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# Mathematical Modeling in Mathematics Education: Basic Concepts and Approaches*

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**Abstract**

Mathematical modeling and its role in mathematics education have been receiving increasing attention in Turkey, as in many other countries. The growing body of literature on this topic reveals a variety of approaches to mathematical modeling and related concepts, along with differing perspectives on the use of mathematical modeling in teaching and learning mathematics in terms of definitions of models and modeling, the theoretical backgrounds of modeling, and the nature of questions used in teaching modeling. This study focuses on two issues. The first section attempts to develop a unified perspective about mathematical modeling. The second section analyzes and discusses two approaches to the use of modeling in mathematics education, namely modeling as a means of teaching mathematics and modeling as an aim of teaching mathematics.

**Keywords**

Mathematics Education, Mathematical Model, Mathematical Modeling, Problem Solving.

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In the last two decades, mathematical modeling has been increasingly viewed as an educational approach to mathematics education from elementary levels to higher education. In educational settings, mathematical modeling has been considered a way of improving students' ability to solve problems in real life (Gravemeijer & Stephan, 2002; Lesh & Doerr, 2003a). In recent years, many studies have been conducted on modeling at various educational levels (e.g., Delice & Kertil, 2014; Kertil, 2008), and more emphasis has been given to mathematical modeling in school curricula (Department for Education [DFE], 1997; National Council of Teachers of Mathematics [NCTM], 1989, 2000; Talim ve Terbiye Kurulu Başkanlığı [TTKB], 2011, 2013).

The term "modeling" takes a variety of meanings (Kaiser, Blomhoj, & Sriraman, 2006; Niss, Blum, & Galbraith, 2007). It is important for readers who want to study modeling to be cognizant of these differences. Therefore, the purpose of this study is twofold: (*i*) Presenting basic concepts and issues related to mathematical modeling in mathematics education and (*ii*) discussing the two main approaches in modeling, namely "modeling for the learning of mathematics" and "learning mathematics for modeling." The following background information is crucial for understanding the characterization of modeling, its theoretical background, and the nature of modeling problems.

## Mathematical Modeling and Basic Concepts

**Model and Mathematical Model:** According to Lesh and Doerr (2003a), a *model* consists of both conceptual systems in learners' minds and the external notation systems of these systems (e.g., ideas, representations, rules, and materials). A model is used to understand and interpret complex systems in nature. Lehrer and Schauble (2003) describe a model as an attempt to construct an analogy between an unfamiliar system and a previously known or familiar system. Accordingly, people make sense of real-life situations and interpret them by using models. Lehrer and Schauble (2007) describe this process as model-based thinking and emphasize its developmental nature. They also characterize the levels of model-based thinking as hierarchical.

Mathematical models focus on structural features and functional principles of objects or situations in real life (Lehrer & Schauble, 2003, 2007; Lesh & Doerr, 2003a). In Lehrer and Schauble's hierarchy, mathematical models do not include all features of real-life situations to be modeled. Also, mathematical models comprise a range of representations, operations, and relations, rather than just one, to help make sense of real-life situations (Lehrer & Schauble, 2003).

**Mathematical Models and Concrete Materials:** In elementary education, the terms mathematical model and modeling are usually reserved for concrete materials (Lesh, Cramer, Doerr, Post, & Zawojewski, 2003). Although the use of concrete materials is useful for helping children develop abstract mathematical thinking, according to Dienes (1960) (as cited in Lesh et al., 2003), in this study, mathematical modeling is used to refer to a more comprehensive and dynamic process than just the use of concrete materials.

**Mathematical Modeling:** Haines and Crouch (2007) characterize mathematical modeling as a cyclical process in which real-life problems are translated into mathematical language, solved within a symbolic system, and the solutions tested back within the real-life system. According to Verschaffel, Greer, and De Corte (2002), mathematical modeling is a process in which real-life situations and relations in these situations are expressed by using mathematics. Both perspectives emphasize going beyond the physical characteristics of a real-life situation to examine its structural features through mathematics.

