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Preservice Teachers' Understanding of Variable

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Abstract

This study examines the research on middle school students' understanding of variables and explores preservice elementary and middle school teachers' knowledge of variables. According to research studies, middle school students have limited understanding of variables. Many studies have examined the performance of middle school students and offered suggestions on how to improve instruction in middle school. This study considers preservice elementary and middle school teachers' knowledge of variables. A total of 73 preservice teachers, all candidates for Early Childhood - 6 certification, were given the same variable assessments completed by middle school students. Many of the misconceptions displayed by middle school students were also present in the results from preservice elementary and middle school teachers. This suggests that another means of improving middle school students' performance in algebra is by strengthening the preservice elementary and middle school teachers' understanding of variables.

Introduction

Since 1990, when the National Council of Teachers of Mathematics (Edwards, 1990) affirmed the teaching of algebraic thinking at the elementary level, there has been limited progress in increasing students' understanding of algebra. For example, eighth-grade performance on the National Assessment of Educational Progress improved from 283 in 2009 to 284 in 2011 (NAEP, 2011). In one multiple choice algebra problem, eighth grade students were asked to identify the equation of a line given a point on the line and that the slope was negative. Only 31% of the eighth grade students were able to correctly identify the equation of the line (NAEP, 2011). If students are to be successful in algebra they must have a well-developed conceptual understanding of variables. According to many researchers (Hill, Rowan, & Ball, 2005; Ma,1999; Borko & Putman, 1996), there is a correlation between teachers' content knowledge and students' knowledge of mathematics. To successfully facilitate students' understanding of algebra, teachers themselves must have an in-depth knowledge of the subject. Educator preparation programs must provide the content background to support preservice teachers in their understanding of algebra and variables in particular. Preservice teachers' understanding of variables has received relatively limited attention from the mathematics education research community. This study investigates both middle school students and preservice teachers' understanding of variables.

Literature Review

Middle School Students' Understanding and Variable

Many studies have examined the ways in which middle school students perceive variables, the common conceptual errors, and possible causes of such misconceptions. A common misconception is using variables to represent physical objects. This has been documented by many researchers (Booth, 1988; Kaput, 1987; Kuchemann,1981; MacGregor & Stacey, 1993; Pimm, 1987; Asquith, Stephens, Knuth, & Alibali, 2007). For example, some students think $3y$ could represent 3 yachts. According to Pimm (1987), when teachers describe $5a + 2b$ as apples plus 2 bananas they are fostering a misunderstanding of variable.

Asquith, Stephens, Knuth, and Alibali (2007) asked the following questions as part of their study: "In the express $2n + 3$, what does the symbol n stand for?" Some students thought n could only stand for one specific number while other students through n stood for a word or an object, for example $2n$ meant 2 nickels.

A second misconception of middle school students who lack a conceptual understanding of variable was to replace the variable letter with the numerical position of that letter in the alphabet (Booth, 1984; MacGregor & Stacey, 1997; Knuth, Alibali, McNeil, Weinberg, & Stephens, 2011). According to Knuth, Alibali, McNeil, Weinberg, and Stephens (2011), "Algebra has been called the study of the 24th letter of the alphabet. Although this characterization is somewhat facetious, it underscores the importance of developing a meaningful conception of variable in learning and using algebra." (p. 262). However, MacGregor and Stacey (1997) found that some curriculum materials are detrimental to students' understanding of variable. For example, in the book *Navigating Through Algebra in 3-5* (Cuevas &

Yeatts, 2001, pp. 39-40), students work with a variable machine. With this material, if *b* has the value of 1, then *a* is 0 and *c* is 2, and if *y* has the value 24, *x* has the value 23, etc. This reinforces the idea that "1 less than the letter *y* is *x*." Working with this variable machine, students make the assumption that the value of one variable is determined by the value of another variable. Booth (1984) found many students replace the letter *n* in a problem with the number 14, its position in the alphabet.

