MOVING TOWARD POSITIVE MATHEMATICS BELIEFS AND DEVELOPING SOCIO-MATHEMATICAL AUTHORITY: URBAN PRESERVICE TEACHERS IN MATHEMATICS METHODS COURSES

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ABSTRACT

This article explores how preservice elementary teachers change their negative beliefs toward mathematics into positive ones after taking a mathematics methods course that follows the Concrete-Pictorial-Abstract (CPA) instructional method. Also explored is the relationship between those beliefs and sociomathematical authority. By administering surveys, using reflective narratives, and informally interviewing 145 preservice teachers who are preparing to work in urban classrooms during two academic years, the study investigated changes in sociomathematical beliefs and mathematical identities. Findings reveal that most of these preservice teachers changed their mathematical identities and beliefs about teaching mathematics in positive ways to be more in accordance with reform oriented practices. Implications for preservice teacher education programs are discussed.

Keywords: Mathematics beliefs; sociomathematical authority; teacher identity; preservice teachers; elementary mathematics education

Introduction

Focusing on preservice teachers' beliefs in mathematics methods courses is a critical issue because teachers' personal beliefs about a subject matter affect the entire classroom climate and the numerous decisions they routinely make (Spillane, 2000). This study focused on the social context of mathematics beliefs which has not been given as much attention as more cognitive aspects (Gates, 2006). In mathematics education, changing teachers' traditional beliefs and practices is thought to be crucial to the success of the reform effort (Battista, 1994). Leder, Pehkonen, and Törner (2002) claim that no consistent pattern has been found within the area of mathematics teachers' belief change. This lack of

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consistency raises questions about underlying causes of teacher change and what teacher educators can do about it (Gates, 2006). In order to support reform-based mathematics, teacher education programs must have a profound understanding of how teachers' beliefs, knowledge, and teaching practices are related (Ball, Lubinski, & Mewborn, 2001; Bray, 2011).

This article documents how preservice elementary school teachers can change their negative beliefs toward mathematics into positive ones after taking a mathematics methods course that utilizes the constructivist Concrete-Pictorial-Abstract (CPA) Method that follows Bruner's (1966) cognitive growth theory of enaction, imagery, and symbolic representation. Our goal is to explore how preservice teachers' beliefs shift through the period of a mathematics methods course, and how this shift in beliefs affects their classroom practices, and their mathematical identities. The shift in beliefs is meaningful to preservice teachers' motivation, sociomathematical authority, and mathematics teaching practices.

Theoretical Framework

This study is grounded in teachers' identity (Gee, 2001; Gresalfi & Cobb, 2011), sociomathematical authority (Cobb, Gresalfi, & Hodge, 2009; Yackel & Cobb, 1996), beliefs and experiences (Fennema et al., 1996; Thompson, 1992), and Concrete-Pictorial-Abstract methodology (Bruner, 1966). Sociomathematical authority is defined as mathematical dispositions and a sense of intellectual autonomy in mathematics (Yackel & Cobb, 1996). According to Thompson (1992), teachers' beliefs about mathematics teaching are comprised of "Personal Philosophies" about mathematics teaching and related mathematics approaches. Teachers' beliefs involve not only their ideas about their mathematical competencies and their ideas about mathematics teaching, but also their motivation to teach in effective/non-effective ways. A teacher's mathematical belief is very important because it influences teachers' perceptions of mathematics and serves as a guiding force in mathematics teaching and mathematics learning (Bray, 2011; Calderhead, 1996; Franke, Fennema, & Carpenter, 1997). Effective mathematics teaching requires teachers' positive beliefs about mathematics, content knowledge, and knowledge of how to teach mathematics, and sociomathematical authority.

