Learning Disabilities: A Contemporary Journal 11(2), 41-49, 2013

Number Line Estimation in Children with Developmental Dyscalculia

Francesco Sella*
University of Padova, Italy

Ilaria Berteletti
Northwestern University, USA

Brazzolotto Martina
University of Padova, Italy

Daniela Lucangeli
University of Padova, Italy

Marco Zorzi
University of Padova, Italy

In the number to position task, several studies have shown that typically developing children shift from a biased (logarithmic) to an accurate (linear) mapping of symbolic digits onto a spatial position on a line. The initial pattern of overestimation of small numbers and the underestimation of larger numbers is compensated by means of age and education. Children with mathematical disability seem to show less accuracy in placing numbers on the line and their mapping tends to be more biased than linear. Here we evaluate to what extent this hypothesis holds for a sample of Italian children who have received a formal diagnosis of developmental dyscalculia (DD). Ten children with DD (M age-months = 123, SD = 25) and ten typically developing (TD) children (M age-months = 121, SD = 23), matched for age and gender, completed two number to position tasks (intervals: 0-100, 0-1000). For the interval 0-100, children with DD obtained a mapping in an intermediate stage between logarithmic and linear whereas the TD group reached a linear mapping. For the interval 0-1000, children with DD exhibited a logarithmic mapping whereas TD children had a linear mapping. These results highlight the presence of basic numerical deficit in children with DD.

Keywords: Developmental dyscalculia, number line estimation, and number to position task.

INTRODUCTION

Successful mathematical achievement can be considered as the by-product of several cognitive, educational, and motivational factors, which can differently interact across a lifetime. Various reasons could be responsible for weak mathematical achievement in children who perform at the lower end on standardized mathematical tests. Beyond educational and motivational aspects, children with math difficulties may present relatively different cognitive profiles thus composing a rather heterogeneous group (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). Therefore,

---

*Please send correspondence to: Francesco Sella, sella.francesco@gmail.com.

it is important to identify which cognitive subcomponent is impaired in children with math learning difficulties and understand at which level the cognitive process fails. Several studies have highlighted that children with developmental dyscalculia (DD) have a specific impairment in basic numerical processing (Landerl, Bevan, & Butterworth, 2004; Mazzocco, Feigenson, & Halberda, 2011; Moeller, Neuburger, Kaufmann, Landerl, & Nuerk, 2009a; Piazza et al., 2010). Therefore, it is important to investigate whether children with math disability are able to estimate numerical quantities relatively to typically developing peers.

Two mechanisms have been individuated as fundamental for fast quantification processes: the Object Tracking System (OTS) and the Approximate Number System (ANS; Feigenson, Dehaene, & Spelke, 2004; Piazza, 2010). The former allows identifying quickly and accurately the numerical quantity of small sets of objects (i.e., up to 3-4 items; Mandler & Shebo, 1982) without the use of counting strategies; the second, the Approximate Number System (ANS), allows approximating the numerical quantity of larger sets. Recent findings have highlighted that both quantification systems are impaired in children with DD. In the subitizing range, they tend to adopt serial counting to determine the numerosity of small sets resulting in longer reaction times (Schleifer & Landerl, 2010; Moeller et al., 2009a; Landerl, Bevan, & Butterworth, 2004). For larger quantities (beyond 4), children with DD show lower efficiency and need a larger numerical difference between two sets of items to be able to precisely identify the one with the larger/smaller numerical quantity (Piazza et al., 2010; Mazzocco, Feigenson, & Halberda, 2011). In Piazza and colleagues' study (2010), performance of 10 year-old children with DD to compare sets based on the numerical quantity (number acuity) was similar to the performance of 5-year younger typically developing children. The low performance shown by children with DD on non-symbolic numerical tasks suggests that their poor math achievement stems from an impaired basic numerical representation.