Kaput (1987, p. 187) discussed students' performance on the popular Students-Professors problem:

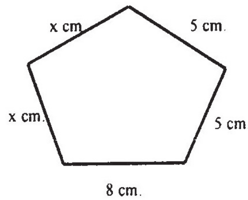
> At a certain university, for every 6 students there is 1 professor. Let *S* stand for the number of students and *P* stand for the number of professors. Write an algebraic equation using *S* and *P* that gives the relation between the number of students and the number of professors at that university.

Kaput (1987) notes approximately 65% of all errors were accounted for by the response $6S = P$. Kaput suggests the cause of this error is the difficulty students have in transferring this information to an algebraic symbol system. The algebraic symbolism is overridden by the students' natural language encoding. Kaput's findings are supported by other researches (Clement, 1982; Kaput & Sims-Knight, 1983).

A third common misconception for middle school students concerned the legitimacy of an answer, which contained a variable (Booth, 1984; Booth, 1988; Chalough & Herscovics, 1984; Stacey & MacGregor, 2000; Asquith, Stephens, Knuth & Alibali, 2007). Middle School students do not want an algebraic answer, they want a specific numerical answer. When Booth (1984, p. 34) asked a student the perimeter of a regular pentagon with sides of length *n,* she replied "Well, you can't give a proper answer, because you don't know what *n* is. If I knew *n,* I could work it out, but as it is, all you can put is 5*n*." Booth (1988) further documented students' difficulty in accepting variable expressions as answers to problems.

Finally, some middle school students were somewhat accepting of algebraic answers, but assumed that what was required was a "single term" answer (Booth, 1984; MacGregor & Stacey, 1997; Stacey & MacGregor, 2000; Stephens, 2006; Kaput, 1987; Kuchemann, 1981; Kieran, 1981). Thus $x + y$ was not an acceptable answer, but *z* was. Stacey and MacGregory (2000) summarized data from separate studies of 14 and 15 year old school students' performance on variable tasks. Most students were not willing to support an expression with two or more terms as an acceptable answer, and they had difficulty simplifying an expression correctly. Students in the MacGregor and Stacey (1997) study were asked to respond to a question similar to the following:

Some students measured "*x* cm" with their rulers. Others assumed $x = 1$



"What is the distance around this shape?"

Figure 1. Perimeter of a Pentagon with Both Constant and Unknown Side Length.

and obtained the answer 20 cm. In interviews, students commented: "Do I have to figure out the number?" "That's the hypotenuse." Some students drew lines to form two triangles, while other students wrote $x^2 + 5^x + 8$. This problem was difficult for students, since they were required to simplify an expression and report an answer that was not a single number but rather an expression.

While there is a large body of research on middle school students' understanding of variables, there is little research on elementary and middle school preservice teachers' understanding of variable. Borko and Putman (1996) have identified teacher knowledge as a determinant to effective teaching of algebraic concepts in elementary school. There has been little research focused on the knowledge base of preservice elementary and middle school teachers with regard to variables. The purpose of this study is to examine preservice elementary and middle school teachers' knowledge of algebraic notation in two areas relating to the concept of variable: how to interpret a variable and how to write an algebraic expression.

Preservice Teachers' Understanding of Variable

Researchers have investigated preservice elementary teachers' understanding of various aspects of algebra. Van Dooren, Verschaffel and Onghena (2003) examined algebra word problem skills and strategies in primary preservice teachers in Belgium. They concluded that primary teachers did not use algebraic strategies because of their lack of algebra understanding. Rule and Hallagan (2007) found that elementary preservice teachers found algebraic content challenging and they had difficulty defining a variable and identifying patterns.

Hansson and Grevholm (2003) asked elementary preservice teachers the meaning of $y = x + 5$. They concluded that the majority of preservice teachers do not connect the concept of function with equations. In another study, Dobrynina and Tsankova (2005) examined the ability of preservice elementary teachers to solve equations with two and three variables and to create guiding questions for elementary students. Their findings pose concerns about the ability of preservice teachers to support elementary students developing algebraic reasoning. In another study related to variables, Hallagan, Rule, and Carlson (2009) investigated elementary school preservice teachers' understanding of algebraic generalizations and found that the preservice teachers had considerable difficulty generating an algebraic rule.

