Coming to Know and Do Mathematics with Disengaged Students

Margaret Marshman
University of the Sunshine Coast

Raymond Brown
Griffith University

This case study explored how students disaffected with their school experience were scaffolded during their participation in a middle-school mathematics classroom. Of particular interest were the level of student engagement in discussion about the mathematics being presented by the teacher and the approach to doing mathematics being displayed by the students. It was found that scaffolding students' participation in middle-school mathematics promoted student engagement in discussion about mathematics and in the doing of mathematics. This was evidenced through increased participation by students in classroom discussions about mathematics, in the making and testing of conjectures related to mathematics tasks, and in the quantifying and modelling of mathematical tasks.

Keywords · scaffolding · collective argumentation· middle years · student engagement · classroom interaction

Introduction

One of the aims of mathematics teachers in general and middle school mathematics teachers in particular is for all students to participate in class lessons and to engage with the mathematics being presented (MCEETYA, 2008). Therefore, a question that confronts teachers is how best to scaffold student engagement in the mathematics of the classroom so that all students can participate with the teacher in coming to know and do mathematics, that is, to engage with tasks using the ways of thinking, speaking, and writing that are privileged in mathematics. This paper explores how one form of scaffolding, "collective argumentation" (Brown & Renshaw, 2006), promoted student participation in a middle-school mathematics classroom.

A number of international and national curricula documents such as the *Guiding Principles for Mathematics Curriculum and Assessment* (National Council of Mathematics Teachers, 2009) and the *Australian Curriculum: Mathematics* (Australian Curriculum, Assessment and Reporting Authority, 2010) are based on the premise that thinking, reasoning, and working mathematically are essential for learning mathematics. When students think mathematically they make meaningful connections with prior knowledge and experiences to plan solution pathways to set problems. They also make decisions about which mathematical knowledge to use when following these pathways. When students think mathematically they are expected to represent, explain, and defend solution pathways in different ways and to challenge the solution pathways of others (Schoenfeld, 1992). Watson (2002) described mathematical thinking as identifying and using patterns, and as abstracting understandings through generalising and manipulating representations.

Mathematical Thinking

Schoenfeld (1992) described mathematics as "the science of patterns" and doing mathematics as "an act of sense-making" (p. 9). Here, knowledge is "socially constructed and socially transmitted" (p. 18). Students need to understand and know mathematics so that they can develop the disposition to quantify and model what is happening in the world around them. As students are doing mathematics, they need to develop skills related to "modelling, abstraction, analysis, inference, and the use of symbols" (Schoenfeld, 1992, p. 33). These skills can be achieved through problem solving, by giving students the opportunity to explore a wide range of problems and situations from closed to open ended problems involving the application of modelling techniques (Schoenfeld, 1992; Romberg, 1994). The consequences for teachers when developing students' skills are that they need to consider: the implications of the different solution pathways represented by students; the timing and degree of assistance to be provided to students in the design of these pathways; and comfort levels in situations where they do not know all the answers (Burkhardt, 1988). Such an approach to teaching mathematics, therefore, needs to acknowledge that knowledge is actively constructed and that learning is a social process. The learning environment in which such skills are developed needs to be one where "... children publicly express their thinking and, more generally, engage in mathematical practice characterised by conjecture, argument, and justification" (Cobb, Wood & Yackel, 1993, p. 91).

Teaching requires the teacher to encourage students to build on each other's ideas and to participate in conversations about mathematics until they have constructed a shared understanding of a concept or skill. During these conversations, students are expected to share not only their solution pathways but their understandings as well (Cobb, Wood & Yackel, 1993; Solomon, 2007). For this to occur successfully, the teacher needs to be able to scaffold student participation in the mathematics classroom.

Scaffolding Participation in Mathematics

Wood, Bruner, and Ross (1976) coined the term *scaffolding* when describing ideal adult-child interaction exhibited in problem solving. The term was used as a metaphor for the process by which an adult assists a child to attain a greater degree of individual mastery of a problem solving task. The process of scaffolding was seen by Wood et al. (1976) as serving a number of important functions for the child, namely, (a) arousing interest; (b) limiting possible solution paths; (c) focusing attention; (d) highlighting salient task features; (e) managing affect; and (f) revealing idealised solution paths (p. 98). However, a limitation of Wood et al.'s metaphor of scaffolding relates to the mechanism whereby responsibility for task completion is transferred to the child (Stone, 1993). Stone (1993) maintains that it is conversation and non-verbal communication between participants, rather than simply adult guidance, that promotes the emergence of a shared understanding of the problem-solving goal and of appropriate means to achieve it. The emergence of this shared understanding within the process of scaffolding is important not only for the responsibility for completing the problem solving task, but also for the establishment of mutuality, trust, and relevance within the teaching-learning relationship. This approach to understanding scaffolding deepens the notion from one of simple transfer of knowledge to one of transforming understanding.

