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Developing a pedagogical problem solving view for mathematics teachers with two reflection programs

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Abstract

The study investigated the effects of two reflection support programs on elementary school mathematics teachers' pedagogical problem solving view. Sixty-two teachers participated in a professional development program. Thirty teachers were assigned to the self-questioning (S_Q) training and thirty two teachers were assigned to the reflection discourse (R_D) training. The S_Q program was based on the IMPROVE self-questioning approach which emphasizes systematic discussion along the phases of mathematical or pedagogical problem solving as student and teacher. The R_D program emphasized discussion of standard based teaching and learning principles. Findings indicated that systematic reflection support (S_Q) is effective for developing mathematics PCK, and strengthening metacognitive knowledge of mathematics teachers, more than reflection discourse (R_D). No differences were found between the groups in developing beliefs about teaching mathematics in using problem solving view.

Keywords: Elementary mathematics teachers; PCK; Metacognition; Reflection support; beliefs

Introduction

Standards of mathematics education pose great challenges for the preparation and Life Long Learning education of mathematics teachers (National Council of Teachers of Mathematics [NCTM], 2000; Program for International Student Assessment [PISA], 2003). In mathematics classrooms aligned with the vision of NCTM standards, teaching is focused on problem solving, mathematical reasoning and communication as part of a

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coherent curriculum. These goals maintain that teachers must cope with the complex dynamic process of teaching mathematics with pedagogical content knowledge (PCK) methods that challenge and guide students to gain mathematical ideas (e.g., NCTM, 2000). It is suggested (e.g., Hill, et. al., 2005; Putnam and Borko, 2000) that meaningful teaching should challenge students to shift toward student-centred learning that encourages knowledge construction through self-regulated learning (SRL). Students are self-regulated to the degree that they are active participants in their own learning process (Zimmerman, 2000).

Unfortunately, elementary school teachers continue enter the teaching field unprepared to teach mathematics in the way envisioned by the NCTM standards (Hill, et. al., 2005; Putnam & Borko, 2000). Most elementary school teachers have not experienced mathematics in this manner; they viewed mathematics in an instrumental way. They perceived mathematics as an unrelated set of facts, rules, and skills, to be used as required, rather than a process of reasoning and generalizing (Ernest, 1989). Studies indicate that these prior beliefs often serve as a lens through which the teachers view the new pedagogical knowledge being taught and the new processes of teaching and learning encountered. Therefore, it is essential that teacher educators consider these prior beliefs in teachers' professional education (Pajares, 1992; Kramarski & Michalsky, 2009; Richardson, 1996; Thompson, 1992).

One promising instructional support to develop a process view seems to be the use of reflection. Zimmerman (2000) argues that self-reflection has a central role in achieving self-regulation in learning. If students are to exert influence over their learning activities, they must be aware of these activities and they must reflect during their learning (Bandura, 1986). Our study investigates the effects of two reflection training programs on teachers' pedagogical view, by measuring: mathematical teachers' pedagogical content knowledge (PCK) in the context of problem solving, metacognitive knowledge, and beliefs. Prior to describing the present exploratory study's design, I present a brief overview of each of the variables utilized in this study.

PCK in mathematical problem solving context

Lester and Kehle (2003), characterize mathematical problem solving as an activity that involves the students' engagement in a variety of cognitive actions: include accessing and using previous knowledge and experience. Successful problem solving involves coordinating familiar representations and patterns of inference, and intuition in an effort to generate new representations. Evidence from empirical studies suggests that the development of process-oriented learning methods, which emphasize mathematical problem solving is the most difficult topic for elementary school students (Verschaffel, et. al, 2000).