Bishop and Stump (2000) found that the students in their algebra class for preservice elementary and middle school teachers had a limited understanding of generalizations. They also found that many preservice teachers do not understand the distinction between arithmetic and algebra, and they view algebra from a procedural perspective.

No research study has focused specifically on preservice teachers' understanding of variables. This study was undertaken to gather insights into preservice elementary and middle school teachers' conceptual understanding of variables. This study will allow the researchers to derive more warranted conclusions about preservice teachers' readiness to facilitate students' learning of algebraic concepts.

Costa's Model of Questioning

In this study, preservice teachers were asked a series of questions related to variables. When teachers ask questions and analyze responses, it is important for them to recognize the cognitive level of the questions. In 1956, Bloom and colleagues (Bloom, B. S., Engelhart, M. D., Furst, E. J., Hill, W. H., & Krathwohl, D. R., 1956) developed a multi-tiered model of classifying thinking by six cognitive levels of complexity: knowledge, comprehension, application, analysis, synthesis, and evaluation. A revised version of the taxonomy was created in 2001 with the categories: remembering, understanding, applying, analyzing, evaluating, and creating (Anderson & Krathwohl, 2001).

According to Costa (2001), questions require differing levels of complexity of thinking. He suggests three cognitive levels of questions. Basically Costa's level one incorporates Bloom's first two levels, Costa level two includes Bloom's Application and Analysis levels, while Costa's level three includes Blooms levels five and six.

In Costa's first level, referred to as "Gathering and Recalling Information," students are asked to use the concepts and information they have acquired in the past and stored in long-or short-term memory. At this level, a teacher might ask students to define, name, convert, describe, identify, etc. At level 1, a student might be asked, "Is this an example of part to whole or part to part?"

In the second level, "Making Sense Out of the Information Gathered," students are asked to process the data gathered and to analyze, classify, compare/contrast, or differentiate the data they have acquired. A level 2 question would be: "What is the difference between a fraction that is a ratio and one that is a rate?" Costa (2001) refers to level 3 as "Applying and Evaluating Actions in Novel Situations." In this level, students are asked to go beyond the concepts they have developed and use the information in a novel or hypothetical situation. At level 3, students might be asked to generalize, hypothesize, infer, predict, interpret, etc. Level 3 questions require students to demonstrate mastery of the knowledge they have learned. An example of a level 3 question is: "If ratio of girls to boys in our class is 3 to 4 and we added six new girls, how many boys should we add to maintain the ratio of 3 to 4?"

Instruction in the Costa's levels is a component of the University of Houston−Clear Lake (UHCL) mathematics methods courses. Inservice teachers have reacted positively to the taxonomy. According to an eighth grade teacher:

> I quickly realized that this new Costa classification was actually much better than Bloom's. With Bloom's, there were all these different levels and it was hard to keep them straight. You never really knew what level of questions you were asking. With Costa it is very simple; there are three levels. Basic questions are level one, middle level questions are level two, and the more complex questions are level three. It is so much easier to identify what type of question you are asking and what level question it is. (A. Maire, personal communication, March 1, 2012)

In this study, the researchers decided to code questions using the Costa levels because of their experience using the model with inservice teachers. Teachers like Maire reported it easy to use and helpful in determining the cognitive level of the questions they ask their students.

Method

Subjects

All participants (N = 73) were preservice teachers seeking Early Childhood through Grade 6 (EC-6) teacher certification from a teacher education program at an established regional university. All participants are required to take nine hours of mathematics: College Algebra, Mathematics for EC-6 Teachers I, and Mathematics for EC-6 Teachers II. These three courses together are framed around the state content standards for mathematics. The content standards are grouped into five strands across the K-12 curriculum: Number, Operations, and Quantitative Reasoning; Patterns, Relationships, and Algebraic Thinking; Measurement; Geometric and Spatial Reasoning; and Probability and Statistics (Texas Education Agency, 2012).

The College Algebra course is prerequisite to entry into the specialized courses for teacher certification candidates. The knowledge and skills from algebra are incorporated into the subsequent courses.

