ACTA DIDACTICA NAPOCENSIA
Volume 2, Number 3, 2009

RESEARCH ON HOW SECONDARY SCHOOL PUPILS DO GEOMETRICAL CONSTRUCTIONS

Iuliana Marchis, Andrea Éva Molnár

Abstract. Communicating on the mathematical language, problem solving, and reasoning are competencies tested on international test. The aim of this research is to study how secondary school pupils do geometrical constructions, how they give mathematical argumentation and use geometrical notions in their explanations.

Key words: Mathematics Education, Mathematics problem, geometrical constructions

1. Introduction

Two of the core competencies in Romanian secondary school curricula are [4]:

- development of the capacity to communicate on the mathematical language;
- development of the ability of problem solving.

The TIMSS (Trends in International Mathematics and Science Study) international test also focuses on these competencies. Between the competencies evaluated in TIMSS 1995 and TIMSS-R 1999 we can find the following two [5]:

- communicating in the language of Mathematics;
- solving problems.

At TIMSS 2003 using concepts, develop explanations, and solve problems are tested too [1]. At TIMSS 2007 solving problems and reasoning had an important place [2].

Thus our research focuses on the following three competencies: solving problems, reasoning, and communicating on the mathematical language. To study this we have chosen, as mathematical content, the geometrical constructions.

For geometrical reasoning arguments based on mathematical properties are required. The axiomatic structure of the Euclidian geometry constitutes a favorable ground for learning reasoning and argumentation.

In [3] the research focuses on how preservice teachers do geometrical constructions, wether they use elements or geometrical properties. The conclusion is that a large proportion of preservice teachers use visual elements and they are trying to use naïve empirical reasoning instead of geometrical one.

The aim of this paper is to study how secondary school pupils do geometrical constructions, how they give mathematical argumentation and use geometrical notions in their explanations.

2. Geometrical constructions with compass and straightedge

We gave the task to the pupils to conctruct an equaliteral triangle, a segment's multiple, and a segment's perpendicular bisector using only compass and straightedge. The straightedge is assumed to be infinite in length, has no markings on it and only one edge. In this section we give the methods of making these three geometrical constructions.

Received 7 September 2009, accepted 25 September 2009, published 30 September 2009

2.1. Construction of an equilateral triangle

The construction has the following steps: construct a segment AB (this will be one of the triangle's sides); draw a circle whose center is A and radius is AB; draw a circle whose center is B and the radius is AB; set C as one of the intersaction point of the two circles; draw the segments AC and BC (see Figure 1).

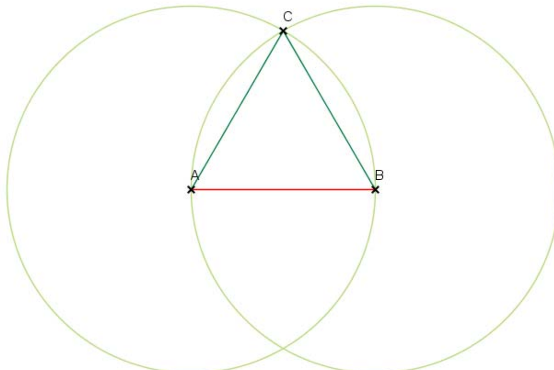

Figure 1. Construction of an equilateral triangle

2.2. Construction of a segment's multiple

The essence of the construction is, that only with the help of a compass we can construct a segment's multiple. After we draw the segment AB, we take into the slot of the compass the segment's endpoints, then we distribute it so many times beside the segment as many times it is necessary (in our case two times). So we obtain the requested segment AB', which is three times longer than the original segment AB (Figure 2).

Figure 2. Construction of a segment's multiple (|AB'|=|AB|)

2.3. Construction of a perpendicular bisector of a segment

In this construction method first we draw the segment AB, for which we want to construct its perpendicular bisector. Then with the help of a compass we draw two arcs with equal radius, whose centers are the segment's endpoints. The line determined by the points of these arc's intersection is the perpendicular bisector of the segment AB (Figure 3).