However, this requires the teacher to promote the goal of talking for understanding within the mathematics classroom (Lampert, 1990). Kershner, Mercer, Warwick and Staarman (2010) emphasised "the influence of classroom social routines and structures, and the productive use of 'talk rules' for conversation and collaborative reasoning" as an important part of the teacher's scaffolding. Building on the work of Cohen and Lotan (1997, cited in Boaler, 2006), Boaler (2006) studied mathematics classes using complex instruction as they solved open mathematical problems. Expecting students to be responsible for each other's learning, by helping someone who needs assistance and asking for help if they need it is one form of scaffolding that can be introduced by giving students in heterogeneous groups specific roles such as facilitator, team captain, recorder/reporter, and resource manager. Collective argumentation is another way to scaffold learning in a mathematics classroom where talking for understanding is used.

Collective Argumentation

Collective argumentation (Brown & Renshaw, 2006) provides a framework that allows students and teachers to work collaboratively to investigate a problem, question, statement, task or issue. This framework supplies support and structure for students to engage in substantive mathematical discussions. The strategies involved in collective argumentation are shown in Table 1.

Table 1
Key Word Format for the Classroom Implementation of Collective Argumentation

Strategy sequence
Represent individually
Compare co-operatively
Explain, justify and agree collaboratively
Validate communally

The students initially work individually to *represent* their solution path to (or ideas about) the mathematics task. This may be through the use of a diagram, drawing, graph, algorithm, numbers, or words that relate to an idea for a solution path. The value of this step is that it requires all students to have thought about the problem and to bring something to the subsequent discussion. In small groups (2–5 persons) the students *compare* their representations with those of other members of the group. The students then *explain* and *justify* their representations so that the group can reach consensus and construct a shared understanding and a shared solution path to a mathematics task. It is important that all members of the group understand and agree on this final solution path. If students do not understand a solution path, it is their responsibility to seek clarification from other group members who, in turn, have an obligation to help them. This agreed solution path is presented to the class for discussion and *validation*. The students are expected to present not only their shared response to the task but also the thinking and steps that led to their selection of this solution path. During this presentation, other students are encouraged to engage in a mathematical discussion with the presenting group.

During the small group phase of collective argumentation when the students are comparing, explaining, justifying and agreeing, the teacher listens to and observes the students before asking questions, or seeking explanations and justifications. In the whole class phase of collective argumentation, when the students are presenting their shared response to the class, the teacher's role is to ensure that the mathematics in the response is drawn out and made public for the class. This may mean that the teacher re-phrases, paraphrases and/or re-represents the students' shared response, and orchestrates opportunities within the discussion to make connections between student presentations and mathematical concepts and procedures previously presented in the class (Brown, 2007).

Throughout the whole process the teacher's role is to encourage the students and to engage with them in the construction of mathematical understandings. S/he needs to be able to question the students about the legitimacy of their conjectures and the usefulness of solution strategies (Lampert, 1990). The teacher must support students as they challenge the statements of others and support students in their use of the symbols of conventional mathematics. In other words, the teacher must scaffold student participation in the authority of the classroom.

Sharing Authority for Problem-solving

Within a classroom, the teacher is often seen as the authority and the one who possesses the knowledge (Oyler, 1996). When students accept their teacher as an expert authority in mathematics and treat some of their peers also as authorities, they may simply listen and agree with the expert or peer. The implications of this are that students fail to engage their own mathematical thinking and cannot participate in mathematical discourse (Amit & Fried, 2005). Teachers can learn to share authority by allowing students to ask questions to clarify their own understanding, and by encouraging students to speak in class discussions and to answer the questions of others (Oyler, 1996). In this case they are, in terms of the dynamics of the authority of the classroom, "shifting the emphasis from domination and obedience to negotiation and consent" (Amit & Fried, p. 164).

In order to scaffold student participation in the authority of the classroom, the implementation of collective argumentation requires that students negotiate with the teacher their own class charter of norms so as to establish the behaviour patterns that may lead to building a classroom where all members feel safe to engage in the discourse. A suggested class charter of norms is given in Table 2.

Table 2
A Suggested Class Charter of Norms (Brown, 2006)

Negotiated by the students, but will usually include norms such as displaying:
the *courage* required to state ideas and opinions to others;
the *humility* necessary to accept that ideas may not always be adequate;
the *honesty* essential to giving accurate feedback and reports;
the *restraint* integral to maintaining social cohesion;
the *persistence* required to pursue ideas and views in the face of opposition; and,
the *generosity* necessary to affirm the achievements of others.

For teachers to encourage students to think, reason, and talk mathematically they need to develop a strong sense of efficacy (Smith, 1996). The teacher needs to ensure that they not only scaffold the students' learning as they make and test ideas and assumptions, but also ensure that they do not force student discussions towards predetermined outcomes (Gregory, 2002). As such, the focus of the scaffolding needs to include both whole class teacher facilitated discussion and small group student discussion, and to be about sense making through, generalising, conjecturing and convincing; individual reflection and self-monitoring; and expecting students to clarify, elaborate and justify their thinking (Goos, 2004).