Shulman (1986), described PCK as the way content, pedagogy, and knowledge of students are blended into understanding about how particular topics are taught, represented, and adapted to students' characteristics, interests, and abilities. In terms of PCK, teachers must to know how to teach their subject matter in a way that engages students extensively in tasks that require understanding. To achieve this objective, teachers must first learn to identify students' reasoning difficulties in the specific subject matter domain. Second, teachers must know how to plan lessons or didactic materials to explicitly treat these difficulties. Third, teachers need to know how to implement a curriculum that addresses higher-order understanding. Finally, teachers should know that successful implementation involves a considerable change in teachers' roles. The traditional teacher-centred role of acting as a "source of knowledge" should be replaced by student-centred learning by highlighting the role of initiating and coaching students' inquiries and problem solving (Zohar & Schwartzer, 2005). Such learning principles require self-knowledge and beliefs, motivation, goals, and strategy knowledge, and is indicative of self-regulation in learning (e.g., Pintrich, 2000; Schoenfeld, 1992; Schraw, et. al., 2006; Zimmerman, 2000).

Integrating SRL into PCK

Self-regulated learning involves a combination of using cognitive strategies, metacognition, and motivation. Researchers believe that the role of metacognition is especially important (Schraw et al., 2006; Zimmerman, 2000). Metacognition refers to knowledge and control of cognitive skills, and usually involves declarative knowledge and beliefs that refer to what the student knows or believes procedural knowledge that refers to how to use the knowledge, and conditional knowledge that refers to when to use it. Unlike the knowledge component, the control component refers to planning, monitoring, and evaluating learning toward the achievement of personal goals. According to Zohar and Schwartzer (2005), developing PCK requires using knowledge on both the cognitive and metacognitive levels. Knowledge of PCK on a cognitive level means that the teacher uses teaching strategies in the specific subject matter domain. Knowledge of PCK on a metacognitive level means that teachers reflect and verbalize processes of a particular pedagogical case; make generalizations about these processes (e.g., identify difficulties in using specific topic); and describe when, why, and how they use PCK.

Supporting reflection in SRL

According to Zimmerman (2000), reflection includes both judgment and reaction components. Judgment refers to students' ability to conduct introspection about their performance by looking back, whereas reaction refers to learners' ability to control and adjust their learning according to their conclusions by looking ahead. Reflection is particularly important for teachers' practice because it helps connect teachers planning to students needs (Schon, 1987). The use of reflection enables teachers to focus attention on their own planning, and on understanding the activities in which they engage during their learning and teaching (Davis, 2003; Kauffman et al., 2008; Nurckles, et. al., 2009). Although research has indicated the importance of reflection for SRL, students do not seem to implement such behaviours spontaneously.

Reflection support programs aim to increase learning competence by means of providing explicit guidance to students as they think and reflect on their tasks. An explicit approach incorporates the ability both to verbalize thinking patterns and to conceptualize and analyze relational structures that are employed while thinking (Veenman et al., 2006). This approach raises the question about the conditions required to support reflection ability in teacher training programs as teachers think and reflect on their tasks. Our study investigates two reflection support programs: Reflection discourse (R_D) and self-questioning (S_Q).

Reflection Discourse (R_D) vs. Self-Questioning (S_Q).

Research indicated the role of reflection discourse with comparable peers for making monitoring and regulation processes overt (Brown & Campione, 1994). Reflection discourse encourages students to share meanings, in order to achieve deeper metacognitive and subject matter understanding. Students must explain their own thinking to other group members and adapt their own thinking to the solutions proposed by other members, which, in turn, may facilitate more efficient use of metacognitive skills. Through critically examining others' reasoning and participating in disagreements' resolution, students learn to monitor their thinking, which in turn strengthens their mathematical reasoning concepts (e.g., Artz & Yaloz-Femia, 1999).

Although the group discourse has potential to develop students' reflection ability research findings indicate that students are often "cognitively overloaded" during the group process, experiencing difficulties in self-observation and reflection, in remembering what they did previously, and in documenting their thinking. This overload prevents from sharing their learning behaviours with other students (Clearly & Zimmerman, 2008; Kramarski & Mizrachi, 2006; Schon, 1987).

Many researchers have emphasized that self-reflection should be attained by systematic support that focuses on promoting learners understanding of the task, planning, monitoring and evaluating through the learning task process (e.g., Clearly & Zimmerman, 2008; Zimmerman, 2000).