The content of Mathematics for EC-6 Teachers I is based upon strands I and II of the standards. The topics covered include: a set theory approach to the real number system, fundamental operations and concepts of arithmetic, and systems of numeration. The content of Mathematics for EC-6 Teachers II addresses strands III, IV, and V of the standards. Topics included are: informal two- and three-dimensional geometry, measurement, probability and statistics, and transformational geometry. Algebraic notation is incorporated in both courses in such areas as functions, pattern development, measurement, and formulae.

At the time of the study, all participants had successfully completed College Algebra and Mathematics for EC-6 Teachers I and were enrolled in Mathematics for EC-6 Teachers II. Upon successful completion of the course, they would take an EC-6 Mathematics Methods course.

Procedure

The participants in this study were not randomly selected. They were members of three sections of Mathematics for EC-6 Teachers II taught by full-time faculty members. There were five sections of the course the semester the study was implemented. The other two sections were taught by adjunct faculty members. At the beginning of one class period, students were asked for the voluntary participation in a study about algebra. They were assured that participation was voluntary and anonymous and would in no way influence their grade. The students were given the questionnaire and given approximately 10 minutes to respond. Because the data collection was not announced in advance and a high percentage of the population of students in the course participated (more than 65%), the data can be consid-

Question 1. Sue weighs 1 pound (lb.) less than Chris. Chris weighs y lbs. What can you write for Sue's weight?

Question 2. The following question is about this expression: $2n + 3$. What does the symbol n stand for?

Question 3. Can you tell which is larger, $3n$ or $n + 6$?

Question 4. What is the distance around this shape?

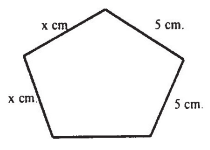


Figure 2. Variable Tasks Completed by Preservice Teachers.

ered representative of the population of students in Mathematics for EC-6 Teachers II that semester.

The items used in testing are shown in Figure 2. The items were presented to the preservice teachers in order. Questions 1 and 4, which pertain to the writing of an algebraic expression were used in the MacGregor and Stacey (1997) study involving students aged 11-15. Questions 2 and 3, which asked for interpretation of a variable, were used in the Asquith, Stephens, Knuth and Alibali (2007) study, which also involved middle school students.

Responses were evaluated using the rubric in Figure 3. Coding was done by the two researchers with cases of disagreements being mediated. This rubric is an adaptation of the rubrics used by Macgregor and Stacey (1997) for questions 1 and 4 and Asquith, Stephens, Knuth and Alibali (2007) for questions 2 and 3. In each case, the first response is the correct response. The second response indicates the replacement of the variable by a specific value. The final response category, "Don't know, No response, Other," is the

Item 1:

- *Variable*. Response indicates understanding of variable. "Sue's weight is $y - 1$."
- *Single Value.* Response expresses Sue's weight by selecting a specific weight for Chris.
- *Incorrect Order of Operation.* $1 - y$.
- *Don't know, No response, Other.* Responses do not fit the above categories.

Item 2:

- *Variable*. Response expresses the idea that the symbol can stand for any number.
- *Single Value*. Response expresses the idea that the symbol can stand for one specific number only.
- *Object*. Response expresses the idea that the symbol is an object or word that begins with the letter n.
- *Don't know, No response, Other.* Responses do not fit the above categories.

Item 3:

- *Variable*. Response expresses the idea that one cannot determine which quantity is larger because the variable can take on multiple values.
- *Single Value*. A single value is tested and a conclusion is drawn on that basis; thus, a student's conclusion might vary depending on the value tested.
- *Operation.* Response expresses the idea that one type of operation leads to larger values than the other. "3 times anything is more than 6 plus anything."
- *Don't know, No response, Other.* Responses do not fit the above categories.

Item 4:

- *Unknown Quantity*. Response expresses the distance around the shape as $2x + 18$ (or as the sum of $8 + x + 5 + 5 + x$)
- *Single Value*. Response expresses the distance around the shape by selecting a specific value for x.
- *Incorrect Content Knowledge*. Response indicates incorrect algebra or geometry knowledge.
- *Don't know, No response, Other*. Responses do not fit the above categories.