However, not all classrooms display the characteristics outlined above. Schoenfeld (1989) showed in his study of 230 mathematics students that the dominant belief of students was that mathematics was about memorising and was best achieved through rote learning and practice. In one particular class, the students were told that, "Practice is the key ... you have to know your constructions cold so that you don't spend a lot of time thinking about them" (Teacher cited in Schoenfeld, 1989, p. 344). Schoenfeld also referred to the regular occurrence where students, although they know particular mathematics in a particular context, do not indicate they know the mathematics in a different context. Moreover, the problem of engaging learners is more difficult with students who are already disengaged with learning mathematics. McFadden and Munns (2002) stated that:

> …. the persistence of culturally supported school resistance intensifies the challenge for educators committed to opening up pathways so that students from educationally disadvantaged backgrounds have greater chances of educational opportunity and success. (p. 359)

Boaler (2008) showed students can do mathematics and make sense of their world when they investigate and solve practical problem. The goals of engaging students in investigations and problem solving are "for students to make sense of a real-world use of mathematics, to get them involved in problem formulation, problem solving, and mathematical reasoning" (Battista, 1994, p. 463) and to give them the confidence to persist at problem solving. Boaler (2008) demonstrated it is possible to engage students in deep mathematical learning by using an investigative pedagogy. This type of pedagogy works particularly well for those students who have been alienated by traditional approaches to mathematics education.

The aim of the study reported in this article was to explore the effectiveness of scaffolding in promoting the engagement of disengaged students in a secondary mathematics classroom. No suitable definition of "disengaged students" is provided in the literature (McGinty & Brader, 2005). The term *disengaged* as used in this article is to be taken as meaning students who experience multiple disadvantages (e.g., economic, physical, cultural, social) in the learning of mathematics. It also includes students who choose to limit their participation in school mathematics—a choice may be made to raise their status with peers (Lundy & Firebaugh, 2005), making students reluctant to change their ways of relating in school for the purpose of achieving in mathematics (César, 2009).

The form of scaffolding used in the study was that provided by collective argumentation. Thus, the aim of the study was examined through investigating the following research question: "How does collective argumentation promote student engagement by scaffolding participation in a secondary mathematics classroom, where learners are predominately disengaged with mathematics?" This question was investigated within a framework of scaffolded learning as suggested by Goos (2004) and, as such, explored the following elaborations of mathematical understanding as they were enacted in a Year 9 classroom that utilised collective argumentation on a regular basis, with an emphasis on sense making, reflection and self-monitoring, and clarifying, elaborating, and justifying assertions.

Method

The study reported in this article was an action research project (Kemmis & McTaggart, 1988) implemented for the purpose of assisting the classroom teacher (the first author of this paper) to change student forms of engagement with mathematics. The teacher had been employing the strategies of collective argumentation as part of her teaching of mathematics for one year. This project involved a Year 9 class (13-14 year-old students; 5 females and 22 males) of a metropolitan state high school set in a lower socio-economic area in Australia. This project was part of a larger study involving a group of teachers from a number of different schools using collective argumentation in their classrooms to teach mathematics (Brown & Renshaw, 2008).

The project reported here followed one class of students and their teacher as they used collective argumentation in the mathematics classroom over the year, and in particular includes a two-week unit of work on indices that was taught three quarters of the way through the school year. The unit involved the rules for manipulation of indices, scientific notation and applications of these. This was a purely mathematical unit centred mainly on revision; however the students within this class, according to their classroom teacher, lacked the mathematical competence to engage with the mathematics being presented. As such, the teacher, in response to the students' request, chose to use collective argumentation for all lessons associated with this unit of work, a planned outcome of which was to achieve greater academic engagement and participation by students in this class.

Data Collection

Student work samples were kept, and students kept journals where they were asked to respond to a number of questions, as seen in Figure 1, after each class.

Collective Argumentation Journal Entries

Date:

Subject:

What did you do in today's lesson?
Why did you do it that way?
Today I worked with …? Why?
Who or what helped you the most in today's lesson?
Where did your ideas come from in today's lesson?
What did you enjoy about today's lesson? Why?
What didn't you enjoy about today's lesson?
What new thing/s do you know or can do after today's lesson?
How do you feel you worked during today's lesson? Why?
What do you think you could do better next lesson?

Figure 1. The stimulus questions for the students' journal writing.

As can been seen in Figure 1, all ten questions were open in nature with the first three questions being designed to elicit students' perceptions of the nature of the activity engaged in (content, motivation, and partners). Questions 4 and 5 were designed to elicit students' perceptions of the resources (human and cognitive) that they may have accessed during the lesson. Affective perceptions of the session were sought through Questions 6, 7, and 9, and perceptions of learning and effort were sought through Questions 8, 9, and 10.

It must be noted that the journal questions addressed each student as an individual and that no bias is evident in the questions regarding the practices or processes that the teacher preferred the students to engage. The teacher kept a journal to reflect on each lesson. This journal required the teacher to record (a) the teaching aims associated with each mathematics lesson where collective argumentation was used; (b) the mathematical problems/tasks presented to the students during each of these lessons; (c) appropriate syllabus/source book references associated with the content of the presented problems; (d) observations of teacher-student/student-student interactions at the group and whole class level; and (e) a summary of the group explanations/presentations and whole class discussions that accompanied such presentations.

Teacher and student journal entries were the main form of data collection in this study because the journal questions required the teacher and students to provide a report on their practices. These reports also provided the teachers involved in the larger study with a means to discuss and reflect upon their own practice when they came together to share their experiences of teaching mathematics using Collective Argumentation. These meetings occurred four times a year over the course of the larger three year study and were often video and audio-taped.