Mevarech and Kramarski (1997), designed the IMPROVE metacognitive self-questioning method that represents the acronym of all classroom teaching steps: Introducing new concepts; Metacognitive questioning; Practicing in small groups; Reviewing; Obtaining mastery; Verification, and Enrichment and remediation. The metacognitive questioning encourages students to actively engage in self-regulating their learning by using four types of questions: Comprehension (e.g., 'What is the problem/task?'); Connection (e.g., 'What is the difference/similarity?'); Strategy (e.g., 'What is the strategy?; 'Why?') and; Reflection (e.g., 'Does the solution make sense?;' "Can the solution be presented otherwise?'). The IMPROVE method questions direct students' thoughts, actions and discourse throughout the SRL processes (Zimmerman, 2000) of planning (what, when, and how), monitoring and reflection (why). In general, research reported that IMPROVE self-questioning (S_Q) demonstrated positive effects on school students' learning outcomes and SRL processes (e.g., Kramarski, 2004; Kramarski, et. al., 2002; Kramarski & Mevarech, 2003; Kramarski & Zoldan, 2008). Recently the IMPROVE method was adapted for pre-service teachers (Kramarski & Michalsky, 2009; Kramarski & Michalsky, in press). However, minimal research exists in investigating such reflection support approaches in professional training programs of elementary school mathematics teachers.

Current study objectives

In the present study, teachers participated in one of two professional training programs (see a detailed description of each program in the Method section): either with the reflection discourse (R_D) or the IMPROVE self-questioning method (S_Q). The purpose of this study was twofold. We compared the effects of R_D versus S_Q on the teachers' pedagogical problem solving view as measured: (a) PCK regarding mathematical problem solving; and (b) metacognitive knowledge and beliefs regarding declarative, procedural, and conditional knowledge. We expected that systematic discussing with pedagogical issues with the IMPROVE method (S_Q) embedded within PCK, would help teachers become more actively engaged in comprehending the PCK of problem solving and more aware of metacognitive considerations for student-centred learning. Thus, we assumed the S_Q group will outperform the R_D group in PCK measures. We also assumed that the S_Q method enhances teachers' high level of perceived metacognitive knowledge (procedural and conditional) more than the R_D method.

Method

Sixty two elementary school teachers from 16 urban schools, participated in this study. These teachers participated in an Israeli government sponsored professional development program for three years. The purpose of the development program was to enhance teachers' mathematical knowledge and pedagogical methods with regard to NCTM standards in mathematics. Teachers were exposed to the Israeli Ministry of Education mathematical curricular: numbers and operations, data, algebra, proportion, space, and shapes. Teachers' knowledge was assessed each year in all professional development centres by uniform government tests based on the topics which were studied. In the beginning of the study there were no significant differences between the two groups in the following variables: Years of experience in teaching mathematics mathematical knowledge and PCK were assessed by government measures.

Professional training program

Shared structure and curriculum. Teachers in both groups (R_D and S_Q) participated in five weekly 4-hour workshops during five weeks (20 total hours of training). The aims were: (a) to strengthen teachers' mathematical knowledge for teaching; and (b) to practice pedagogical means for enhancing mathematical understanding (Hill et. al., 2005). Teachers studied the new curriculum standards for early childhood (problem solving, mathematical reasoning, and communication) as an integral part of facilitating students' mathematical understanding (NCTM, 2000). During training, teachers practiced arithmetic exercises (e.g., numbers and operations) and discussed algebraic ideas (e.g., symbols, expressions, patterns, and representations). In addition, teachers studied theories based on student-centred learning (e.g., Brown & Campione, 1994), such as learning by inquiry and participating in discussion.

All four workshops in both groups contained the same structure. First, the instructor presented the lesson's subject and contents to the in-service teachers. Second, the teachers practiced the tasks collaboratively in pairs. Practice was based on (a) solutions of various complexities requiring comprehension of mathematical knowledge and pedagogical episodes, and (b) analysis and evaluation of lesson plans, video-captured lessons, or pedagogical events. Third, each pair of teachers presented their summary of the task solution or lesson evaluation to the class. Finally, teachers participated in class discussions regarding the interpretation of mathematical ideas and pedagogical events, understanding difficulties, and proposing solutions and explanations for problems. In addition, a discourse related teachers' attitudes, beliefs and feelings regarding the training program was organized for the entire class. As part of their training, teachers conducted their actual school lessons while practicing various mathematical and pedagogical activities with their students, and then reflected on and discussed their experiences with their peers in the workshop.