Figure 3. Coding Scheme for Test Items.

same for each item. For purposes of discussion, Question 1 will be referred to as WEIGHT, Question 2 as SYMBOL, Question 3 as LARGER, and Question 4 as DISTANCE

Results and Discussion

Preservice Teachers' Performance

The researchers anticipated that the majority of participants would be successful on at least three of the variable tasks. This expectation was premised on two factors. First, the variable tasks were originally designed for middle school students with limited experience in algebra. The participants in this study had previously completed a college algebra course and, therefore, should be well equipped for the tasks. Second, these preservice teachers were in the middle of their final required mathematics course for certification. Therefore, they would understand how to write an algebraic expression and how to interpret expressions containing a variable.

Table 1 reports the percentage of preservice teachers who answered each question correctly. Only 42.5% of the participants responded correctly to 3 or 4 tasks, and 23.3% performed at an unacceptable level, with 0 or 1 correct response. Clearly many of these preservice teachers exhibit a weak understanding of variables. A closer analysis of the test responses to individual items provides more specific information concerning these weaknesses.

Table 1.
Percentage of Preservice Teachers with Correct Responses (n=73)

Number of Correct Responses	Percentage of Population
4	13.7%
3	28.8%
2	34.2%
1	21.9%
0	1.4%

Preservice Teachers and Writing Algebraic Expressions

Table 2 presents the responses of the preservice teachers in each response category for each of the four variable tasks.

Table 2.
Preservice Teachers' Responses to Each Question in Each of Four Response Categories (N=73)

Category	Percentage of population
Weight	
Variable	94.5%
Single Value	1.4%
Incorrect Order of Operations	1.4%
Don't Know, No response, Other	2.7%
Symbol	
Variable	43.8%
Single Value	32.9%
Object	6.8%
Don't Know, No response, Other	16.5%
Larger	
Variable	54.8%
Single Value	4.1%
Operation	34.2%
Don't Know, No Response, Other	6.9%
Distance	
Unknown Quantity	41.1%
Single Value	12.3%
Incorrect Content Knowledge	24.7%
Don't Know, No Response, Other	21.9%

The striking success of WEIGHT is immediately of interest. Approximately 95% of the preservice teachers responded correctly to this item. This level of success was not repeated in any other test item. What makes the respondents so successful on this particular item? There are several possible explanations. An informal survey of high school algebra books indicates that this item is very typical of the questions asked when writing an algebraic expression is first introduced. The form of the question, "What can you write?" is also a typical of these introductory exercises. So the item is one that is familiar to the preservice teacher.

Applying Costa's cognitive levels of questioning, the WEIGHT problem is a level 1 question. It falls into the category of defining and describing and does not require any higher order thinking skills. Using only the results of WEIGHT, it would seem that the preservice teachers are adequately prepared to write algebraic expressions.

DISTANCE is the second item involving the writing of an algebraic expression. However, the results for DISTANCE are significantly different from those of WEIGHT, with only 41% responding correctly. The Chi-Square Test of independence indicates that a statistically significant relationship does not exist between the questions and correct responses, χ^2 (1, N = 73) = 47.24, p = 6.3 x 10^{-12}. Correctness of response is dependent upon the question asked. A closer analysis of DISTANCE sheds additional information on what preservice teachers actually understand about writing algebraic expressions.

Almost half of the incorrect responses (24.7% of all responses) exhibited errors in preservice teachers' content knowledge. In some cases, the error was in the understanding of "distance around" or perimeter. Several respondents confused perimeter with angle measures, as in "x + x + 8 + 5 + 5 = 360^0", or "2x + 18 = 540^0" (the sum of the interior angles of a pentagon). In other cases, the error was made during the simplification process. For example, "x + x + 5 + 5 + 8 = 2x + 18 so x = 9". These types of errors were also present in the responses of the middle school students in the MacGregor and Stacey (1997) study. For example, when middle school students were interviewed, some students responded: "Do I have to figure out the numbers? That's the hypotenuse."