Lesson Context

Collective argumentation was introduced to these students early in the year, initially by discussing the steps involved. An A3 poster of the principles of collective argumentation was displayed prominently in the classroom. The students were given problems or questions to solve, and the main focus was for the groups of students to discuss their solution or interpretation of the problem or question, and to reach consensus about a solution so that they produced a mutually agreed upon response to share with the class. Initially questions/tasks were designed to stimulate mathematical discussion for students who were not familiar with discussing mathematics and having to explain their thinking. These tasks included**:**

- Place the fractions a half ($\frac{1}{2}$), a third ($\frac{1}{3}$), and a quarter ($\frac{1}{4}$) on a number line. Justify your placement.
- Multiplication makes things bigger and division makes things smaller. Do you agree? Why / Why not?

After a group had presented their solution to the class, the students invited the class to ask them questions, thereby putting their ideas up for discussion. The teacher asked questions and rephrased statements to clarify student claims and to ensure that the response was truly a collaboratively built group response and not just the work of a few.

After students achieved an understanding of the desired outcomes of building mathematical understanding of the problems/questions through discussion, more emphasis was placed on the initial stage of *representing* as being undertaken individually. The students were asked to represent individually so as to give them time to consider the task. The student group reported in this article found this part of the process challenging, as their initial reaction when placed under pressure was to talk and chat about anything not relevant to the problem or question that they had been given.

These students also found the discussion about values (the norms such as those shown in Table 2) difficult and reduced their values to one word, "Respect". They felt that this included respect for ideas—both their own and those of others; respect for people—both themselves in having the courage to have a go, as well as for others in that they needed to listen quietly and take turns and to challenge the ideas of others but not to attack the person presenting the idea.

Learning Tasks

Data from the following learning tasks are examined in this article. These tasks were spread throughout the year and used to encourage the students to think about specific mathematical concepts.

1. The area of a trapezium. Following developing the formulas for area of a rectangle, triangle, and parallelogram using grid paper, students were asked to develop the formula for the area of a trapezium. The aim of this task was for students to determine the formula for themselves and to give them confidence in being able to work things out for themselves.

2. Drawing a concept map for length, area and volume. At the completion of a unit on perimeter, area, and volume of numerous shapes, students were asked to draw a concept map to represent their understanding of the concepts. The aim of this was to determine the connections that the students had made between these concepts.

3. Exploring indices. The indices unit began with a chessboard problem. One grain of rice is placed on the first square of the chessboard. Two grains are placed on the second square and four grains on the third square. If this pattern is continued how many grains will there be on the sixty-fourth square and how many grains will there be on the chessboard in total?

The unit of work reported in this paper involved students developing rules for the manipulation of indices using the scaffolding strategies of collective argumentation. The students were asked to come up with rules, for example $a^m \times a^n =$ __.

To assess students' understanding and to determine whether they were able to apply their rules in the context of scientific notation they were asked, "How many drips are in the Hinze Dam?" (a large, nearby weir) as an open-ended investigation. During the time of this research, large parts of Australia were in drought, including the region in which the school was situated, and this investigation built on ideas the students were studying in Studies of Society and Environment (a combined History and Geography subject that often studied local issues) and in the 'Waterwise' media campaign that was prominent on television networks due to low dam levels being experienced in the local area.

Data Analysis

An interpretative data analysis (Stringer, 2008) was used to identify significant experiences and events from the teacher's journal. These were deconstructed to reveal the features and elements that comprise them. The corresponding entries in the students' journals were analysed to determine their individual accounts. These were brought together in a collective account (Stringer, 2008) for each experience or event. The criteria in Goos' (2004) framework were used to support the analysis of the students' journals.

Findings and Discussion

In this section, we discuss they key findings in terms of sense making; clarifying; elaborating and justifying; reflection and self-regulation; and promoting student engagement.

Sense Making

If the students are to make sense of the mathematics they need to consider the mathematics and ask the questions they need answered. Goos (2004, p. 269) expressed this form of mathematical thinking as being, "an act of sense-making (that) rests on the processes of specialising, generalising, conjecturing, and convincing." So, when teaching mathematics, it becomes necessary initially to develop a student's ability to think mathematically. Consequently, the teacher in this study was attempting to scaffold her students' ability to think and participate in mathematical discussions so as to make sense of the mathematics. This is shown by a comment in the teacher's journal: "I am still struggling trying to encourage/teach/get students to think" (4th month of school year).

This desire to develop the students' ability to think mathematically was often a motivating factor for this teacher to use collective argumentation, where the teacher first required the students to think about the problem, think about the mathematics, and produce an individual response to the task. The students were then required to engage in small group discussions, initially to share their individual responses but, as they justified and argued their different points of view about a task, to build a group response. This co-construction was promoting a goal of this teacher, namely, to have the students talk for understanding (Lampert, 1990).