Each group received training from one of two female expert instructors. Both instructors held an MA degree in mathematics education, had 10+ years of teaching experience, and were considered experts in pedagogical development and training programs. For this study, each instructor was trained separately; the instructor assigned to the S_Q group practiced exercises and tasks using the IMPROVE method (see the next section), whereas the other instructor practiced these tasks with the R_D group.

Figure 1 summarizes the main components in each training program.

[a] Learning Approaches	Self-Questioning S_Q	Reflection Discourse R_D
Theoretical teaching and learning framework	New curriculum standards (NCTM, 2000) for early childhood education: Problem solving, mathematical reasoning, and communication; numbers and operations and algebraic ideas; Teaching methods for *student-centred* learning; Research review	
Teacher training (for 2 instructors)	One-day, 5-hour in-service training seminar, and teachers' observations	
Workshop structure	Five weekly 4-hour workshops for a period of five weeks that included 6 main activities: (a) Instructor presented the lesson's subject and contents (b) Practice was based on task solutions, analysis of lesson plans, video-captured lessons or teachers' actual school lessons (c) Teachers practiced tasks collaboratively in pairs (d) Each pair presented their summary of the task solution, lesson evaluation or pedagogical event (e) Teachers participated in class discussion regarding their activities in the workshops and their actual school lessons (f) Instructors implemented procedures for debriefing regarding attitudes, beliefs, and feeling toward the program	
Guidance	(a) IMPROVE self-questioning method: Comprehension, connection, strategy and reflection; (b) Systematic practice in both perspectives as *student* and as *teacher*	Reflection discourse on NCTM standards of teaching and learning: (a) Problem solving (b) Mathematical understanding (c) Teaching methods (d) Pedagogical considerations

Figure1: Summary of the mathematics training program by reflection support

Self-Questioning (S_Q) group

Teachers in this group received reflection support based on the IMPROVE metacognitive self-questioning model (Kramarski & Mevarech, 2003; Mevarech & Kramarski, 1997). In previous applications of this model for students, we utilized a series of four metacognitive self-guided questions on comprehension, connection, strategy, and reflection. In the present study, we expanded the model to incorporate two perspectives of practice for teachers: as a student (i.e., regarding solving problems) and as a teacher (i.e., in planning lessons involving those problems). In both perspectives, teachers used the metacognitive self-guided questions before, during, and after the solution process, whether or not the given solution involved a task, a lesson plan, or a pedagogical event.

The comprehension questions were designed to prompt teachers to reflect on the problem before solving it, plan a lesson or analyze a pedagogical event. In addressing comprehension questions, the teacher was required to focus on the basic features of the problem (e.g., givens, terms) or the event. For planning lessons or pedagogical events, teachers had to demonstrate the lesson's topic, mathematical knowledge, and the explanations needed in the lesson. The connection questions were designed to prompt teachers to focus on similarities and differences among problems, explanations, lessons, or pedagogical events that the teachers had already used or planned, and to explain why. In addressing the connection questions, teachers had to focus on prior knowledge, and to define the structural features of the task and the information provided. The strategic questions were designed to prompt teachers to consider which strategies were appropriate for solving or teaching the given problem/task/pedagogical event and the basis for doing so, and for what reasons. In addressing the strategic questions, teachers had to describe "what" strategy they selected, "how" they suggested it should be implemented, and "why" the strategy was the most appropriate one for solving or teaching the problem/task. The Reflection questions were designed to prompt teachers to control their problem solving and lesson planning. In addressing the reflection questions, teachers monitored and evaluated their understanding and different ways to solve problems or using teaching approaches. The metacognitive questions were embedded in the teachers' workshop materials. The teachers were encouraged to use these questions explicitly in solving their tasks, when providing explanations, planning their lessons and conducting team and class discussions. Teachers were asked to provide written responses to metacognitive questions.