Another group of incorrect responses (12.3% of all responses) substituted a specific value for *x* in the problem. The most frequent substitution was the value 5 cm, with 4 cm as another popular choice. Compared to WEIGHT, in which only 1.4% of respondents chose to use a specific value for *x*, this group of respondents is interesting. Perhaps, since the problem provided specific values for some of the sides, students were encouraged to use specific values for the unknown sides. In addition, the question format "What is the distance?" may have encouraged the need for a "real answer" to the problem. This phenomena has been documented by Chalouh and Herscovics (1984), Booth (1988), and Stacey and MacGregor (2000). MacGregor and Stacey (1997) also reported students substituted a specific number for *x* and drew inferences from new learning. Finally, another group of incorrect responses (21.9% of all responses) were either "Don't Know, No Response, or Other."

If Costa's Levels of Thinking and Questioning are applied to DISTANCE, this question falls into the category of application and analysis or level 2. This may explain the greatest source of variance in success rates between DISTANCE and WEIGHT, since WEIGHT was categorized as a level 1 question. Given these findings, it appears that preservice teachers have a basic knowledge of writing algebraic expressions but are not as strong in applying that knowledge.

Preservice Teachers and Variables

Items 2 and 3, SYMBOL and LARGER respectively, involve interpreting a variable. Referring again to Table 2, there is no significant difference in the performance levels on these two items. Correctness of response is independent of the question asked, $\chi 2$ (1, N = 73) = 1.75, p = .019. Each of these items would be categorized as Coasta level 3 questions, which require higher order thinking skills such as synthesizing, integrating, and predicting. Under these circumstances, the success rates on these questions were not unexpected. Once again, an analysis of the incorrect responses provides additional information about the preservice teachers' understanding of variable.

In item 2, SYMBOL, the greatest proportion or incorrect responses was of those that implied that the symbol can stand for only one specific number (32.9%). Responses such as "a number," or a specific value were counted as incorrect. In each case, a definite article implied a single value for the symbol. Correct responses usually included the word "variable," or "any number." This narrow interpretation of the responses certainly impacted the results. It is also possible that the responses to this item were influenced by the apparent need of preservice teachers to find a "real, numerical" answer to every question.

Of greater concern in SYMBOL are the responses in the last category, *Don't Know, No Response, Other* (16.5%). Respondents in this category seemed to lack the prerequisite knowledge to recognize the multiple values for a variable. Their responses included restatements of the question, making the expression into an equation, naming *n* as a coefficient or exponent, and claiming that there was not enough information to answer the question. These responses strongly parallel the results Asquith et al. (2007) found with their 6^{th}, 7^{th}, and 8^{th} grade students.

In Item 3, LARGER, the greatest proportion of incorrect responses (34.2%) was based on the misconception that the operation of multiplication always leads to larger values than addition. This result is consistent with the results of the Asquith, Stephens, Knuth and Alibali study (2007). In that study, between 22% and 54% of the responses, depending upon the grade level of the student, were justified using an operation. For 6th, 7th, and 8th grade students, who have less experience with the real number system, this is not an unexpected response. For preservice teachers with algebra experience, this result is troubling. A relatively small percentage (4.1%) of the responders chose a single value to evaluate the two expressions and made a conclusion based upon that result. It is encouraging that most preservice teachers recognized the limitations of a single test case approach to solving a problem.

If one considers SYMBOL and LARGER using Costa's Levels of Thinking and Questioning, they are level 3 questions. They definitely fall into the category of generalizing, hypothesizing, and interpreting. We would expect that the preservice teachers would find these items more difficult, which was the case in this study. Given these findings, it appears that preservice teachers have somewhat limited ability in interpreting an algebraic expression.

This study indicates preservice teachers have some difficulty accepting an algebraic expression as an answer, that algebraic expressions involving more than one operation are less acceptable, and that effective interpretations of algebraic symbols are often impeded by natural language coding. These patterns are parallel to those of middle school students.