In this article, the concept of identity has been developed within an amalgam of the frameworks of Gee (2001) and Gresalfi and Cobb (2011). Accordingly, we conceptualize mathematics identity as encompassing (a) an individual teacher's self-perception with regard to their knowledge of mathematics; (b) their confidence level to teach mathematics; and (c) a set of practices and expectations that shape individual teacher's beliefs about their mathematics teaching competencies. In this article, we view mathematics identity in the context of mathematics teaching and learning, and competence to teach mathematics in school settings (Calderhead, 1996; Franke et al., 1997; Leatham, 2006). Preservice teachers' instructional strategies and their motivation to provide

meaningful mathematics instruction are driven by their mathematics identity, sociomathematical authority, and their confidence to teach mathematics. We stress that the process of preservice teachers' mathematical identity formation is profoundly influenced by the norms, values, and practices of the specific context of the mathematics method course(s), their prior mathematics experiences, and their early field experiences.

Our stance is that teachers' beliefs can be modified through positive experiences in undergraduate and graduate mathematics methods courses so that preservice teachers learn to grapple with mathematics concepts and can teach reform-based mathematics (CCSSO, 2010; National Council of Teachers of Mathematics, 2007). The reform-based mathematics standards suggest that teachers develop constructivist learning and teaching environments in which children learn by "doing mathematics." Given the demands of reform-based high quality mathematics education, teachers must teach and "engage in forms of mathematical activity that differ from those they experienced as students and also requires that they reconceptualize what it means to do mathematics" (Gresalfi & Cobb, 2011, p. 272). In this context, teacher education programs have the responsibility to prepare teachers to teach reform-based mathematics, adapt a constructivist approach of teaching, and teach mathematics effectively. However, teacher education programs are often faced with challenges of teaching individuals who learned mathematics during their formative years in a traditional environment and developed negative attitudes and beliefs of mathematics and view mathematics teaching as an arduous and unpleasant task. In order to prepare effective mathematics teachers, it is important to understand how teachers' beliefs and attitudes influence their reform-based teaching practices and what intervention(s) are needed to bring positive shifts in their beliefs and identities.

Concrete-Pictorial-Abstract (CPA) Methodology

The mathematics methods course at the center of this study follows the Concrete-Pictorial-Abstract (CPA) methodology. Cognitive growth and educational learning theorist Bruner (1966) believes that children pass through concrete-pictorial-abstract stages of cognitive development, and children learn best by constructing their own knowledge through concrete-pictorial-abstract activities. CPA methodology is based on Bruner's three stages of learning: *enactive*, *iconic*, and *symbolic*. The *enactive* representation denotes that children learn by actively engaging in experiences and associating the concrete experiences with past experiences and information stored in memory. The *iconic* representation is of mental/visual images of an activity. These visual/pictorial images of concrete experiences help children make meaning of complex mathematics concepts. According to Bruner, the *symbolic* representation is the final stage of mathematics learning. At this stage, individuals connect the mathematical concepts with symbols or language.

Methodology and Research Design

This ethnographic study was conducted over two academic years and was contextualized in a constructivist paradigm. It was designed to examine shifts in preservice elementary school teachers' beliefs toward mathematics. The study followed designed-based research (DBR) methodology for education research that is situated in a genuine educational context. According to Anderson and Shattuck (2012), DBR is designed for educators to translate education research into improved practices. They define DBR as being "situated in a real educational context provides a sense of validity to the research and ensures that the results can be effectively used to assess, inform, and improve practices" (Andersib & Shattuck, 2012, p. 16).

The DBR methodology is designed specifically to bring improvement in local practices and create changes (Brown, 1992; Cobb, 2002; Cobb, Stephan, McClain, & Gravemeijer, 2001). We used DBR to design and organize our instructional strategies and learning activities. The practical aspect of DBR focuses on selecting and applying an intervention in the form of pedagogical design or technology (Anderson & Shattuck, 2012). In this study, we have used CPA instructional methods, teaching models, and teaching strategies as interventions to support preservice teachers' understanding of mathematics and bring changes in negative beliefs and perceptions of mathematics.

Context of the Study

The sites purposefully selected for this study were two colleges, Old College and Songsville College (pseudonyms), located in the northeast United States. Old College is situated in an urban lower-income and predominantly African American neighborhood while Songsville College is located in a predominantly White middle class community. The study was contextualized in undergraduate and graduate elementary mathematics methods courses with 145 participants during the 2010-2011 and 2011-2012 academic years. These participants ranged in age from 20 to 60 with 124 of them being female and 21 being male. The racial/ethnic composition of the participant population was as follows: 50 participants were White, 22 were Hispanic, 66 were Afro/Caribbean, and seven were of another ethnicity. The majority of the participants intend to work in urban elementary schools when they complete their degrees.