The instructor also explicitly presented and discussed research findings about the effects of the IMPROVE method on students' problem solving, mathematical reasoning, pedagogical knowledge and SRL (e.g., Kramarski, et, al., 2002; Kramarski, 2008; Kramarski & Michalsky, 2009). In class, the instructor also discussed with teachers how to use metacognitive self-questioning during classroom learning and teaching.

Reflection Discourse (R_D) group

The aim of reflection in this program was to improve teachers mathematical problem solving and actual practice according to the NCTM standards. Teachers were expected to solve mathematical problems, plan lessons, make provisions for classroom learning and teaching and participate in a reflection discourse. Teachers were asked to discuss in small groups and in the whole class their experience on (a) mathematical problem solving; (b) teaching in their actual classes focusing on enhancing mathematical understanding (e.g., ways of solutions, mathematical explanations), (b) teaching methods (e.g., sharing knowledge), and (c) pedagogical considerations of their actual experiences with their students (e.g., task demands, levels of thinking, and students difficulties). The R_D program encouraged teachers to be critical about their work, with the intention that they can effect change in their teaching.

Teachers practiced the same tasks with the workshop materials. The instructor discussed the main research literature in reference to principles and standards of mathematical reasoning and teaching techniques to promote young children's understanding in student-centred learning approaches (Brown & Campione, 1994). The instructor presented Zimmerman's (2000), approach to effective reflection. The approach often involves making a judgment and then to suggests techniques to modify teaching. The instructor also discussed with teachers how to implement the NCTM principles in the class, and modeled how to reflect on the problem solution, lesson plan, or a pedagogical event in actual teaching. The instructor explained that by sharing methods, discussing written work, and reflecting on problems and solutions, teachers could improve the understanding of goals for student learning.

Supervision of workshops

During the period of the study, an assistant researcher visited all of the workshops and observed how teachers were engaged in the process. Particular attention was paid to the requirements of participating in reflection discourse. Observations in both groups indicated that 84% of the teachers were involved in reflection discourse.

Measures

Three quantitative measures assessed teachers' outcomes (PCK assessment and delayed test, metacognitive knowledge, and beliefs).

Teachers' PCK assessment

At both the beginning and the end of the study, we administered a 12-item test adapted from Teo et. al., (2007), to all teachers. The test covered pedagogical issues ranging from the elementary level teaching unit on numbers, operations, and basic algebraic reasoning, referring to pedagogical planning or suggestions for teaching including: (1) presenting the topic (demonstrations, representations and justifications); (2) developing students' understanding (connecting concepts; identifying difficulties and justifications); and (3) fostering student-centered learning (active learning, self-opinion and theoretical justifications). The Appendix presents two PCK tasks. For each item, teachers received a score of either 3 (full answer-referring to the three criteria), 1 or 2 (partial answer-referring to one or two criteria) or 0 (incorrect answer), and a total score ranging from 0 to 36. We translated the scores to percentages. The Cronbach alpha reliability of the test coefficient was .86. The pre-test and post-test versions of these pedagogical knowledge tests shared similar but not identical contents and structure. The scoring criteria across time were consistent.

PCK-delayed test

The annual Israeli Ministry of Education end-of-year teacher assessment, administered 5 months after the intervention, assessed a large range of PCK tasks. The mathematical PCK test (14 open tasks) assessed various pedagogical skills referring to the same mathematical topics like: suggesting a way to explain the topic; identifying students' errors and explaining reasons for them; identifying and analyzing alternative problem solving strategies; building connections between math concepts; and using different representations and demonstrations to teach a mathematical concept (see example items in the Appendix). The Ministry provided a total score in percentages for each teacher.

Metacognitive knowledge and beliefs questionnaire

A pre/post 54-item questionnaire assessed metacognitive knowledge and beliefs. The questions were based on the questionnaires of Montague and Bos (1990), Kramarski et al., (Kramarski & Mevarech, 2003; Kramarski & Mizrachi, 2006; Kramarski, 2008; Kramarski & Michalsky, 2009) and Schoenfeld (1992). The questionnaire contains three components: declarative, procedural and conditional knowledge. The declarative component referred to beliefs in teaching for understanding (e.g., 'Lessons in mathematics should based on formulating conjectures, not just performing exercises'); the procedural (e.g., 'during the problem solving process, I asked students for self-opinions and conclusions); and the conditional component (e.g., 'In the class discourse, I asked students to referre others' solutions').