Implications

According to the National Council of Teachers of Mathematics *Principles and Standards for School Mathematics,* in grades 3-5 all students should "represent the idea of a variable as an unknown quantity using a letter or a symbol" (NCTM, 2000, p. 394). In grades 6-8 all students should "develop an initial conceptual understanding of different uses of variables; and use symbolic algebra to represent situations and to solve problems, especially those that involve linear equations" (NCTM, 2000, p. 395). The *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* (NCTM, 2006, p. 18) states that in grade 6, "They (students) understand that variables represent numbers whose exact values are not yet specified, and they use variables appropriately." This study attempted to determine if preservice teachers have the content knowledge to support students in developing their conceptual understanding of variables.

The current study surveyed 73 preservice teachers and found they were able to successfully solve the Costa level 1, WEIGHT question, and write an algebraic expression. However, when asked to solve the remaining three questions, which were cognitively more difficult problems involving the variables, they were not able to do so. Approximately 44% percent of preservice teachers answered the SYMBOL question correctly. This is similar to the performance of middle school students where 46% of sixth grade students gave a correct answer for the SYMBOL question (Asquith, Stephens, Knuth, & Alibali, 2007). The LARGER question was answered correctly by approximately 55% of the preservice teachers, while 46% of sixth grade students and 76% of eighth grade students answered the question correctly (Asquith, Stephens, Knuth, & Alibali, 2007). Approximately 41% of the preservice teachers answered the DISTANCE problem correctly. This compares with a 27% to 53% success rate students in the MacGregor and Stacey study (1997).

This study found preservice teachers, in general, do not have an adequate understanding of variables and algebraic expressions. They, in fact, have a knowledge base similar to middle school students. How will these "soon to be" teachers support students in developing not only procedural understanding, but more importantly, conceptual understanding of variables?

Limitations of this study must be noted. As previously stated, each researcher independently coded each question and then met to compare their codings. Cases of disagreement were mediated. This raises the question of possible researcher bias. Despite the limited sample size included in this study, the results yield insights into preservice teachers understanding of variables. Replicating this study in other locations would provide additional data to further develop the contributions of this work.

Teacher preparation programs typically require teacher certification candidates to complete several mathematics content courses: college algebra or an equivalent, one or two mathematics content courses specifically designed for certification candidates, and a mathematics methods course. Each of these courses contributes to the development of preservice teachers' understanding of variable. In each of these courses faculty need to consider more effective methods to develop preservice teachers' conceptual understanding of variable. In this study preservice teachers were able to solve problems requiring a low level of cognitive thinking but were not able to solve problems requiring greater complexity of thinking. University faculty should consider the questions they ask to determine if they pose questions requiring preservice teachers to analyze, generalize, infer, predict, etc.

In a mathematics methods course, the instructor could present problems about variables to illustrate common misconceptions and to discuss possible interventions. Stephens (2006) found few preservice teachers realize many elementary students hold misconceptions about algebraic concepts. Proactively sharing common misconceptions in a methods class would address this issue. As a result of examining research on students' thinking related to variables, preservice teachers might develop a checklist of do's and don'ts. For example, do not use the old, "you cannot add apples to oranges" verbiage; be careful in the selection of variable names, for example, do not describe 5 apples plus 2 bananas as $5a + 2b$; do not use "variable machines" where if a has the value 1, then b is 2, c is 3, etc. The "do" list might include such things as present problems where an expression rather than a single variable is the answer to the problem.

Also it is important for preservice teachers to realize that questions require differing levels of complexity. As they work with students, preservice teachers should categorize their questions so that they address differing levels of cognitive thinking. This could also occur in a mathematics methods course. For example, if students have studied variables, preservice teachers could be asked to identify a level 1, 2, and 3 question they could use with students. A possible level 1 question might be: evaluate $3x^2$ if $x = 4$. A level 2 question would be: It is possible for $x^2 = 2x$? A level 3 question would be:

Describe a situation where the perimeter of a pentagon is 3x + 12.

If preservice teachers are expected, in their educator preparation program, to think about variables not only at a basic level but also at higher cognitive levels, will they increase their conceptual understanding of variables and outperform middle school students on variable tasks? Results of studies to answer this question will lend additional insight into the issues in this article.

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