The specific focus of the mathematics methods course was to engage preservice teachers in reform-based quality mathematical activities and require them to design lesson plans and other instructional activities following CPA methodology. An Early Field Experience (EFE) was attached with this course. The EFE course required preservice teachers to spend 10-12 hours in an elementary classroom during mathematics periods as participant observers, to teach one mathematics lesson in the elementary classroom, and demonstrate one mathematics lesson in their methods classroom. Most of the participants conducted their EFE in urban elementary classrooms.

For consistency and to create similar communities of practice, both professors teaching the elementary mathematics methods courses and conducting this study—one at Old College and one at Songsville College—utilized the same textbook, *Elementary and middle school mathematics: Teaching developmentally* (Van de Walle, Karp, & Bay-Williams, 2010), and taught using the same CPA method.

Data Sources and Analysis

Since this study focused on how constructivist mathematics methods courses influence preservice teachers' beliefs and development of sociomathematical authority, the analysis centered on ten mathematics methods courses and observations of students teaching mathematics lessons in predominantly urban elementary school classrooms within the Fall 2010, Spring 2011, and Fall 2011 semesters.

Data for this study consisted of reflective narratives, surveys, transcripts of informal interviews, and field notes. To explore teachers' beliefs before and after taking the mathematics methods courses, Table 1 showcases the data sources we utilized, when the data sources were collected, and a description/sample prompts for each data source.

Data analysis was an ongoing process. Following qualitative research design (Guba & Lincoln, 1989), we began to analyze qualitative data as soon as we collected them. Data were analyzed utilizing Grounded Theory (Strauss, 1987) where theorizing grows from the data rather than from a pre-existing framework used to confirm or disconfirm a theory. Through document analysis of reflective narratives, surveys, transcripts of informal interviews, and field notes, codes were developed based on categories which emerged within mathematics identities, conceptions of the nature of mathematics, and best practices in mathematics teaching and learning.

In order to strengthen the trustworthiness of our findings, we followed a framework of prolonged engagement, member checking, and triangulation of data from multiple sources (Guba & Lincoln, 1989). The member-checking process provided us opportunities to verify data with participants who provided them (Guba & Lincoln, 1989). During the collection and analysis period, we constantly asked participants to verify their responses for accuracy and consistency between what was recorded and what was intended to communicate because "the most certain test is verifying those multiple constructions with those who provided them" (Guba & Lincoln, 1989, p. 239). The member checking provided us chances to correct errors, and it allowed participants to confirm data and judge the adequacy of their responses. The study took place over two years, but we were only with each of the ten groups of methods course participants for four months at a time; this time presented us with "fronts" to "establish the rapport and build the trust to uncover construction" (Guba & Lincoln, 1989, p. 237).

Table 1

Data sources and sample prompts

Data Source	Timeframe During Each Semester of the Study	Description/Sample Prompts
Reflective essays about mathematical experiences (e.g., mathematics autobiographies and walking down memory lane activities)	Month 1	Prompts for these essays included: *How would you define mathematics? *What is your relationship with mathematics? What experiences led you to this relationship? *Who or what do you believe have been your greatest influences on how you define mathematics and how you feel about the content area? *What do you believe are best practices for teaching mathematics? How did you arrive at those beliefs?
Open-ended surveys	Month 1	Participants had to briefly write about their beliefs about what mathematics is, what learning mathematics requires, what constitutes good mathematics teaching, and self-analysis of their mathematics knowledge.
Mathematics reflective journal writing	Months 1 through 4	Sample prompts included: *What are your strengths/weaknesses in learning mathematics concepts and procedures? *Does the CPA method make a difference in learning mathematics? *How, if at all, was the enactment of your lesson different than your expectations?
Informal interviews	Months 1 through 4	Informally interviewed methods course participants about their experiences of learning mathematics and teaching mathematics to children.
Class discussions	Months 1 through 4	Sample prompts included: *How did you incorporate the CPA method into your lesson planning and instruction? *What strategies and activities did you plan for each stage of the CPA method?
End of the semester participant survey questions and reflections	Month 4	Sample questions included: *How, if at all, have you changed your beliefs about yourself in relation to mathematics? *How, if at all, have you changed your beliefs about what mathematics is? *How, if at all, have you changed your beliefs about what constitutes best practices in mathematics teaching?