Lesh and Doerr (2003a) describe mathematical modeling as a process in which existing conceptual systems and models are used to create and develop new models in new contexts. Accordingly, a *model* is a product and *modeling* is a process of creating a physical, symbolic, or abstract model of a situation (Sriraman, 2006). Similarly, Gravemeijer and Stephan (2002) state that mathematical modeling is not limited to expressing real-life situations in mathematical language by using predetermined models. It involves associating phenomena in the situation with mathematical concepts and representations by reinterpreting them. To be able to express a real-life situation in mathematical language effectively, students must have higher-level mathematical abilities beyond just computational and arithmetical skills, such as spatial reasoning, interpretation, and estimation (Lehrer & Schauble, 2003).

**The Mathematical Modeling Process:** No strict procedure exists in mathematical modeling for reaching a solution by using the given information (Blum & Niss, 1991; Crouch & Haines, 2004; Lesh & Doerr; 2003a). Researchers agree that modeling is a cyclical process that includes multiple cycles (Haines

& Crouch, 2007; Lehrer & Schauble, 2003; Zbiek & Conner, 2006). In the literature, a variety of visual references describe the stages of the cyclic nature of the modeling process (Borromeo Ferri, 2006; Hıdıroğlu & Bukova Güzel, 2013; Lingefjard, 2002b, NCTM, 1989). For instance, the modeling process described in the earlier *Standards* document by NCTM (1989, p. 138) emphasizes that mathematical modeling is a non-linear process that includes five interrelated steps: (*i*) Identify and simplify the real-world problem situation, (*ii*) build a mathematical model, (*iii*) transform and solve the model, (*iv*) interpret the model, and (*v*) validate and use the model. Such types of diagrams can help readers and teachers understand the probable stages that students may experience during the modeling processes.

**Mathematical Modeling and Problem Solving:** Mathematical modeling is often confused with traditional word problems. From the view of Reusser and Stebler (1997), traditional word problems cause students to develop some didactic assumptions about problem solving. Moreover, the real-life contexts in these problems are often not sufficiently realistic and thus fail to support students' abilities to use mathematics in the real world (English, 2003; Lesh & Doerr, 2003; Niss et al., 2007). While working on such problems, students often simply focus on figuring out the required operations (e.g., Greer, 1997; Nunes, Schliemann & Carraher, 1993). Some studies focus on reorganizing word problems to enable students to gain competence in thinking about real-life contexts while solving them (Greer 1997; Verschaffel & De Corte, 1997; Verschaffel, De Corte, & Borghart, 1997; Verschaffel et al., 2002). Such versions of word problems can be used as warm-up exercises in preparation for modeling (Verschaffel & De Corte, 1997).

While Lingefjard (2002b) argues that it is unreasonable to compare problem solving and modeling, the similarities and differences between them can be useful (Lesh & Doerr, 2003a; Lesh & Zawojewski, 2007; Mousoulides, Sriraman, & Christou, 2007; Zawojewski & Lesh, 2003). The following table briefly describes a few of the important differences between the two concepts.

### Mathematical Modeling Approaches

Different approaches have been proposed with different theoretical perspectives for using modeling in mathematics education, and no single view is agreed upon among educators (Kaiser, Blum, Borromeo Ferri, & Stillman, 2011; Kaiser & Sriraman, 2006). To clarify the different perspectives on this issue and reach a consensus, these similarities and differences should be elaborated (Kaiser, 2006; Kaiser & Sriraman, 2006; Sriraman, Kaiser, & Blomhoj, 2006). Kaiser's (2006) and Kaiser and Sriraman's (2006) classification systems for presenting modeling approaches can be considered the leading perspective. According to this scheme, the perspectives are classified as (*i*) realistic or applied modeling, (*ii*) contextual modeling, (*iii*) educational modeling, (*iv*) socio-critical modeling, (*v*) epistemological or theoretical modeling, and (*vi*) cognitive modeling. Generally, modeling is also classified by its purpose in mathematics education, such as (*i*) modeling as the purpose of teaching mathematics or (*ii*) modeling as a means to teach mathematics (Galbraith, 2012; Gravemeijer, 2002; Julie & Mudaly, 2007; Niss et al., 2007).