The students needed to be encouraged to think mathematically. An example of this occurred when students were asked to consider the statement "Multiplication makes things bigger and division makes things smaller". This statement was given after students had completed a unit on trigonometry that had included use of ratios, decimals, and fractions. After each group in the class had presented their results and engaged in the ensuing discussions, the teacher was not satisfied with the depth of thought or meaning given by the class to the questions asked by herself and by students. So the class was asked to extend their group responses to show they had made sense of the mathematics. This purpose is demonstrated in the following journal extract.

> Students agreed the statement was false but would only come up with one justification even when pushed. ... So after all presentations I asked students to think a bit more about the statement and write/add to their group decision. (Teacher journal, 4th month of school year).

One group's response, Jack's, was shared with the class (see Figure 2).

This is false because
multiplication doesn't always
make it bigger, e.g.

```
     5
 x 0.5
   2.5
```

So it gets smaller.

Division doesn't always make
a number smaller, e.g.

```
    10
5 ) 0.5
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So it becomes bigger.

Figure 2. Jack's group's initial response.

In their initial presentation, Jack's group had evaluated 5 × 0.5 and 5 ÷ 0.5, identifying only one example of multiplication making the number smaller and only one example of division making the number bigger. However, Figure 3 shows Jack's reflections about this presentation after they took note of the teacher's request.

Not always true	eg
less than one (0 ⟷ 1)	0.5 × 20 = 10 smaller
negative numbers (one for ×)	10 × -5 = -50 smaller
both negative for ÷	-10 ÷ -5 = 2 bigger
multiply by 0 if not negative	3 × 0 = 0 smaller
times one = the same	20 × 1 = 20 same

Figure 3. Jack's group's journal response after further prompting.

In the reflection, Jack identified: (a) the range of fractional values between 0 and 1, (b) that if one of the numbers is negative and one is positive then the product may be less, (c) that if both the numbers are negative then the quotient will be larger, and (d) multiplying a positive number by zero will make it smaller and a negative number by zero will make it bigger. He also considered that multiplication by 1 results in the number remaining unchanged. Jack acknowledged his additional thinking with a comment in his journal in response to the question, "What new thing/s do you know or can do after today's lesson?" His written response was "Think harder more often".

Thinking harder more often, as voiced by Jack, was echoed by other students in this class in journal reflections for this particular task. For Jack, as for others, it signalled the potential for change to occur in their approach to knowing and doing school mathematics—a change that would not occur randomly. For disengaged students, this would have to be a deliberate choice because they would need to value improvement in mathematics over popularity with their peers (César, 2009). However, the students in this class, as illustrated by Jack's group's response, were prepared to "think harder" so as to make sense of the mathematics (Goos, 2004); to re-think their representation of a mathematical task. Re-presentation is an important function of engaging in collective argumentation where students are required in a group situation to generate, compare, and discuss multiple representations of a mathematical task (Brown & Renshaw, 2006).

Over the following weeks, the teacher continued to work with small groups of students in this class to scaffold their thinking. Some students took longer than others to develop their mathematical thinking. For example, when the students were developing concept maps, in response to reporting "Any moment which challenged or extended a student mathematically?" the teacher wrote in her journal, "... discussions with some students about the connections, e.g., how the formulas build on each other e.g., square $A = s^2$, cube $A = A_{base} \times h = s^3$ (6th month). By making this statement, the teacher was indicating the lack of building on previous learning by students to be challenging. The teacher journal entry continued, "These students are very much stuck in ruts. They are used to thinking about maths in one way and it is very difficult to change their thinking." The students were resisting the teacher's desire to prompt their deeper thinking of the tasks with students' comments reported in their journals such as, "Why can't we just learn the formulas? Why do we have to think? What is the purpose?"

The teacher wanted these students to think and make sense of the mathematics through problem solving and investigations (Boaler, 2008). For students to focus on making sense of the mathematics they need to conjecture, generalise, and convince (Goos, 2004). The teacher needed to select problems/questions that encouraged students to make sense of the mathematics. Students in an inquiry task oriented classroom need to explore and to take ownership of the problems and to critique and question classmates' responses to problem tasks. The teacher in an inquiry classroom will ask questions to encourage students to consider their assumptions upon which their responses are based and to locate errors. If students are to make sense of their mathematics they need to be able to publicly admit confusion and to ask questions to aid understanding (Goos, 2004).

This ownership and teacher practice was evidenced by the class in this study towards the end of the year when a unit on indices was commenced with an investigation of the chessboard problem. The reason for using the chessboard activity was that it **"...** introduced the idea of why you might want to use indices" (teacher journal, 9th month of school year). The aim of the lesson was to give the students a reason to use index notation by providing them with a context for the mathematics and to avoid the student comment, "When are we ever going to use this (mathematics)?" The students were not told to use collective argumentation but at the commencement of the task, a number of students turned to their classmates and said, "Shh! We have to do (represent) it ourselves first". This was an important moment in these students' engagement with mathematics because it demonstrated that they were taking ownership of 'doing' the mathematics by choosing to use collective argumentation without being instructed to do so by the teacher.

