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Gamma-Ray Telescope and Uncertainty Principle

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Abstract
Heisenberg's Uncertainty Principle is one of the important basic principles of quantum mechanics. In most of the books on quantum mechanics, this uncertainty principle is generally illustrated with the help of a gamma ray microscope, wherein neither the image formation criterion nor the lens properties are taken into account. Thus a better illustration is required for the clear understanding of this principle. In this article we illustrate Heisenberg's uncertainty principle with a new thought experiment that could be called as gamma ray telescope with the inclusion of properties of the lens and quantum nature of light.

Introduction

The well known Heisenberg's uncertainty principle (Heisenberg, 1925) is generally illustrated with a thought experiment using a gamma ray microscope ((Shankar, 2010), (Bransden and Joachain, 2004). But, such illustrations are rather sketchy (Popat Savaleram Tambade, 2012) and do not take into account the properties of the lens or position of the image etc. In this short article, we present an illustration of the uncertainty principle considering the well-known properties of a lens and the quantum nature of light, so that the beginner in quantum mechanics should have a clear understanding of this principle. We would like to call the arrangement as gamma-ray telescope.

The Experimental Arrangement

Let us consider a point particle P located at a distance L from a lens of diameter D and focal length f as shown in the figure 1.

The natural question that arises here is, how best one could localise the particle at Q?. The location of the particle has to be inferred from its image formed by the lens in its focal plane. Let us assume that the particle is far away from the lens, that is to say, L >> f. Hence the arrangement is that of a 'telescope' and not a microscope. Monochromatic light of wavelength λ is shined on the particle. The light scattered from the particle is collected by the lens, forming an image in its focal plane.

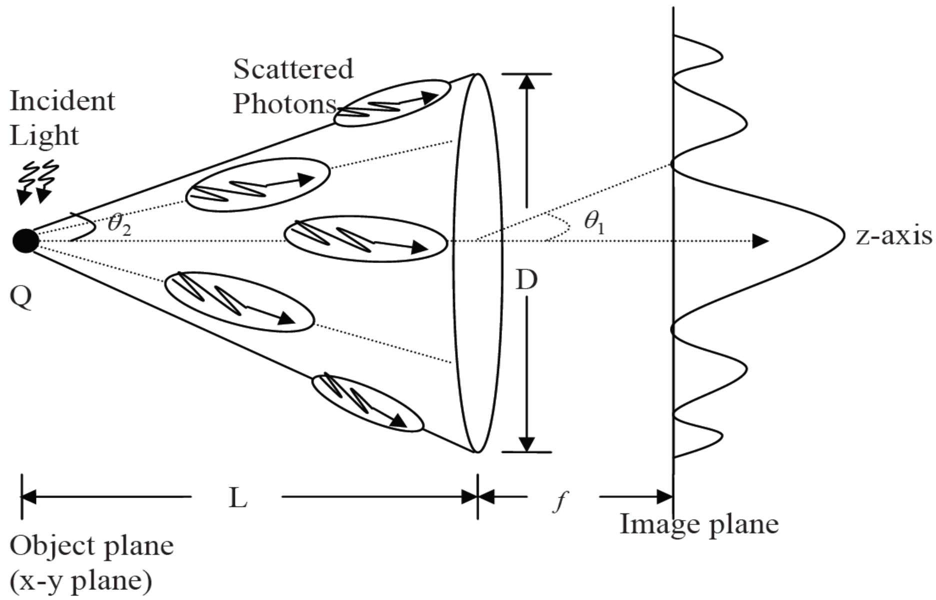

Figure 1.Schematic ray representation of a gamma-ray telescope

Uncertainties in position and momentum

The image of the point particle consists of circular bright and dark rings according to diffraction theory (Max Born and Emil Wolf, 1999). The angular radius of the central bright ring is given by,

$$\theta_1 = \frac{1.22\,\lambda}{D} \qquad (1)$$

An observer would conclude that the uncertainty in the angular position of the object is $2\theta_1$ (This is essentially taken as the angle between the first minima on either side of the central maxima from the lens). Hence the uncertainty in the transverse position (x- coordinate) of the particle is equal to

$$\Delta x \cong 2L\theta_1 = 2.44\frac{\lambda L}{D} \qquad (2)$$

Photons that are elastically scattered by the particle at Q have a momentum of the magnitude

$$P = \frac{h}{\lambda} \tag{3}$$

The angular spread in the directions of scattered photons entering the lens has a value $2\theta_2$. Where

$$\theta_2 \cong \frac{D}{2L} \tag{4}$$

(Since the point particle is far away from the lens, we use small angle approximation $\sin\theta_2 \cong \tan\theta_2 \cong \theta_2$ to obtain equation (4))
On account of momentum conservation, the spread in the x - component of momentum (ΔP_x) of the particle is expected to have the same values as that of the scattered photons. Hence,

$$\Delta P_x \cong P.2\theta_2 = \frac{h}{\lambda}\left(\frac{D}{L}\right) \tag{5}$$

Therefore, the product of the position and momentum uncertainties (spreads) will be given by,

$$\Delta x.\Delta P_x \cong \left(\frac{2.44\lambda L}{D}\right)\left(\frac{Dh}{L\lambda}\right) \tag{6}$$

Thus,

$$\Delta x.\Delta P_x \cong 2.44h \tag{7}$$

This is independent of the wavelength of light used, diameter of the lens as well as the focal length of the lens, as it should. Of course the factor 2.44 results from a particular choice of characterizing the image in the focal plane and hence is not unique.

Discussion

We have illustrated the validity of Heisenberg's uncertainty principle with the help of the well known image forming properties of a lens. The arrangement may be rightly called a gamma-ray telescope. Hence we claim that Heisenberg's uncertainty principle could be better illustrated with the help of our thought experiment, which we would like to call as gamma-ray telescope instead of gamma-ray microscope that is used in most of the books on basic quantum mechanics. Thus this piece of information will surely help the students

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and other readers who begin their first course in quantum mechanics.

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