To document changes in mathematics beliefs, comparisons were noted between initial assessments of participants' beliefs through the beginning of the semester activities during month one and their final assessments through the end of semester survey questions and reflections during month four. Data from the beginning of the semester and the end-of-semester survey questions and reflections were used to analyze each participant's responses in relation to changes in mathematics identities, changes in the nature of mathematics, changes in best practices in mathematics teaching, and the development of sociomathematical authority.

Findings

Findings reveal that although the majority of the preservice teachers entered the mathematics methods courses with negative beliefs about their mathematics identities and very fixed conceptions about what mathematics actually is based on their past experiences and influences, by the end of the semester, they had positive shifts in their mathematics beliefs in three areas: mathematics identities, the nature of mathematics, and best practices in mathematics teaching.

Change in Mathematics Identities

Mathematics identity encompasses an individual's self-perception of their knowledge of mathematics, confidence level to teach mathematics, and beliefs about their mathematics teaching competencies. The top noted change in mathematics identities reported by the majority of the participants was their comfort level and confidence in mathematics by the end of the mathematics methods course. They believed that they were much more confident in doing and teaching mathematics because they not only refreshed their knowledge of elementary mathematics, but experienced how much it is relatable to and useful in their real lives. The higher level of confidence increased their sociomathematical authority and enabled them to construct positive mathematics identities. Many of the preservice teachers noted that the comfortable atmosphere within the methods courses also aided in this shift because they were free to really ask questions to deepen their mathematics knowledge. As one preservice teacher wrote in her end of semester survey in fall 2010, "I know now that math is not impossible for me; it just takes practice and thinking about numbers in a different way than how I was taught growing up." This belief change seemed to occur because the participant experienced CPA methodology of mathematics teaching and was "engaged in forms of mathematical activity that differed from those they experienced as students" (Cobb & Gresalfi, 2011, p. 272). Similar to this participant, many participants reconceptualized mathematics teaching and learning during the course.

The participants also felt more comfortable and confident because they now had a deeper understanding of the conceptual processes rather than mere

procedural knowledge. They understood the "why", the "bigger picture" behind the concepts; these were rarely explained to them in their past mathematics courses/experiences. As another preservice teacher mentioned,

> I've learned that conceptual knowledge is more important than procedural knowledge when it comes to math. If you can conceptualize ideas and see the interrelationship of numbers, and strategies that can be used, solving problems will be more enjoyable and less scary. (End of semester survey, Spring 2011)

This key shift in the importance of developing conceptual understanding was echoed by the majority of the participants. They articulated that understanding mathematical concepts was more powerful than fluency of mathematical procedures.

Change in the Nature of Mathematics

Many of the preservice teachers entered the mathematics methods course with an instrumentalist view (Ernest, 1988) of mathematics as being equivalent to an accumulation of facts, rules, and skills to be used for some external end. This traditional approach is what many of the teachers grew up with. At the end of the course, however, participants began to shift their initial traditional conceptions toward more reform-oriented views as mathematics is a process or inquiry, coming to know and adding to the sum of knowledge. Mathematics, for many of them, was now viewed as being more dynamic and less fixed, more creative and less rigid, more relative to life, and deeper than just numbers. This is echoed by preservice teacher candidates' comments:

> Prior to this course, I felt mathematics was fixed and lacked creativity. However, I learned that mathematics is dynamic and can be taught in an exciting and beneficial way. Mathematics is much more than memorization; it is a subject area that allows for active inquiry and engagement.(End of semester survey, Fall 2011)

> I think that I always felt like math was about rules and formulas. After this course, I realize that math is ever changing and for many problems there isn't only one way to come to the solution. Math is also so much more involved in our everyday lives than I ever realized. I now understand the look on my teachers' faces when students said 'why are we even learning this?' (End of semester survey, Spring 2011)

Overall, preservice teachers in the methods courses began to see mathematics as much more flexible since they experienced, first-hand, multiple ways to approach and solve problems.