The advantages of beginning the unit with an investigation were that it reminded students what indices are, and enabled the teacher to listen to the students during group discussions so as to assess the students' knowledge and understanding of indices. It also provided a platform on which to build new knowledge. For example, in a lesson embedded in this unit students were asked to explain the rule for $a^m \times a^n =$? and $a^m \div a^n =$?. When the students were unable to begin an explanation, the teacher scaffolded their thinking by suggesting that they substitute numbers for the letters and look for patterns.

The presentations arising from this task were organised by the teacher so that the simpler representations of the task were presented first, with subsequent presentations building on these representations. The teacher observed that often after an initial presentation students would volunteer to show their work if they considered that they had taken their ideas to a higher level of sophistication than the initial presenting group. This showing of ideas became the stimulus for discussion, and during these discussions students were able to support each other and to show where errors were being made.

For example, the students needed to make connections with prior knowledge to evaluate $3^2 \times 3^4$ and $3^4 \div 3^2$ and other numerical combinations, and to look for the patterns leading to a generalisation. When attempting this task, although all groups of students attempted a number of different numerical combinations to determine a pattern, none had shown the class their patterns by identifying more than one example to demonstrate how their rule worked.

For example, the first group to present (see Figure 4) had chosen to present one particular example for displaying the rules $a^m \times a^n =$? and $a^m \div a^n =$?. As a consequence, the group had not been able to represent their work in a fashion that would allow them to see a pattern. In addition, errors in the group's order of operation precluded their provision of an acceptable solution. (They had made a mistake with the order of operations when evaluating the division example having not put a bracket around 3^2 when they had expanded it.) During their presentation other students questioned them as to why they had different answers.

After the discussion, the students—whose work is illustrated in Figure 4—acknowledged that their calculation was incorrect, as instead of dividing by 3^2 they had divided by 3 and then multiplied by 3.

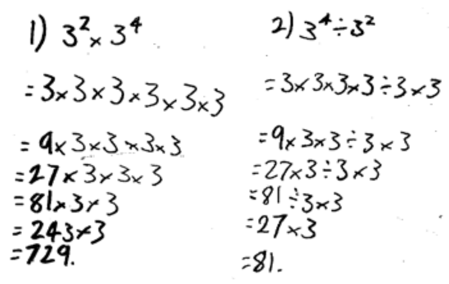

Figure 4. The solution by the first group.

The second group to present offered to show their results (see Figure 5) to the class, as they considered that they had found one of the rules. They had written their equation in index form, having been given it in index form. This allowed them to identify a pattern for multiplying exponential numbers with a common base as "you add the powers together".

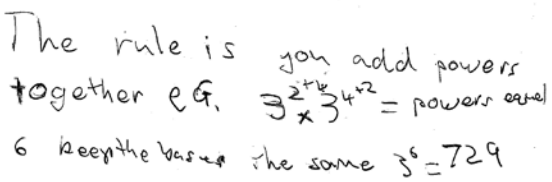

Figure 5. The solution by the second group.

The third group had justified their rules by showing the addition of indices for the multiplication case and the subtraction of indices for the division case. These generalisations needed to be written in a symbolic form and they too did not write a generalised form of the rule so further discussion was encouraged by the teacher to determine these rules. That is, the students were specifically asked by the teacher to complete $a^m \times a^n =$? and $a^m \div a^n =$?. With this assistance the third group was able to write their response in the symbolic form and added the two mutually agreed upon rules into an ellipse drawn on their overhead (see Figure 6). All students then recorded these rules in their books for future reference.

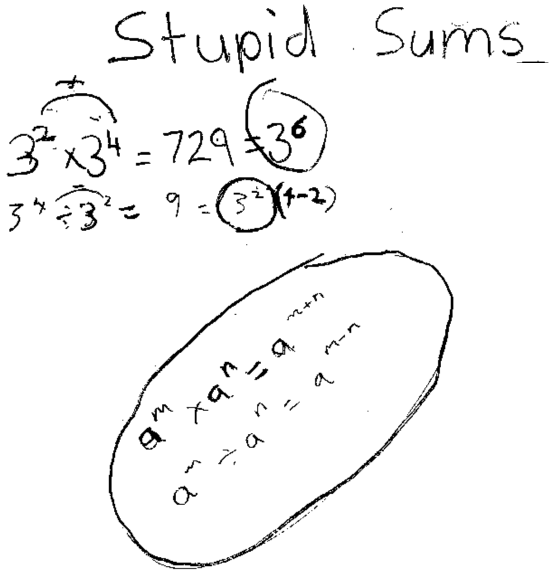

Figure 6. The solution by the third group.

The significant point raised by summarising the above group presentations is that the students were the ones engaging in discussion and questioning to make sense of the mathematics. In so doing, the students were coming to the belief that success in mathematics follows attempts to 'make sense of things' rather than attempts to memorise the teacher's method of solution (Cobb, Wood & Yackel, 1993). In this case, students were sharing not only their solutions but their understandings (Cobb, Wood & Yackel, 1993; Solomon, 2007) thus enabling them to see errors and suitably adapt their thinking.

On other occasions, the students acknowledged that making sense of the mathematics was challenging and time consuming. Examples of this thinking were provided in student responses to the journal question: "Why did you do it (the mathematics task) that way?" after students had been exploring the area of a trapezium (5th month of the school year). For example, John wrote, "Because Miss Y believes it is the best way to learn" and Annie wrote, "Because that is the way I understood it best (eventually) … I worked average, because it took me a while to understand".

Clarifying, Elaborating, and Justifying

As the year progressed, the students became more confident with working together collaboratively and engaging in mathematical discussions where they were clarified, elaborated on, and justified their assertions. The presentations provided the students with the opportunity to explain their ideas to their peers as they made sense of the mathematics.

> Working in collaborative peer groups, students have the opportunity to own the ideas they are constructing and to experience themselves and their partners as active participants in creating personal mathematical insights. (Goos, 2004, p. 263).

The students enjoyed presenting to the class and enjoyed the lively discussions that came with the presentations. In response to the questions "What did you enjoy about today's lesson? Why?" Brett repeatedly wrote, "Using the overhead projector and talking to the class" (4th month, 7th month, 10th month, 10th month of school year).