**Table 1**
*A Comparison between Problem Solving and Mathematical Modeling (Adapted from Lesh & Doerr [2003a] and Lesh & Zawojewski [2007])*

| Traditional Problem Solving | Mathematical Modeling |
|---|---|
| Process of reaching a conclusion using data | Multiple cycles, different interpretations |
| Context of the problem is an idealized real-life situation or a realistic life situation | Authentic real-life context |
| Students are expected to use taught structures such as formulas, algorithms, strategies, and mathematical ideas | Students experience the stages of developing, reviewing, and revising important mathematical ideas and structures during the modeling process |
| Individual work emphasized | Group work emphasized (social interaction, exchange of mathematical ideas, etc.) |
| Abstracted from real life | Interdisciplinary in nature |
| Students are expected to make sense of mathematical symbols and structures | In modeling processes, students try to make mathematical descriptions of meaningful real-life situations |
| Teaching of specific problem-solving strategies (e.g., developing a unique approach, transferring onto a figure) transferable to similar problems | Open-ended and numerous solution strategies, developed consciously by students according to the specifications of the problem. |
| A single correct answer | More than one solution approach and solution (model) possible |

**Modeling as the Purpose of Teaching Mathematics**

In this perspective, mathematical modeling is seen as a basic competency, and the aim of teaching mathematics is to equip students with this competency to solve real-life problems in mathematics and in other disciplines (Blomhøj & Jensen, 2007; Blum, 2002; Crouch & Haines, 2004; Haines & Crouch, 2001; Izard, Haines, Crouch, Houston, & Neill, 2003; Lingefjard, 2002a; Lingefjard & Holmquist, 2005). In this approach, initially, mathematical concepts and mathematical models are provided and later these ready-made concepts or models are applied to real-world situations (i.e., mathematics → reality) (Lingefjard, 2002a, 2002b, 2006; Niss et al., 2007). Mathematical models and concepts are considered as already existing objects (Gravemeijer, 2002). Researchers adopting this perspective focus on the issue of conceptualizing, developing, and measuring the modeling competencies (e.g., Haines & Crouch, 2001, 2007). In the literature, different viewpoints exist on this issue (Henning & Keune, 2007). While Blomhøj and Jensen (2007) adopt a holistic approach, other studies address this issue at the micro level (Crouch & Haines, 2004; Haines, Crouch, & Davis, 2000; Lingefjard, 2004). Furthermore, some studies focus on teaching mathematical modeling (Ärlebäck & Bergsten, 2010; Lingefjard, 2002a). Fermi problems, for example, are regarded as appropriate kinds of problems for teaching of modeling (Ärlebäck, 2009; Ärlebäck & Bergsten, 2010). Sriraman and Lesh (2006) contend that Fermi problems can be used as warm-up and starting exercises in preparation for modeling.

**Modeling as a Means for Teaching Mathematics**

In this approach, modeling is considered a vehicle for supporting students' endeavors to create and develop their primitive mathematical knowledge and models. The Models and Modeling Perspective (Lesh & Doerr, 2003a) and Realistic Mathematics Education (Gravemeijer, 2002; Gravemeijer & Stephan, 2002) are two examples of this approach.

**Models and Modeling Perspective (MMP)**

The models and modeling perspective is a new and comprehensive theoretical approach to characterizing mathematical problem-solving, learning, and teaching (Lesh & Doerr, 2003a; 2003b) that takes constructivist and socio-cultural theories as its theoretical foundation. In this perspective, individuals organize, interpret, and make sense of events, experiences, or problems by using their mental models (internal conceptual systems). They actively create their own models, consistent with the basic ideas of constructivism (Lesh & Lehrer, 2003). Moreover, for productive use of models for addressing complex problem-solving situations, they should be externalized with representational media (e.g., symbols, figures).