Each item was constructed on a 5-point Likert type scale ranging from 1 (never) to 5 (always). The Cronbach alpha reliability of the questionnaire coefficient was .91; .87; 89, respectively for each component).

Results

Teachers' PCK in the context of problem solving

The first purpose of the study was to examine mathematics teachers' PCK. Table 1 presents the mean scores, adjusted mean scores, and standard deviations based on teachers' PCK, by reflection approach. Effects of the training programs were observed with regard to their view in: Presenting the topic; developing students understanding; and fostering student-centred learning.

MANOVA results indicated that prior to the beginning of the study no significant differences existed between the two treatment groups in teachers' PCK. However, the post-test MANCOVA results indicated that teachers in the S_Q group significantly outperformed their peers in the R_D group, $F(3, 58) = 19.17$; $p < .0001$. Further ANCOVA results indicated differences between the two groups in two of the PCK criteria: Fostering student-centred learning, $F(1, 59) = 4.58$, $p < .01$ ($d = .65$), and developing students understanding, $F(1, 59) = 8.74$, $p < .01$ ($d = 1.14$). However, no significant differences emerged on using ways to present the topic, $F(1, 61) = .57$, $p > .05$ ($d = .24$).

Table 1. Means, adjusted means, standard deviations, F values, and Cohen's effect sizes (d[a]) on teachers' PCK, by treatment group[1] (S_Q vs. R_D) and time (pre-test/post-test)

	Self-Questioning (S_Q) group *n* = 30		Reflection Discourse (R_D) group *n* = 32	
	Pre	*Post*	Pre	Post
Presenting the topic				
M	84.50	88.38	85.36	87.02
Adjusted *M*		87.63		86.28
SD	2.45	2.76	2.87	5.76
		d = 0.24		
Student-centered learning				
M	76.62	89.41	75.83	82.34
Adjusted *M*		88.27		81.29
SD	15.21	7.89	15.97	10.72
		d = 0.65		
Students' understanding				
M	77.16	86.43	75.39	73.06
Adjusted *M*		85.56		72.63
SD	14.32	11.75	15.80	18.71
		d =1.14		

Note. Range: 0-100.

Teachers' metacognitive knowledge and beliefs

Secondary purpose of the study was to investigate metacognitive knowledge and beliefs in the area of problem solving among elementary mathematics' teachers (S_Q and R_D). Table 2 presents the mean scores, and standard deviations based on metacognitive knowledge, by treatment group and time. Effects of the two training programs were observed regarding their declarative, procedural, and conditional knowledge.

Table 2. Means and standard deviations of metacognitive knowledge and beliefs by treatment group [2] and Time

	Self-Questioning (S_Q) group *n* = 30		Reflection Discourse (R_D) group *n* = 32	
	Pre	Post	Pre	Post
Declarative knowledge and beliefs				
M	3.72	3.84	3.68	3.81
SD	0.38	0.32	0.34	0.31
	d = 0.32		*d* = 0.38	
Procedural knowledge				
M	3.84	4.10	3.86	4.03
SD	0.28	0.32	0.31	0.30
	d = 0.93		*d* = 0.55	
Conditional knowledge				
M	3.86	4.10	3.80	3.89
SD	0.36	0.33	0.35	0.34
	d = 0.67		*d* = 0.26	

[1]Range: 1-5.

The data was analyzed by MANOVA for the pre-test, and Repeated Measures for the post-test. In addition, the effect-size (*d*) was calculated by the difference between the means of the pre-test and post-test divided by the standard deviation of the total pre-test.