Warren's journal entries expressed enjoying the collaborative nature of the work, as witnessed in three journal entries: "We worked in a group and solved the problem together" (4th month of school year); "We got to listen to other people's ideas" (7th month of school year); and "I enjoyed presenting to the class" (10th month of school year). When students collaborate, they build ideas together and they need to explain their thinking to the others. Explaining allows group members to clarify their thinking, but to take ideas forward students need to elaborate and build on ideas: "Students take increasing responsibility for suggesting strategic steps and making links to prior knowledge … [Students] begin to offer conjectures and justification without the teacher's prompting" (Goos, 2004, p. 269). As affirmed by Goos, this is an essential part of the pedagogy of a teacher who strongly believes in establishing a classroom community of inquiry that enables students to make sense of the mathematics.

> [H]e demonstrated through a commitment to personal sense-making and by his willingness to deal with more abstract ideas concerning conjecture, justification, and proof. Typically, he modelled the process of inquiry by presenting the students with a significant problem designed to engage them with a new mathematical concept, eliciting their initial conjectures about the concepts, withholding his own judgement to maintain an authentic state of uncertainty regarding the validity of these conjectures, and orchestrating discussion or presenting further problems that would assist students to test their conjectures and justify their thinking to others. (Goos, 2004, p. 282)

During the validation stage of collective argumentation, the students in this study justified their decisions and made changes appropriately. During the whole class discussions of a group presentation, they were challenging the thinking of others, asking them to clarify and elaborate ideas whilst defending their own. This was evidenced in the presentation represented in Figure 4 when students in the class challenged the presenting group when they did not agree with the answer for $3^4 \div 3^2$ and hence their thinking was different.

The teacher in this class helped students to clarify, elaborate and justify their thinking with questions such as "Why did you do it that way? Can you explain …?" These questions were used to keep the students on task and to stimulate further discussion and thought. This is important, as noted by Goos (2004), "Yet the teacher did not abandon the students to work alone: on the contrary he saw it as one of his responsibilities as modelling and scaffolding mathematical thinking" (p. 270).

Because the students in this class also needed to publicly validate their work, they had to produce a suitable solution to the question and justify ideas they discussed with the class. This public validation helped to develop a taken-as-shared understanding (Yackel and Cobb, 1996) in which, through the discussion, the students built their personal understanding. Teacher questions such as, "Why did you chose that solution?" reinforced the need for the group to understand not only their solution but also all of their solution method.

The teacher used questions to scaffold student thinking. For example when students were developing a rule for $a^m \times a^n =$?, the teacher suggested the students insert some numbers and look for a pattern. When the students had not found a general rule, she reminded them of the initial question and asked whether someone could determine the rule. By using collective argumentation, the teacher delivered to these students a scaffolding format that encouraged them to participate in discussions and to develop taken-as-shared understandings (Yackel & Cobb, 1996) in the classroom. It was during these discussions that students had the opportunity to conjecture and to justify their thinking to others. Students also became comfortable about asking to use others' ideas. These discussions between the students, as well as the role of the teacher, were highlighted in the transcript of the teacher talking at a professional development day:

> The whole class would then discuss and decide, "Okay that's going to be the answer, the rule that we are all going to write in the section of our books". So one of the best things with [the class discussions] was the substantiative conversation, because they really got into trying to come up with it and talking about it; and, yes, I was questioning them. "Why have you done it that way?" [For example $3^4 \times 3^2$] "Can you write it a different way?" Because they had, they could tell that you had to add indices, [that is $3^4 \times 3^2 = 3^{4+2} = 3^6$] but they had great trouble writing it mathematically. So just that questioning and getting them to come up with the answers, actually probably did more. Because they don't like it when I just stand up the front and talk.

Reflection and self-regulation

The ability to plan, monitor, and adjust our thought processes as we construct knowledge is necessary for successful learning (McInerney & McInerney, 2010). As Goos (2004) advised, "The processes of mathematical inquiry are accompanied by habits of individual reflection and self-monitoring. The teacher asks questions that encourage students to question their assumptions and locate their errors" (p. 269). Teachers need to support students as they plan and reflect on their learning. Within the classroom that is the focus of this article, directions and questions of the form, "Explain why you have done it this way. Is that the only way you could answer the question? Is there another way of doing it?" were posed to encourage the development of student reflection. Evidence of these types of discussions was displayed particularly during the validation stage of collective argumentation. For example on their own, the groups of students were unable to develop the rules for the index laws using symbolism. However with teacher support through the asking of questions and through requests for explanations as to whether their way was the only way to answer the question and by asking students to explicitly complete $a^m \times a^n =$?, the groups of students were able to determine the conventional rules.