Model-eliciting activities (MEAs) are specially designed for use within the MMP. In MEAs, students are challenged to intuitively realize mathematical ideas embedded in a real-world problem and to create relevant models in a relatively short period of time (Carlson, Larsen, & Lesh, 2003; Doerr & Lesh, 2011). Lesh, Hoover, Hole, Kelly, and Post (2000) offered six principles to guide the design of MEAs: (*i*) the model construction principle, (*ii*) the reality principle, (*iii*) the self-assessment principle, (*iv*) the construct-documentation principle, (*v*) the construct shareability and reusability principle, and (*vi*) the effective prototype principle. In the implementation of MEAs, students work in teams of three to four. They are expected to work on creating shareable and reusable models, which encourage interaction among students. Therefore, the social aspect of learning is another component of the MMP (Zawojewski, Lesh, & English, 2003). According to Lesh et al. (2003), MEAs should not be used as isolated problem- solving activities. They should be used within model development sequences, where warm-up and follow up activities are also important.

**The Modeling Approach in Realistic Mathematics Education**

Similar to the MMP, the modeling approach assumed by RME is based on constructivist and socio-cultural theories (Freudental, 1991; Gravemeijer, 2002). In this approach, modeling goes beyond translating real-life problem situations into mathematics. It involves revealing new relations among phenomena embedded in the situations by organizing them (Gravemeijer & Stephan, 2002). In modeling, students initially work on real-life situations and create their primitive models, which are called *model of*. The term "model" describes not only the physical or mathematical representations of the phenomena, but also the components of students' conceptual systems, such as their purpose and ways of thinking about the situation (Cobb, 2002). With the help of carefully designed real-life problems and learning environments that encourage students to discover sophisticated mathematical models, students proceed to create more abstract and formal models, which are called *model for* (Doorman & Gravemeijer, 2009). Accordingly, modeling is characterized as a process of moving from "model of" to "model for," which is called as *emergent modeling* (Doorman & Gravemeijer, 2009; Gravemeijer & Doorman, 1999). Besides describing students' learning process, this perspective also assumes principles about how a learning environment should be designed to support students' emergent modeling processes.

## Discussion and Conclusion

In recent years, using modeling in mathematics education has been increasingly emphasized (NCTM, 1989, 2000; TTKB, 2011, 2013). A variety of different perspectives have been proposed for the conceptualization and usage of modeling (Kaiser & Sriraman, 2006). These perspectives can be grouped into two main categories: (*i*) modeling as a means for teaching mathematics and (*ii*) modeling as the aim of teaching mathematics (Blum & Niss, 1991; Galbraith, 2012). In the first perspective, students are provided with predetermined models and are expected to apply these models to real-life situations. The ultimate goal is to improve students' modeling competencies (Haines & Crouch, 2001, 2007; Izard et al., 2003; Lingefjard, 2002b). In the second perspective, the underlying assumption is that students can learn fundamental mathematical concepts meaningfully through a modeling process in which they need and intuitively discover mathematical concepts while addressing a real-life problem-solving situation (Lesh & Doerr, 2003a).

In summary, the second approach (i.e., modeling as a means for teaching mathematics) seems more developed for pedagogical purposes. However, whatever approach is preferred and used, integrating modeling into mathematics education is important for improving students' problem-solving and analytical thinking abilities. However, few studies have been conducted in Turkey on using modeling in mathematics education. Furthermore, there are insufficient resources (e.g., modeling tasks) for teachers who want to integrate modeling into their teaching. Thus, there is a need for more research on using modeling for different levels of education. This can enable the production of resources that can be used in pre-service and in-service teacher education programs. Sources including good examples of modeling tasks are needed for teachers.

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