Change in Best Practices in Mathematics Teaching

The sentiment echoed by the majority of the preservice teachers was that they learned how valuable using manipulatives are to teach mathematics. Many never realized what a difference they can make and noted that if they had used them in their formative years, their outlook on mathematics may have been more positive from the onset. As one preservice teacher asserted, "After having taken this course, I strongly believe in the use of manipulatives. I never realized what a difference they can really make in developing conceptual understanding" (End of semester survey, fall 2010). The "use of manipulatives" is also interpreted as active engagement.

These preservice teachers also learned how important it is for teachers to offer students several different techniques/strategies to approach/solve a problem; there is not just one avenue or approach and that teachers themselves, need to be equipped with many strategies/approaches to problems. As one preservice teacher wrote,

> I now truly appreciate the importance of exploration and student discovery. If you allow your students to come up with their own problem-solving methods, you give them a tool that helps them solve any problem. (End of semester survey, Spring 2011)

The importance of differentiated instruction was also reported. Furthermore, the idea that teachers must promote conceptual understanding via discovery activities was critical.

Discussion and Implications

Teachers are at the forefront of this study. Although current reform efforts can enumerate desired mathematics changes, no curriculum teaches itself. Therefore, it is important to look at the affective aspects of teaching and social influences on mathematics in order to help bring about the desired changes through additional support systems and/or look at challenges which can hinder change or high-quality mathematics teaching.

This study focused on the social context of mathematics beliefs which has not been given as much attention as more cognitive aspects have been explored (Gates, 2006). Findings from this study reveal that it is possible for teachers to change beliefs to be more aligned with reform oriented practices in the course of a semester. However, they need to be exposed to constructivist methods consistently, be given the chance to reflect on their beliefs and practices in a variety of ways, and come to these new beliefs on their own through experience.

Through the preservice teachers' changes in mathematics beliefs and

reflections on those changes during the semester, they reported feeling prepared to teach the content when they assume their own classrooms since they now had a much greater conceptual understanding of mathematics. They thought about mathematics differently, and their newfound confidence has made many preservice teachers actually exited to teach mathematics. Overall, most participants noted that mathematics was no longer a "scary subject" or an impossible task; they adopted a more positive outlook toward mathematics and deeper appreciation for it as a result of the CPA approach taken in the mathematics methods courses. This is echoed in comments by preservice teachers in the end of semester survey in Fall, 2011:

> After this course, I gained a more positive outlook on mathematics. I no longer see mathematics as boring and absolute. Instead, I feel mathematics is full of opportunities to be creative and have fun. I no longer feel intimidated about teaching mathematics due to the knowledge I gained about teaching mathematics and the opportunity to teach my own lesson.

> This course has exposed me to amazing ways of teaching math. I must admit that I wish I was introduced to math through these methods of teaching. My guess is that I would have learned to love math from the very beginning. Therefore, my belief about best practices in teaching math are the ones that teach you to understand, appreciate, and love math rather than just memorizing rules and facts.

Teachers in this study shifted their beliefs which can only set them on more positive teaching journeys when they enter their own classrooms. These positive shifts in beliefs can likely translate into more positive experiences for students as they will not perpetuate a negative cycle of mathematics affect. With this shift in beliefs, they are also more able to develop sociomathematical authority and can become autonomous mathematics teachers. Given the importance of the social context of mathematics beliefs, the significance of participants' change in their beliefs is important as they are the ones who will be teaching according to their beliefs.

This study can also be beneficial to teacher education programs because it highlights the importance of incorporating CPA methods into practice and providing avenues for continuous teacher reflection such as the use of mathematics autobiographies, mathematics journals, and essays. With these practices, preservice teachers are more likely to develop sociomathematical authority which can potentially translate into meeting the expectations in the reform movement and increased student achievement in urban settings.

References

Anderson, T., & Shattuck, J., (2012). Designed-based research: A decade of progress in education research? *Educational Researcher, 41*(1), 16-25.