A multilevel analysis of variance (MANOVA) indicated that prior to the beginning of the study no significant differences were found between the S_Q and R_D teachers in metacognitive measures, $F(2, 59) = 1.82$, $p > .05$. Further analysis of two-way Repeated Measures of variance [groups (2) by time (2)] indicated significant differences in the main effect of time for the declarative knowledge, $F(1, 59) = 12.43$, $p < .01$, procedural knowledge $F(1, 59) = 9.34$, $p < .01$, and conditional knowledge, $F(1, 59) = 6.81$, $p < .01$).

However, a significant interaction was found between groups and time for procedural knowledge, $F(1, 59) = 9.68$, $p < .01$, and conditional knowledge, $F(1, 59) = 7.86$, $p < .01$). The findings indicated that at the end of the study the S_Q teachers improved significantly more in their application of procedural knowledge and conditional knowledge ($d = .93$; .67), compared to the R_D teachers ($d = .55$; .26 respectively for procedural and conditional knowledge). We found no significant differences between treatment group and time in improving the use of declarative knowledge in the area of metacognitive knowledge and beliefs in teaching with problem solving approach, $F(1, 59) = .35$, $p > .05$).

PCK–delayed test

ANOVA results indicated significant differences between the two groups at the delayed test. The S_Q teachers outperformed the R_D teachers on PCK (S_Q: $M = 82.75$, $SD = 13.45$; R_D: $M = 70.69$, $SD = 13.42$; $F(1, 60) = 10.25$, $p < .01$; $d = .89$).

Discussion

Findings indicated that systematic reflection support based on IMPROVE's self-questioning (S_Q) is effective for developing PCK in the context of mathematics problem solving. In addition, systematic reflection supports strengthen metacognitive knowledge and beliefs of mathematics teachers more than reflection discourse support. These findings support previous conclusions that self-questioning strengthens pre-service teachers' metacognition and pedagogical ability (Kauffman, et al., 2008; Kramarski & Michalsky, 2009). Our findings highlight the importance of self-questioning support in the different stages of life long learning of mathematics teachers, in the preparation stage as pre-service teachers, and in professional development training as in-service teachers (PISA, 2003).

We suggest two reasons to explain the beneficial effects of S_Q support. Self-questioning encourages students to reflect on their goals, their understanding, making links, and restructuring ideas. Perhaps the systematic explicit use of the questions enable the student to walk through the activities step-by-step, thereby helping students monitor and evaluate their learning processes. This ability, in turn, affected their metacognitive knowledge and PCK (Davis, 2003; Kramarski & Michalsky, 2009; in press). This conclusion is in line with other researchers who argued that directed support may act as a "more able other," prodding students to consider issues they may not have considered otherwise (Nuckles, et al., 2009; Vygotsky, 1978). In contrast, the reflection discourse exposed the teachers to an open dialogue that enabled them to learn new ideas and pedagogical solution strategies; however such open dialog might increase their cognitive load thereby causing difficulties in integrating ideas and solutions that were raised in the group.

PCK in mathematical problem solving context

The current findings concerning PCK indicated that teachers of the S_Q were more successful in integrating content with pedagogy in a deeper level than the R_D group, as found in a test administered immediately after the end of the study, and in a delayed test (government assessment). The S_Q group teachers based their pedagogical considerations and beliefs about student-centred learning (i.e., self-opinions, conclusions, and theoretical justifications) and learning for understanding (i.e., connecting between concepts and identifying difficulties). No differences were found between the two groups on ways of presenting the topic in the class (demonstration, representations and justifications). In contrast, the R_D group teachers focused more on how to transmit the topic without explicit emphasis on students understanding and justifications about their choices.

We suggest two reasons for the beneficial effect of S_Q support on PCK. First, discourse on why and how questions seemed to foster teachers' understanding of task demands and pedagogical decisions. Second, the explicit opportunity to elaborate on different perspectives of problem solving, as both students and teachers, prompted teachers to focus more on deep understanding of task demands and on a student-centred teaching approach in their pedagogical approach. These findings are in line with previous studies that emphasized the importance of using self-questioning in multiple perspectives in learning and teaching (Kramarski et. al., 2001; Kramarski & Michalsky, in press; Kramarski & Revach, in press; Xiaodong et. al., 2005).