Beyond the approximate representation, numerical quantities may be represented in an exact way by means of numerical symbols. Zorzi and Butterworth's model (1999) postulates that numerate children and adults are able to linearly map Arabic digits to the corresponding numerical internal magnitude (also see Verguts, Fias, & Stevenson, 2005). In a seminal study, Siegler and Opfer (2003) have used the number to position task (NP-task) to show that children shift from an intuitive to an exact representation of numbers with age and greater numerical skill. Participants from grades two and six were required to place Arabic numbers (i.e., 25) onto a black horizontal bounded line going from 0 to 100. This task entails transcoding a numerical value into a spatial position on a visual line. Performance of younger children was characterized by an overestimation of small numbers and an underestimation of larger numbers, yielding a logarithmic pattern. Because smaller numbers are over-represented on the mental number line, according to the ANS, it suggests that younger children facing an unfamiliar numerical range rely on an intuitive and logarithmic mapping to solve the task. Nevertheless, other theoretical perspectives suggested different interpretations regarding the pattern of biased estimates in younger children. According to the Familiarity model, the pattern of estimates is more consistent with a bilinear fit separating familiar and non-familiar numbers (Ebersbach, Luwel, Frick, Onghena, & Verschaffel, 2008; Moeller, Pixner, Kaufmann, & Nuerk, 2009b). Other authors, instead, interpreted the positioning of a number as a consequence of a proportional judgment (Barth & Palladino, 2011).

Despite this theoretical issue, with increasing age and numerical proficiency, children shift from an immature mapping to a formal and linear one by placing numbers in correspondence of the correct position. Interestingly, at a same developmental time point, a child may use both mappings depending on the scale of the line interval: Preschoolers show a linear mapping for small intervals such as 1-10, whereas their mapping is still logarithmic for a larger scale such as 0-100 (Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010); during the first two years of elementary school, the linear mapping is progressively acquired for the 0-100 interval (Siegler & Booth, 2004), whereas linearity is mastered around the 4th grade for the 0-1000 interval (Booth & Siegler, 2006) and around the 6th grade for the 0-10000 interval (Thompson & Opfer, 2010). With increasing numeracy, a child will position numbers linearly on progressively larger intervals. It is critical to report that being perfectly able to name and recite the entire sequence of an interval does not grant linearity (Berteletti, Lucangeli, & Zorzi, 2012). Thus, children's logarithmic mapping is not merely an artifact of the task itself or poor knowledge of the items in the interval presented but it entails a specific maturation of the understanding of numerical quantities. Finally, supporting the diagnostic importance of the NP-task, studies have shown that performance correlates with other estimation tasks (Booth & Siegler, 2006), memory for small versus large numbers (Thompson & Siegler, 2010) and future mathematical achievement (Booth & Siegler, 2008).

Geary, Hoard, Nugent, and Byrd-Craven (2008), using standardized mathematical achievement tests, classified 1st and 2nd grade children into mathematical learning disability (below the 11th percentile), low math achievement (between 11th and 25th percentile), and typical achievement groups. In the number line task with the interval 0-100, grade 1 pupils with math disability displayed a logarithmic representation compared to the other groups, who showed a linear mapping. Only by grade 2, children with math disability displayed a representation at an intermediate stage between the logarithmic and the linear mapping. In a subsequent study, Landerl, Fussenegger, Moll, and Willburger (2009) analyzed performance in the NP-task of typically developing, dyscalculic, dyslexic, and dyslexic-dyscalculic children between 8- and 10-years of age. Children categorized as dyslexic had a score below 1 standard deviation (SD) in a reading fluency test and an adequate score in the arithmetic test. Conversely, children with dyscalculia had a score below 1 SD in the arithmetic test but had an adequate score in the reading test. Children with performance below 1 SD in both, the reading and the arithmetic tests, were categorized as dyslexic-dyscalculics. In the 0-100 interval, only the control group had a reliable linear positioning and the dyslexia and dyscalculia groups approximated a linear mapping whereas the dyslexia-dyscalculia showed no difference in precision between the two fits. In the 0-1000 interval, only the control group was close to a linear fit whereas for all the other groups the logarithmic model provided a better explanation of the data. The difference in favor of a logarithmic model was reliable only for the dyslexia-dyscalculia group.

In the present study, we replicate and extend results of the previous studies by investigating the ability to translate numbers into a spatial position in Italian children with DD. Because the NP-task has the potential for becoming a diagnostic tool for low math achievement, it is important to test its reliability with several groups from different cultural and educational systems. Moreover, its simplicity makes it a task administrable to children prior to formal schooling (Berteletti et al., 2010) and therefore a tool for an early diagnosis of low achievement. To this aim, the estimates of children with DD in two NP-tasks (intervals: 0-100, 0-1000) were compared to those of a typical developing (TD) group. We expect children with DD to show a less accurate mapping as compared to the TD group and to show longer reaction times (RT) for placing numbers. Lower precision and longer RT confirm a reduced basic numerical knowledge in children with DD.

