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Building Students′ Reasoning Skills by Promoting Student-Led Discussions in an Algebra II Class

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Current research and professional organizations call for greater emphasis on reasoning and sense making in algebra (Chazan, 2000; Cuoco, Goldenberg, & Mark, 1996; Harel & Sowder, 2005; National Council of Teachers of Mathematics [NCTM], 2009, 2010). This paper illustrates how students in an Algebra II class had opportunities to develop their reasoning and sense making skills while discussing a problem about piecewise linear functions in small groups. In particular, students displayed a capacity for provoking each other to extend their prior knowledge, for making use of multiple representations, and for making connections with their non-mathematical prior experiences. We discuss the teacher's work in encouraging specific reasoning skills to support individual groups' work on the problem. We argue that a teacher's implementation of tasks that allow students the autonomy to work productively and promote student discussions of a problem can provide an avenue through which students in algebra may develop their reasoning and sense making skills.

Current research emphasizes the importance of students engaging in reasoning and sense making in all mathematical disciplines (Schoenfeld, 1992; Stein, Grover, & Henningsen, 1996). The National Council of Teachers of Mathematics [NCTM] (2009, 2010) has increasingly made calls for reasoning and sense making skills to be incorporated throughout the mathematics curriculum, including in high school algebra. Moreover, the Common Core State Standards

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for Mathematics [CCSSM] (National Governors Association Center for Best Practices, Council of Chief State School Officers [NGAC], 2010) identify the ability to reason abstractly and quantitatively as one of the eight core practices that students should develop through their study of mathematics. According to the CCSSM, students in all grades should develop abilities to contextualize problems as well as to abstract mathematical ideas, and they should be able to perform operations flexibly in conjunction with careful reasoning about a problem. Reasoning and sense making are not only important skills for learning new mathematical content, but also can be considered a primary goal of mathematics education.

Reasoning and sense making skills have been specifically emphasized in the geometry classroom (National Council of Teachers of Mathematics [NCTM], 2009, p. 18). Incorporating reasoning and sense making habits into an algebra class presents a unique challenge, as algebra has traditionally been characterized by a focus on symbolic manipulation (Smith, 2003; Kilpatrick & Izsák, 2008). Given the recent emphasis on reasoning and sense making in all mathematical disciplines, it is important to consider ways that teachers may support students' development of such skills in their classrooms. In this paper we examine how group discussions during a problem-based lesson in Algebra II afforded opportunities for students to develop their reasoning and sense making skills, as well as the teacher's actions towards supporting those opportunities. We use examples from a lesson about piecewise linear functions taught by a teacher in three Algebra II classes to show that, when given the opportunity to discuss the problem in small groups, students provoked one another to reason about the problem. Moreover, we identify teaching actions that promoted students' use of reasoning and sense making skills in algebra.

Perspectives

Reasoning and Sense Making in Algebra

We consider reasoning as "the process of drawing conclusions on the basis of evidence or stated assumptions" and sense making as "developing understanding of a situation, context, or concept by connecting it with existing knowledge" (NCTM, 2009, p. 4). Taken together, the focus on reasoning and sense making reflects an emphasis on the processes by which students build mathematical knowledge. The implication of this emphasis is that the reasoning skills that students develop through their study of mathematics may be as important as the mathematical content of their study. This view reflects a shift in focus from the content of mathematical knowledge to the process by which mathematics is done. Through a focus on the reasoning skills that students develop as they build mathematical knowledge, the activity of school mathematics may reflect more closely the activity of the mathematical discipline (Cuoco, Goldenberg, & Marc, 1996; Harel & Sowder, 2005; Lampert, 1990).

Reasoning and sense making in mathematics education provide a way for students to participate in the activities of the discipline of mathematics. Lakatos (1976) argued that, contrary to what many students have come to know, mathematical knowledge is not immediately and inevitably deduced from a small set of axioms. Rather, what we know to be true in mathematics comes from an iterative process of making conjectures, "proving" conjectures, finding counterexamples, and re-examining proofs. The notion of reasoning has come to include a broad range of skills and habits such as identifying patterns, making conjectures, and providing non-proof arguments (Stylianides, 2008). According to the National Research Council (1989), the process of doing mathematics "involves observations of patterns, testing of conjectures, and estimation of results" (p. 31). A critical aspect of doing mathematics is the ability to make an argument based on some evidence, check the reasonableness of that argument, and revise when necessary. This perspective brings to light the ways in which reasoning and sense making is an integral aspect of all mathematical disciplines, including algebra.