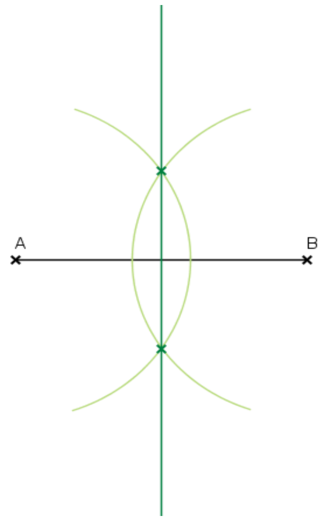

Figure 3. Construction of a perpendicular bisector

3. Research methodology

Two schools have participated in this research, in total 26 sixth grade (11-12 years old) pupils. The pupils have completed the test. They had to do three constructions:

- an equilateral triangle;
- a segment's multiple;
- perpendicular bisector of a segment;

They were asked to describe in words how they have done the geometrical constructions.

4. Analyzing the pupil's work on the test

In this section we analyze the work done by the pupils in the three mathematical construction.

2.1. Construction of an equilateral triangle

Analizing pupils' work, we have identified three approaches to solve the problem: to draw a triangle which seems to be equilateral (visual construction), to draw the equilateral triangle by measuring the sides (measurement), and to construct the equilateral triangle. Table 1 contains the number of pupils using this three type of approach: 8 pupils used visual construction, 4 pupils measurement, and 14 pupils mathematical construction.

Table 1. Results of the construction of an equilateral triangle

	Correctness of the drawing	
Method of the construction	**Correct**	**Incorrect**
Visual construction	0	8
Measurement	0	4
Attempt of mathematical construction	6	1
Mathematical construction	0	7

It is interesting that all the pupils who used visual construction or measurement have drawn not an equilateral triangle. In Figure 4 we see the construction given by a pupil by measuring the length of the edges. We observe, that only two edges have equal length, thus we suppose that the pupil constructed an *isosceles triangle* instead of an *equilateral triangle.*

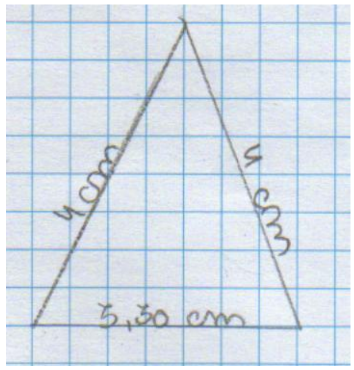

Figure 4. Construction for equilateral triangle given by a pupil

We have identified two clusters for that works which used geometrical construction:

The first type of construction (attempt of mathematical construction) have the following idea: draw the horizontal segment AB, find the midpoint of it (by measuring or visually), draw a perpendicular line through this middlepoint to AB (using a perpendicular ruler or visually), find the third vertex of the triangle on this line (measuring the edge or visually). This construction doesn't satisfy the requirements of the task given to the pupils.

The second type of construction is that one given in Section 2.1.

In the following we analyze some explanations of the pupils.

"I have taken a line and I have found visually the middle point of it, then I have drawn the edges."

Commnets: The pupil writes *line* instead of *segment*. The line can't have middle point. He didn't say, how the middle point of the fisrt edge helps him to draw the edges.

"I made the triangle in the following way: I looked on the cubes and draw the lines."

Comments: Pupils use mathematics copybooks on the lessons. Ususally, instead of the notion *square* they use *cube,* so their copybook is with cubes. So the pupil, who wrotes the above explanation, probably tried to orientate looking on the little squares in the copybook.

I have taken a line, I have found the middle point of it, I have drawn a line from top to bottom, then I have joined the line with the perpendicular line."

Comments: The pupil uses the notion *line* instead of segment and *line from top to bottom* probably for a vertical line. We can see, that this pupil can't use properly the mathematical notions and can't explain the construction done.

"I measured 5 cm on the ruler and I drew the base. I marked the middle point of the base and I drew a line with the same length until I met in a point."

Comments: This pupil uses the first type of construction described above, but can't communicate properly on mathematical language.

"I draw a 4 cm long line, I divide it in 2, going to up at 4 cm I join it with with two endpoint of the line."