METHOD

Participants

Ten children between 8- and 13-years of age with DD (2 boys; *M* age-months = 123, *SD* = 25, range: 96 - 163) were recruited from the Regional Center for Research in Learning Disabilities (Padova, Italy). They all received a formal diagnosis of DD by an expert clinician with a specific specialization in learning disabilities and scored in the normal range for IQ (>85), had neither sensory deficits nor comorbidity with Attention Deficit Hyperactivity Disorder. Ten TD children from middle-socioeconomic schools in Northern Italy were matched in age and gender to the DD group (2 boys; *M* age-months = 121, *SD* = 23 months, range 98 - 158). Children in the TD group were free from learning or attentional disabilities. The DD and TD group did not differ in terms of age ($p = .83$).

Task and Procedure

Children were met individually, in a quiet room, and completed the two computerized versions of the NP-task (Siegler & Opfer, 2003). Tasks were presented as games, no time limit was given and items or questions could be repeated if necessary but neither feedback nor hints were given to the child. Students were free to stop at any time. The Number-to-Position task (NP-task) was a computer adaptation from Siegler and Opfer's (2003). An approximately 17 cm black line was presented in the center of the screen with a mild yellow background. In the 0-100 interval, the left end was labeled 0 and the right end was labeled 100. Children were required to estimate the position on the line of ten numbers (2, 3, 4, 6, 18, 25, 42, 67, 71, 86; set A and B from Siegler & Opfer, 2003). In the interval 0-1000, the left end was labeled 0 and the right end was labeled 1000 and there were twenty-two numbers to estimate (2, 5, 18, 34, 56, 78, 100, 122, 147, 150, 163, 179, 246, 366, 486, 606, 722, 725, 738, 754, 818, 938; sets A and B from Opfer & Siegler, 2007). Answers were given by clicking on the line using the mouse; however, the range of movement of the cursor was constrained to the area covering the line as to avoid collecting unreliable responses. For each trial, the number to position was presented in the upper left corner of the screen. Children first completed the 0-100 interval task and then the 0-1000 interval task. At the beginning of the experiment, children were asked to place the numbers 0, 100 and 50 in the interval 0-100 and 0, 1000 and 500 in the interval 0-1000. This ensured that children understood the task and the interval range, and that they were capable of using the mouse to respond. Moreover, when a response was given, a small red circle appeared in the selected position as visual feedback. After the practice phase, the other numbers were presented randomly. Both estimates and reaction times were recorded.

RESULTS

We removed responses under 200 ms (less than 0.002% of all trials) and above 2 standard deviations (less than 0.01% of all trials) across groups. Analyses followed the method recommended by Siegler and colleagues (Siegler & Booth, 2004; Siegler & Opfer, 2003) and Bonferroni's correction was applied to all post-hoc comparisons. Estimation accuracy was assessed using the percentage of absolute error of estimation (PAE = |estimate - target number|/ interval*100) for each participant in each condition. We analyzed the PAE in a mixed ANOVA with Group as the between-subject factor (TD and DD) and Interval size as the within-subject factor (0-100 and 0-1000). Mean PAEs in the 0-100 interval were 7% for TD children and 11% for children with DD. In the 0-1000 interval, the mean PAEs were 12% for TD children and 25% for children with DD (see Figure 1). Both main effects were significant ($F(1, 18) = 51.28$, $p < .001$ and $F(1, 18) = 11.12$, $p = .004$, for Interval and Group, respectively). Because the interaction was also significant ($F(1, 18) = 11.2$, $p = .003$), we performed separate t-tests to compare groups' performance in each interval. In the 0-100 interval, the two groups did not differ significantly ($t(18) = 2.1$, $p = .05$); whereas in the 0-1000 interval, DD showed lower accuracy in placing the number compared to the TD control group ($t(18) = 3.58$, $p = 0.002$). Mean RT were also analyzed in a mixed ANOVA with Group as the between-subject factor (TD and DD) and Interval as the within-subject factor (0-100 and 0-1000). Mean RTs in the 0-100 interval were 5.9 s ($SD = 2.4$ s) and 4.7 s ($SD = 1.8$ s) for TD and children with DD, respectively. In the 0-1000 interval, mean RTs were 5.3 s ($SD = 2.5$ s) and 5.3 s ($SD = 2.8$ s) for TD and children with DD, respectively. The main effects of the Group and of the Interval as well as the interaction Group x Interval did not reach significance ($p = .58$, $p = .98$, and $p = .13$, respectively).