The teacher's use of collective argumentation also encouraged students to reflect on their learning. When students present their work to the class they open up their ideas for discussion. The ensuing questions from fellow students and the teacher forced them to reflect on their thinking, evaluate it and perhaps make changes. This also happens in the group discussions. In fact, Goos (2004) discussed how the teacher in her study helped his students to develop the ability to self-monitor within the classroom community of inquiry when the teacher "scaffolded the students' learning by providing a predictable structure for inquiry through which he enacted his expectations regarding sense-making, ownership, self-monitoring, and justification" (p. 282).

Does Collective Argumentation Promote Student Engagement?

By establishing a community of inquiry using the components of mathematical thinking as suggested by Goos (2004), student engagement with mathematics appears to have been promoted by this teacher's use of collective argumentation. In the fourth month, Jack voiced the desire to 'think harder more often', a desire which was expressed by other students too. This signified the potential for change by members of this class. By the ninth month, students were asking to use collective argumentation as this gave them a space to think and to engage in mathematical discussions, initially within small groups where they developed a group response and then within whole class discussions. This sharing and talking about mathematics gave students opportunities to make sense of the mathematics and to develop shared understandings. After students shared their response with the class, the ensuing discussion encouraged metacognition through students needing to explain and justify their thinking and to refine and extend or change their thinking.

In particular, the use of collective argumentation in this classroom appeared to lead to a productive approach to the students volunteering to present work and to open their work for all members of the class to discuss whilst working towards a class solution in a manner that may "close with a consensus" (Cobb, Wood & Yackel, 1993, p. 104). As the teacher reflected at the end of the school year:

> It had given them a strategy that allowed them to feel comfortable doing mathematics; a strategy where they felt comfortable taking a risk, where it was okay to make a conjecture and then after discussion revise their thinking, where they felt that they had a share in the outcome and that no one was going to tell them it had to be achieved in a certain way. It provided a strategy in which all students could work within a community of inquiry. (Teacher journal at the end of the school year)

By counting the number of positive, negative, and neutral responses in the students' journals it was possible to gain a sense of student engagement. This was done by focussing on the following questions:

- What did you enjoy about today's lesson? Why?
- What new thing/s do you know or can do after today's lesson?
- How do you feel you worked during today's lesson? Why?

The numerical analysis showed that 81% of the journal entries were positive, 2% were negative and 17% were either neutral or did not include enough information to make a judgement.

Conclusion

The students in this study were initially disengaged with mathematics. They were reluctant participants in class and either worked with the idea in mind to finish as quickly as possible with the minimum amount of effort or with taking so long to start a task that they would run out of time before doing anything substantial. At the beginning of the year, these students believed that mathematics was about learning and using formulae and methods from the textbook. The teacher motivation for using collective argumentation was to encourage the students to think and make sense of the mathematics (Schoenfeld, 1992; Goos, 2004).

A community of inquiry (Goos, 2004) was established in this Year Nine mathematics classroom where students were expected to engage in mathematical discussions to make sense of the mathematics and to develop a taken-as-shared understanding (Yackel & Cobb, 1996). In doing this, students were expected to clarify, elaborate, and justify their assertions. The teacher worked with the students so that they could communicate using conventional mathematical language and symbols. Collective argumentation was found to provide a structured framework that encouraged this. The students were provided with a supportive environment that allowed them to build on prior experiences, and which gave them opportunities to discuss their ideas and to think and justify the choices they had made. They were provided with meaningful problems, issues, statements, questions or tasks to investigate so that they could make conjectures to test, discuss, and refine. The questions asked during the lessons were designed to encourage students to think mathematically so as to make sense of the mathematics. There was the expectation that they reflect on their ideas and make changes to their thinking as necessary. During the discussions, when students were validating, they needed to be actively thinking so as to respond to questions and statements.

In this study, by requiring the students and teacher to work collaboratively, collective argumentation provided the students in this classroom a scaffolding framework. It allowed them to participate in mathematical discussions where personal understandings could be expressed, re-considered, shared and co-authored. Collective argumentation provided a structured framework for students to work collaboratively. As students worked through the steps, represented, compared, explained, justified, and validated, they became involved in mathematical discussions that appeared to help them to develop understandings and work mathematically.

Students found it empowering to be able to present their work and to have the opportunity to justify their choices in an environment where everyone was valued. The previously disengaged students preferred this method over conventional approaches to teaching and learning mathematics. This has significant implications for teachers disillusioned by traditional teaching methods who need to re-engage their students.

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Author

Margaret Marshman
University of the Sunshine Coast, Locked Bag 4, Maroochydore DC, Qld, 4558.
Email: mmarshma@usc.edu.au

Raymond Brown
Griffith University, 176 Messines Ridge Road, Mt Gravatt, Qld, 4122
Email: ray.brown@griffith.edu.au