	4	0	5
F	0	0	0	0	2	0	2
G	0	1	1	2	4	0	8
H	0	0	1	0	5	0	6
I	0	1	0	2	0	0	3
J	0	1	1	0	4	0	6
Total	0	7	5	12	35	0	59

From the above table Item 1 on level curves recorded 100% failure. All learners failed to identify that the parent curve y= x^2 +k-4 represented a family of parabolas translating parallel to the x-axis. Item 6 also recorded a 100% failure. All learners failed to transform the double integral from Cartesian coordinate system to polar coordinate system. Solving the problem in polar coordinate system was more easier than attempting in Cartesian coordinate system. 70% scored correctly on Item2 concerning sketching regions of integration. On Item 3 all learners got partial marks on ability to substitute correctly but all failed to realise that $\lim_{x\to 0}\frac{\sin x}{x} = 1$, a concept which was acquired in Calculus1. 60% scored correctly on Item 4 concerning partial derivates,a concept which relies most on ability to derive single variable functions. On Item 5 all learners got partial marks on ability to compute the partial derivatives but all failed to locate the local minimum or maximum or saddle points. Learners were not able to use the criterion for locating these critical points, a concept which originates from Calculus1.

Table3: Post –Calculus Test

	Topic	Question	Common errors
1	Functions of several Variables (level curves)	Let x^2-y+z=4, if we let Z=k. Sketch the family of curves For k=0,2,4,6.	9 students plotted linear curves of the form y=mx+c instead of quadratic curves.
2.	Functions(Region sketching)	Draw the region represented by the relation: R={(x,y): 0≤x≤2; 0≤y≤$\sqrt{4-x^2}$}	3 students failed to sketch the curve $y=\sqrt{4-x^2}$ hence could not draw the required region. They failed to realise that the curve was a ¼ circle in the 1^{st} quadrant.
3.	Limits	Evaluate $\lim_{(x,y)\to(0,0)}\frac{(1+y)\sin x}{x}$	All students failed to realise that the limit was 1. This is a misconception carried over from Calculus 1.
4.	Partial Derivatives	Let z=$\frac{x}{y}$ where x=e^t, y=ln t Calculate dx/dt	3 students were not able to identify that the derivative of e^t is e^t.
5.	Maxima and Minima	Let f(x,y)=x^3 -3xy +y^2, compute f_x,f_y, f_{xx}, f_{xy}, f_{yy}.Locate the local minimum, maximum, saddle points.	All students were not able to use the criterion for locating critical points.
6.	Multiple Integration	Transform $\iint_{(0;0)}^{(2;\sqrt{4-x^2})}(x^2+y^2)\,dydx$ into polar coordinate system using x=rcosθ and y=r sinθ and hence calculate the integral.	All students were not able to change the double integral to polar coordinate system and also to compute the integral even in the given coordinates.

Errors on reversing the order of integration were in the following 3 categories namely A, B and C.

A. Some learners did not understand the process.
B. Some could not sketch the regions of integration correctly.
C. A small number of learners could not understand keeping y constant and integrating x.

Transformations of multiple integrals had errors/ misconceptions in the following categories:

A. Some learners did not understand the process.

B. Some learners could not simplify Jacobians which were in the form of determinants of 2x2 matrices.

C. Some learners could not link double integrals to problems in space geometry

4. Conclusion

In this paper we analysed some errors and misconceptions made by undergraduate learners in Calculus. Results from tests show that some learners had a weak background of mathematics such as a low pass at "A" level and yet most pre-calculus concepts such as algebra, limits, basic differentiation and integration have a strong link with High school mathematics. Performance in the first test on basic calculus concepts also reflect the same problem, and some errors on limits of functions of a single variable were observed again in limits of functions of several variables. Learners also seem to have difficulties with analytic concepts. Felder and Henriques (1995) suggest that students learn more when information is presented in a variety of modes than when a single mode is used. E.g. the limit of a function concept could be explained using the graph of a function, algebraic methods, a table of values or verbal descriptions and this can help the learner to develop concepts from intuitive stage to analytic stage. Greeno and Hall (1977) also maintain that different forms of representation are useful as they aid in constructing understanding and communicating information. The research also show that misconceptions are a result of poor understanding of the basic themes of calculus like limits of functions and their representation. Bezuidenhout (2001) also obtained that some difficulties and misconceptions in calculus were a result of teaching approach that emphasises to a large extend procedural aspects of the calculus and neglects a solid grounding in the underpinnings of calculus. For some of these findings certain analytic concepts should be developed early in a calculus course.

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Acknowledgement

We acknowledge the assistance we got from the Department of Curriculum Studies, Faculty of Education, Great Zimbabwe University.