In order to understand the pattern of estimates, we fitted the linear and the logarithmic functions first on group medians and subsequently individually for each child (Siegler & Opfer, 2003).

Median estimates and the best fitting models are reported separately for each group in Figure 3. The difference between linear and logarithmic models was tested with paired-sample t-tests on absolute distances between children's median estimate for each number and the predicted values according to the linear model and the logarithmic model. If the t-test was significant, the best fitting model was attributed to the group (Figure 2). In the 0-100 interval, the linear model had the highest R^2 for both groups and was significantly different from the logarithmic model for the TD group ($t(9) = 4.34$, $p = .002$, R^2 lin = 99%, vs. R^2 log = 87%) but not for the DD group ($t(9) < 1$, R^2 lin = 97% vs. R^2 log = 93%). In the 0-1000 interval, the linear model had the highest R^2 and was significantly different from the logarithmic model only for the TD group ($t(21) = 7.18$, $p < .001$, R^2 lin = 97% vs. R^2 log = 73%) whereas for the DD group the logarithmic model had the highest R^2 and was significantly different from the linear model ($t(21) = 3.13$, $p = .005$, R^2 lin = 67% vs. R^2 log = 96%).

Figure 1. **Percentage of absolute error (PAE) in TD and children with DD for the two NP-tasks. Children with DD showed lower accuracy in placing numbers in the 0-1000 interval as compared to the TD group. Error bars correspond to 95% CI. ** *p* < .01**

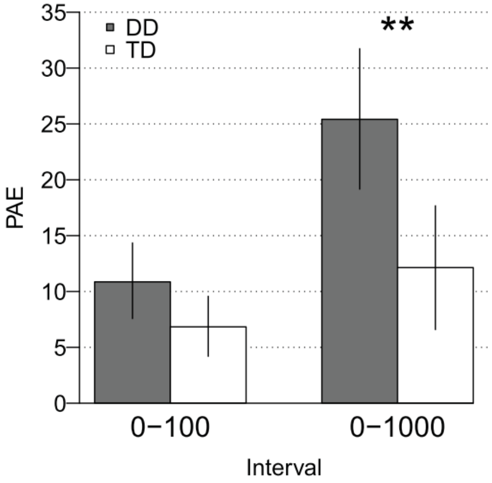
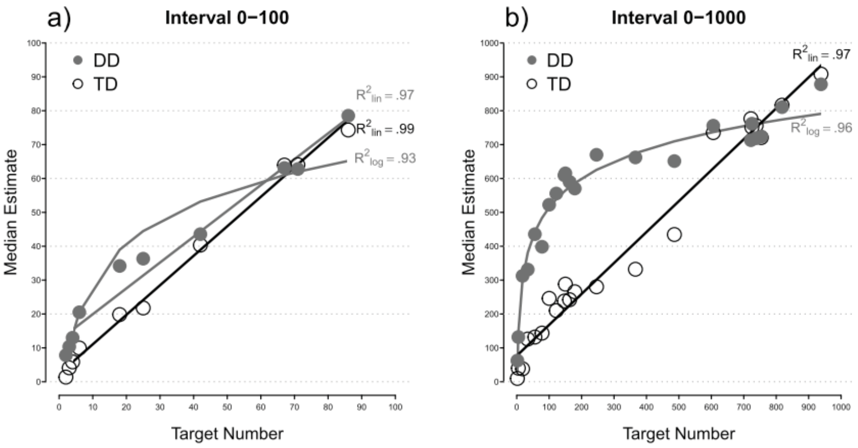


Figure 2. **Children estimates and best fitting models for the DD and TD group separately in (a) the 0-100 interval and (b) the 0-1000 interval. The TD group obtained a linear representation in both NP tasks, whereas the DD group showed an intermediate stage, between logarithmic and linear mapping, in the 0-100 interval and a logarithmic mapping in the 0-1000 interval.**



We ran linear and logarithmic regression analyses also on individual data, the child was assigned to a linear or logarithmic category based on the highest R^2. Whenever both models were not significant, the child was considered unable to perform the task properly and classified as not having a numerical mapping (Table 1).