Ball, D.L., Lubinski, S.T., & Mewborn, D.S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching* (4th ed.) (pp. 433-456). Washington, DC: American Educational Research Association.

Battista, M. (1994). Teacher beliefs and the reform movement in mathematics education. *Phi Delta Kappan, 75*(6), 462-470.

Bray, W.S. (2011). A collective case study of the influence of teachers' beliefs and knowledge on error-handling practices during class discussion of mathematics. *Journal for Research in Mathematics Education, 42*(1), 2-38.

Brown, A. (1992). Design-experiments: Theoretical and methodological challenges in creating complex interventions in classroom settings. *Journal of Learning Science, 2*(2), 141-178.

Bruner, J.S. (1966). *Towards a theory of instruction.* Cambridge, MA: The Belknap Press of Harvard University Press.

Calderhead, J. (1996). Teachers: Belief and knowledge. In D.C. Berliner & R.C. Calfee (Eds.), *Handbook of educational psychology* (pp. 709-725). New York: Macmillan.

Cobb, P. (2002). Modeling, symbolizing, and tool use in statistical analysis. In K. Gravemeijer. R. Lehrer, B. van Oers, & L.Verschaffel (Eds). *Symbolizing, modeling, and tool use in mathematics education* (pp. 171-195). Dordrecht, The Netherlands: Kulwer Academic Publishers.

Cobb, P., Gresalfi, M. S., & Hodge, L.L. (2009). An interpretive scheme for analyzing the identities that students develop in mathematics classrooms. *Journal for Research in Mathematics Education, 40*(1), 40-68.

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in classroom mathematical practices. *The Journal of Learning Sciences, 10*(1), 113-164.

Council of Chief State School Officers [CCSSO]. (2010). Common Core State Standards for Mathematics. Washington, DC: CCSSO.

Ernest, P. (1988). *The impact of beliefs on the teaching of mathematics.* Paper presented at the Sixth International Congress of Mathematical Education, Budapest, Hungary.

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V.R., & Empson, S.B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education, 27*(4), 403-434.

Franke, M.L., Fennema, E., & Carpenter, T. P. (1997). Changing teachers: Interactions between beliefs and classroom practice. In E. Fennema & B.S. Nelson (Eds.), *Mathematics teachers in transition* (pp. 255-282). Mahwah, NJ: Erlbaum.

Gates, P. (2006). Going beyond belief systems: Exploring a model for the social

influence on mathematics teacher beliefs. *Educational Studies in Mathematics, 63*(3), 347-369.

Gee, J. P. (2001). Identity as analytic lens for research in education. *Review of Research in Education, 25*(1), 99-125.

Gresalfi, M. S., & Cobb, P. (2011). Negotiating identities for mathematics teaching in the context of professional development. *Journal for Research in Mathematics Education, 42*(3), 270-304.

Guba, E. G., & Lincoln Y.S. (1989). *Fourth Generation Evaluation*. California: Sage.

Leatham, K.R. (2006). Viewing mathematics teachers' beliefs as sensible systems. *Journal of Mathematics Teacher Education, 9*(1), 91-102.

Leder, G., Pehkonen, E., & Törner, G. (2002). Setting the scene. In G. Leder, E. Pehkonen & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education* (pp. 1-10). Dordrecht, The Netherlands: Kluwer Academic Publishers.

National Council of Teachers of Mathematics [NCTM]. (2007). *Mathematics teaching today.* Reston, VA: NCTM.

Spillane, J. C. (2000). A fifth-grade teacher's reconstruction of mathematics and literacy teaching: Exploring interactions among identity, learning, and subject matter. *Elementary School Journal, 100*(4), 307-330.

Strauss, A. L. (1987). *Qualitative analysis for social scientists*. Cambridge, MA: Cambridge University Press.

Thompson, A.G. (1992). Teachers' belief and conceptions: A synthesis of research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York, NY: Macmillan.

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2010). *Elementary and middle school mathematics: Teaching developmentally* (7th ed.). Boston, MA: Allyn & Bacon.

Yackel, E. & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458-477.