Metacognitive knowledge and beliefs

Findings on the three components of metacognitive knowledge, self-reported questionnaire indicated that providing teachers with reflection support in both groups (S_Q and R_D) was beneficial in promoting metacognitive knowledge components (declarative, procedural and conditional). However, the effects of S_Q support was remarkable in the metacognitive higher order knowledge (procedural and conditional knowledge). Furthermore, the findings indicated that both approaches were less effective in changing the declarative knowledge and beliefs component, in comparison to the procedural and conditional components. The slight improvement of beliefs support other conclusions that beliefs are stable and difficult to change (Pajares, 1992; Richardson, 1996; Thompson, 1992).

We found no differences between the two reflection supports in their declarative knowledge and beliefs. At the end of the study, teachers in both groups perceived mathematics as a process of problem solving and reasoning, rather than a set of unrelated facts, rules, and skills. This finding is in line with previous research on pre-service teachers in a Web-based learning environment that shows that IMPROVE's self-questioning strengthens teachers' perceptions about students' ability to construct knowledge with student-centred learning approach (Kramarski & Michalsky, 2009).

The findings indicated minimal improvement of metacognitive conditional knowledge (d = .26) among the R_D teachers. This finding suggests that although the R_D teachers improved their pedagogical beliefs (d =.38), they didn't know when and how to transfer such beliefs into practice. The findings support the conclusion that, teachers' simple sharing of methods, discussion of written work, and reflection on problems, solutions, and beliefs (based on standards in mathematics learning as observed in the R_D group), do not ensure that teachers understand how those standards benefit performance. Obviously, such understanding is critical for optimal use of those standards in classroom instruction (e.g., Kramarski & Revach, in press; Xiaodong et. al., 2005). Current metacognitive knowledge and beliefs outcomes for the two approaches substantiate previous research which concluded that explicit support is necessary for combining beliefs and construction of new knowledge in teachers training programs (Pajares, 1992; Richardson, 1996; Thompson, 1992).

Implications, further research, and limitations

This study contributes to teachers' theoretical research by examining the role of reflection support in mathematics professional training. To support the vision of the NCTM standards in the mathematics classroom, teachers' professional training programs are being called upon to model different teaching support for understanding and to help teachers develop their knowledge of content, and pedagogy. In particular, it is suggested that educators focus on the importance of helping teachers to become more aware of how their knowledge, beliefs, and actions influence students' learning (Kramarski & Revach, in press).

There are two inherent limitations in this study. First, the present implementation of each support by only one group could be confounded with the instructional support. To strengthen the current claims, further research should examine the effects of reflection support on larger samples of teachers and should expand observations of teachers' class practice. To generalize the present findings, future studies should follow up on the long-term effects of reflection support by explicitly investigating teachers' professional knowledge on various topics and tasks. These tests should be implemented at different time intervals (e.g., one or two years after intervention).

Second, our study investigated relations between reflection support and cognitive variables (pedagogical and metacognitive knowledge). In addition, the study investigated the development of teachers' beliefs toward problem solving. Future research should be conducted to investigate the proposed relationship between other kinds of beliefs and affective variables and teachers' professional development (Farmer, et. al., 2003).

The present research findings add complementary perspectives to the literature on teachers' professional knowledge, by associating teachers' PCK with metacognitive knowledge and beliefs under reflection support. However, the study does not supply data about student outcomes obtained by the participating teachers. Future studies would do well to examine the assumption that teachers' SRL is extremely important to their success in teaching (Perry et. al., 2006; Randi & Corno, 2000). Toward this end, teachers with varying levels of SRL should be observed, and the data should be correlated to students' understanding, achievement data, and attitudes towards mathematics.

Furthermore, considering the complex nature of professional development in mathematics teaching (Hill et al., 2005), it may be useful to pay attention to the measurement of quality in assessing professional development by using different kinds of complementary methods and styles of coding. Offline (questionnaires) and online (actual teaching) methods such as thinking aloud, observations, and interviews, may shed further light on the benefits of reflection support. In conclusion, we underscore the need to further investigate how to enhance mathematical professional development with reflection support in professional training programs.

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