Table 1. ***Cell values represent number of children (no children were classified as showing a non-numerical mapping)***

	Type of Mapping	
Interval	Logarithmic	Linear
0-100 Interval		
TD $_{(N = 10)}$	2	8
DD $_{(N = 10)}$	5	5
0-1000 Interval		
TD $_{(N = 10)}$	2	8
DD $_{(N = 10)}$	8	2

Individual analysis confirmed the group analysis results. In the 0-100 interval, TD children mostly displayed a linear mapping, whereas half of children with DD were classified as linear and the other half as logarithmic. In the 0-1000 interval, the individual analysis confirmed a predominant linear mapping for the TD group, whereas most of the children with DD displayed a logarithmic mapping. These results therefore reinforce the developmental delay of children with DD to properly estimate the position of numbers on the NP-task. Finally, no child, in both intervals, was categorized as having a non-numerical mapping, thus suggesting that all participants possessed sufficient knowledge of numbers to properly accomplish the tasks.

DISCUSSION

Several studies have demonstrated that children, from as early as preschool, progressively shift from a logarithmic to linear mapping in the NP-task (Berteletti et al., 2010; Siegler & Opfer, 2003). The logarithmic mapping is considered a direct evidence that children assign more space on the mental number line to small numerosities than to larger numerosities, following a logarithmically compression that is a signature of the ANS (Siegler & Booth, 2003; for different accounts, see Barth & Palladino, 2011; Eberbasch et al., 2008; Moeller et al., 2009b). With education, children learn to linearly translate numbers into the correct spatial position. Such fine mapping correlates both with other numerical tasks and more importantly also with math achievement as measured by standardized tests (Booth & Siegler, 2006; Booth & Siegler, 2008). Accordingly, children with math disability have lower estimation accuracy positioning numbers onto the physical line, thereby displaying an intuitive logarithmic representation instead of a formal linear representation (Geary et al., 2008; Landerl et al., 2009). In the present study, we tested the mapping between numbers and the spatial position onto the line in a selected sample of primary school Italian children with formal diagnosis of DD as compared to a group of TD children matched for gender and age. It is worth noting that children with DD displayed a time response that was similar to TD children. This result excludes that children with DD were more impulsive and that lower performance was the consequence of a speed-accuracy trade-off. Furthermore, all children were able to map numbers confirming the easiness in understanding task instructions. Poor accuracy in children with DD can be reliably ascribed to a specific deficit in representing numbers formally.

In line with previous studies, group and individual results indicate that children with DD mainly relied on an immature and biased-logarithmic mapping compared to TD controls. Half of the children with DD showed a logarithmic and less accurate mapping on both interval sizes. Compared to the group tested by Landerl and colleagues (2009), our sample of children with DD included a larger age range in which children were approximately one-year older. This might suggest that the deficit in the spatial mapping of numbers is still present in one-year-older children and not normalize for the 0-1000 interval. This supports the delayed development of an accurate numerical representation in children with DD. Indeed, 3- to 4- year younger TD children tested by Opfer and Siegler (2007) were able to perform the 0-1000 interval task linearly. The finding that children with DD have a performance that is delayed compared to TD peers is also in agreement with a previous study showing a basic deficit of the non-symbolic numerical representation (Piazza et al., 2010).

In contrast to more sophisticated mathematical tests to diagnose children with DD, the number line task only requires a core knowledge of numerical magnitudes and excludes possible deficits in more general or higher-order cognitive process such as working memory, attention or procedural knowledge. Furthermore, the task instructions are easy to understand and the materials (i.e., paper and pencil) are minimal, making it easy for teachers and clinicians to apply. The possibility of using different interval ranges makes it a potential tool for early diagnosis. Indeed, it has been shown that children prior to formal education already show linearity on the 1-10 interval (Berteletti et al., 2010), therefore making it possible to highlight at-risk children prior to the start of formal teaching. The NP-task has the potential for being a tool to assess early numerical skill in both TD and children with DD.

In summary, the present study highlights the specific delay in basic numerical processing in Italian children with DD: they display an immature representation of numbers compared to TD children with performances comparable to 3- to 4- year younger peers, and confirms the reliability of the NP-task to assess a delay in the representation of numerical symbols.