Comments: Again we can see the notion *line* instead of *segment.* In the pupils view, a line could have middlepoints and endpoints. The 4 cm measured *"going to up"* is the height of the triangle. In an equilateral triangle the height is not equal with the edge of the triangle, so the pupil made an important geometrical mistake in his/her construction.

2.2. Construction of a segment's multiple

Aparently, this is the simplest construction from the test. Pupils had to construct a segment, which is three times longer, than a given segment.

Three pupils had an interesting construction, they have drawn a circle and take four diameters – see Figure 5. We couldn't find out, what they wanted to construct. Another strange construction is given in Figure 6.

Six pupils have drawn one segment, measure it then measure another two times that segment. Two pupils used the same method, but instead of measuring two times, they measured three respectively four times.

The number of pupils using different approaches for the construction is presented in Table 2.

Table 2. Results of the construction of a segment's multiple

Solution given	Number of pupils
Strange construction	6
Measurement	8
Mathematical construction	8
Didn't solve the problem	4

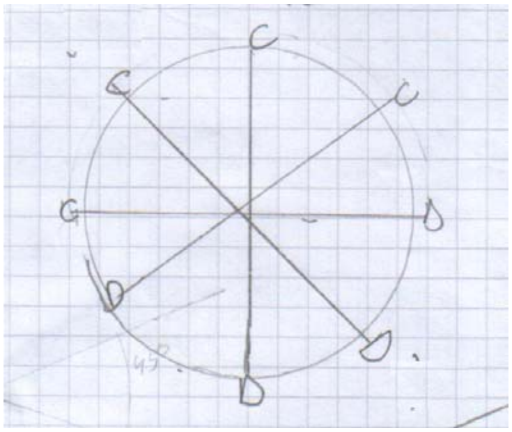

Figure 5. Construction of a segment's multiple given by a pupil

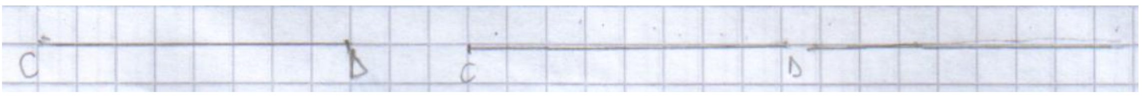

Figure 6. Construction of a segment's multiple given by a pupil

2.3. Construction of a perpendicular bisector of a segment

To construct the perpendicular bisector of a segment, pupils have to know that this is the set of those points, which have equal distance from the endpoints of the segment. In order to construct this line we need to find two points of it.

We have identified four type of solution for this problem.

There were 2 pupils who didn't know what is *perpendicular*, so they have constructed not a perpendicular line (Figure 7).

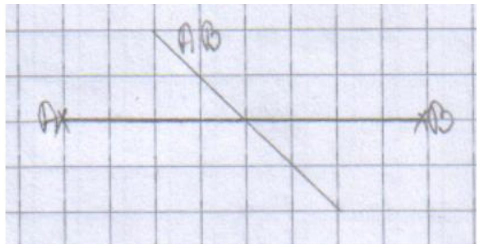

Figure 7. Construction of a segment's perpendicular bisector given by a pupil

Others have found the middle point of the segment (by measurement or visually) and have drawn a perpendiculat line to the segment through this point (visually or using a perpendicular ruler or following the lines in the copybook). An explanation given by a pupil:

I have taken a line, I have found the middle point of it, I have drawn a line from top to bottom, then I have joined the line with the perpendicular line."

Comments: The pupil uses the notion *line* instead of *segment*. Probably he/she has found the middle point by measurement, and drawn a "short segment" to mark this point. The question is, how he/she has drawn the perpendicular line.

There are few pupils, who have found the middle point and didn't draw the perpendicular line (Figure 8). The question is wether they didn't continue the construction or they thought that the *perpendicular bisector* of a segment is the *middle point* of the segment. Seems, that some of the pupils think that *perpendicular bisector"* is a point:

"I have drawn a 5 cm long line and measured the perpendicular bisector of it."