References

Barth, H. C., & Paladino, A. M. (2011). The development of numerical estimation: Evidence against a representational shift. *Developmental Science, 14*, 125-35.

Berteletti, I., Lucangeli, D., & Zorzi, M. (2012). Representation of numerical and non-numerical order in children. *Cognition, 124*, 304-313.

Berteletti, I., Lucangeli, D., Piazza, M., Dehaene, S., & Zorzi, M. (2010). Numerical estimation in preschoolers. *Developmental Psychology, 46*, 545-551.

Booth, J. L., & Siegler, R. S. (2006). Developmental and individual differences in pure numerical estimation. *Developmental Psychology, 42*, 189-201.

Booth, J. L., & Siegler, R. S. (2008). Numerical magnitude representations influence arithmetic learning. *Child Development, 79*, 1016-1031.

Ebersbach, M., Luwel, K., Frick, A., Onghena, P., & Verschaffel, L. (2008). The relationship between the shape of the mental number line and familiarity with numbers in 5- to 9-year old children: evidence for a segmented linear model. *Journal of Experimental Child Psychology, 99*, 1-17.

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences, 8*, 307-314.

Geary, D. C., Hoard, M. K., Byrd-Craven, J., Nugent, L., & Numtee, C. (2007). Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. *Child Development, 78*, 1343-1359.

Geary, D. C., Hoard, M. K., Nugent, L., & Byrd-Craven, J. (2008). Development of number line representations in children with mathematical learning disability. *Developmental Neuropsychology, 33*, 277-299.

Landerl, K., Bevan, A., & Butterworth, B. (2004). Developmental dyscalculia and basic numerical capacities: A study of 8-9-year-old students. *Cognition, 93*, 99-125.

Landerl, K., Fussenegger, B., Moll, K., & Willburger, E. (2009). Dyslexia and dyscalculia: two learning disorders with different cognitive profiles. *Journal of Experimental Child Psychology, 103*, 309-324.

Mandler, G., & Shebo, B. J. (1982). Subitizing: An analysis of its component processes. *Journal of Experimental Psychology, 111*, 1-22.

Mazzocco, M. M. M., Feigenson, L., & Halberda, J. (2011). Impaired acuity of the approximate number system underlies mathematical learning disability (dyscalculia). *Child Development, 82*, 1224-1237.

Moeller, K., Neuburger, S., Kaufmann, L., Landerl, K., & Nuerk, H.-C. (2009a). Basic number processing deficits in developmental dyscalculia: Evidence from eye-tracking. *Cognitive Development, 24*, 371-386.

Moeller, K., Pixner, S., Kaufmann, L., & Nuerk, H.-C. (2009b). Children's early mental number line: Logarithmic or decomposed linear? *Journal of Experimental Child Psychology, 103*, 503-515.

Opfer, J., & Siegler, S. (2007). Representational change and children's numerical estimation. *Cognitive Psychology, 55*, 169-195.

Piazza, M. (2010). Neurocognitive start-up tools for symbolic number representations. *Trends in Cognitive Sciences, 14*, 542-551.

Piazza, M., Facoetti, A., Trussardi, A. N., Berteletti, I., Conte, S., Lucangeli, D., Dehaene, S., & Zorzi. M. (2010). Developmental trajectory of number acuity reveals a severe impairment in developmental dyscalculia. *Cognition, 116*, 33-41.

Schleifer, P., & Landerl, K. (2011). Subitizing and counting in typical and atypical development. Developmental Science, *14*, 280-291.

Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Development, 75*, 428-44.

Siegler, R. S., & Opfer, J. E. (2003). The development of numerical estimation: evidence for multiple representations of numerical quantity. *Psychological Science, 14*, 237-243.

Thompson, C. A., & Siegler, R. S. (2010). Linear numerical-magnitude representations aid children's memory for numbers. *Psychological Science, 21*, 1274-1281.

Verguts, T., Fias, W., & Stevens, M. (2005). A model of exact small-number representation. *Psychonomic Bulletin & Review, 12*, 66-80.

Zorzi, M., & Butterworth, B. (1999). A computational model of number comparison. In M. Hahn & S. C. Stoness (Eds.), *Proceedings of the Twenty-First Annual Conference of the Cognitive Science Society* (pp. 778-783). Mahwah, NJ: Erlbaum.