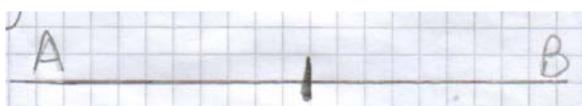

Figure 8. Construction of a segment's perpendicular bisector given by a pupil

Pupils in the third category used a compass to find one point of the perpendicular bisector then draw a perpendicular line to the segment through this point (using a perpendicular ruler or deciding visually that it is perpendicular) – see Figure 9. Observe, that the pupil hasn't drawn a line.

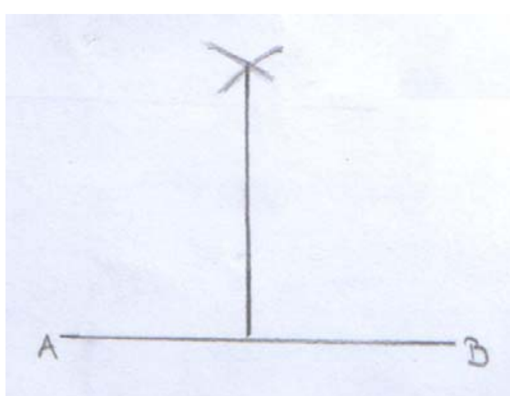

Figure 9. Construction of a segment's perpendicular bisector given by a pupil

In the last category there are those pupils who done a correct construction.

In Table 3 we have collected the above-described methods.

Table 3. Results of the constuction of a segment's perpendicular bisector

Way of solving the problem	Number of pupils
Drawing a non-perpendicular line	2
Finding the middlepoint of the segment	4
Constructing the perpendicular bisector by finding the middlepoint of the segment and drawing a perpendicular line to the segment through this point	17
Constructing one point of the perpendicular bisector using a compass then drawing a perpendicular line to the segment through this point	2
Correct construction	1

5. Conclusion

Geometrical construction problems seem to be difficult for sixth grade pupils. Those, who already learnt about these constructions, try to remember the algorithm; in many cases they don't do it correctly. See for example in the case of the perpendicular bisector: 17 pupils constructed only one point instead of two, so they remembered that they have to draw some arcs with the compass, but didn't do it correctly. Generally, pupils doesn't think about mathematical properties when doing geometrical constructions, they have only two approaches: remembering the algorithm or drawing (not constructing) the required "picture".

They don't use correctly the mathematical notions, for example use *line* instead of *segment*, *cube* instead of *square*.

Pupils don't know basic geometrical notions, as *perpendicular line*, *equilateral triangle*, *perpendicular bisector*.

Literature

[1] I.V.S. Mullis, M. O. Martin, T. A. Smith, R. A. Garden, K. D. Gregory, E. J. Gonzales, S. J. Chrostowski & K. M. O'Connor (2001). TIMSS Assessment Frameworks and Specifications 2003. International Study Center, Lynch School of Education, Boston College.

[2] I.V.S. Mullis, M.O. Martin, J.F. Olson, D.R. Berger, D. Milne, M.G. Stanco (2008). TIMSS 2007 Encyclopedia: A Guide to Mathematics and Science Education Around the World (Volumes 1 and 2), Edited by, Publisher: TIMSS & PIRLS International Study Center, Lynch School of Education, Boston College.

[3] M. S. Tapan, C. Arslan (2009). *Preservice teachers' use of spatiao-visual elements and their level of justification dealing with a geometrical construction problem*, US-China Education Review, vol. 6. no. 3, 54-60.

[4] CNC, 2003. Programa scolară revizuită de matematică. Clasele a VII-a si a VIII-a (Reviewed Mathematics syllabus for 7th and 8th grade). Bucuresti: MECT

[5] International Association for the Evaluation of Educational Achievement (IEA), 1997. Third International Mathematics and Science Study Technical Report Volume I: Design and

Development, Michael O. Martin, Dana L. Kelly (eds.) Publisher: Center for the Study of Testing, Evaluation, and Educational Policy, Boston College.http://timss.bc.edu/

Authors

Iuliana Marchis, Babes-Bolyai University, Cluj-Napoca, Romania, e-mail: marchis_julianna@yahoo.com

Andrea Éva Molnár, master student, Babes-Bolyai University, Cluj-Napoca, Romania, e-mail: mr_andi16@yahoo.com

Acknowledgement

The first author was partially supported by the research grant PNII – IDEI - ID_2418, financed by UEFISCSU.