To illustrate how reasoning and sense making can become a regular feature of mathematics classrooms, NCTM (2009) highlighted specific categories of habits that should become routine across different content areas, classrooms, and schools. The first category includes habits for analyzing a problem, (e.g., identifying the relevant concepts, looking for patterns, and applying and extending previously learned concepts to new situations). The second category includes habits for implementing a strategy (e.g., making purposeful use of procedures and monitoring one's progress towards a solution). The third category includes habits for seeking and using connections (e.g., making connections between different domains of mathematics or different representations of a problem). The fourth category includes habits for reflecting on a solution to a problem (e.g., interpreting how a solution solves a problem, considering the reasonableness of that solution, and refining arguments). The four categories of reasoning and sense making habits suggest that students should be using reasoning and sense making throughout all phases of working on a problem, and these habits apply to all problems in all content areas.

In this paper we have highlighted three of NCTM's (2009) specific habits that surfaced during students' work on a problem about piecewise linear functions and rates of change. The first of these habits is applying previously learned concepts. This habit is part of the work of analyzing a problem. By applying previously learned concepts, students should recognize when mathematical ideas they have already learned can help towards solving new problems. Previously learned concepts provide an entry point when working on a new problem, and these concepts can be expanded to accommodate new situations. The second habit is using connections between different representations, which is one way that students would seek and use connections while working on a problem. To use connections between different representations, students recognize how different representations highlight different aspects of the same problem and inform each other. The third habit that we have highlighted is using non-mathematical prior knowledge to consider the reasonableness of a solution. This habit is part of reflecting on a solution to a problem. To consider the reasonableness of a solution, students may use their own prior experiences to check whether the conclusions they draw through mathematical means make sense in light of their real-world experiences. The examples we provide are not meant to be an exhaustive list of the skills that students developed or used during the lesson, but rather they serve as evidence of the opportunities that students had to develop reasoning and sense making skills.

Discourse

We are aligned with perspectives that assume that learning mathematics involves becoming part of a community of practice through discourse (Lave & Wenger, 1991; Sfard, 2001) and that communication is both a goal and a means of mathematics instruction (Lampert & Cobb, 2003). For example, Slavit (2006) found that not only are students' collaborative problem solving practices a product of their individual sense making, but also the process of collaboration provokes further sense making in each individual and in the group as a whole. Talking about mathematics is a way for students to formulate ideas and make sense of their own understanding; at the same time, group conversations make students' thoughts public, giving other members of a class a chance to check their own understanding, ask questions, or refute claims (Pimm, 1987). Students develop their mathematical ability through their attempts to communicate ideas.

Students' participation in classroom discourse plays a large role in their initiation into the reasoning practices and dispositions of mathematically literate adults (Forman, 1996). For example, Lampert (1990) fostered social interactions in which students formed arguments in response to their peers' conjectures, teaching students to "regard themselves as a mathematical community of discourse, capable of ascertaining the legitimacy of any member's assertions using a mathematical form of argument" (p. 42). When working on a problem to which there is no clear solution method, talking about and refuting one another's arguments made students' thinking a collaborative activity. A study of students' conversations may provide valuable insight into their developing reasoning and sense making skills and habits. For our purposes, an analysis of students' discussions offers insight into how students developed their reasoning skills at the moment they were working on a problem. The classroom discussions act as a snapshot into the processes that students went through in order to develop these skills.

Data and Analysis

Participants and Classroom Context

The setting for our study was Coral High School[1], a rural Midwestern school in the United States. The teacher in this study, Mr. Taylor, taught three sections of a regular Algebra II class made up primarily of juniors and seniors. Mr. Taylor did not use problem-based instruction regularly in his classes. He had elected to participate in the study as an opportunity to incorporate more problem solving into his teaching. The students had not participated in a problem-based lesson in Mr. Taylor's class previous to this study. Working in groups was an unusual activity for the students in Mr. Taylor's classes, and students in Mr. Taylor's class were unaccustomed to working on a single problem over an entire class period. Mr. Taylor and his students offer an example of how students in a traditional American high school classroom may employ reasoning habits through work on a problem-based lesson when this is a new activity in their classroom.

The "Going to Yellow Park" Problem

The "Going to Yellow Park" problem is about a group of friends traveling from different places to a common destination—Yellow Park (Figure 1). Through a variety of media, the friends communicate to each other about their distances from the park and their traveling speeds at different points in their journey; students must use the clues provided by each of the friends' messages to determine at what time each friend will arrive at Yellow Park. In this article we focus on students' work on messages from two of the friends—Karl and Isabel (see Figures 2 and 3). The Yellow Park problem assumes that students have studied linear functions, and the goal of the problem is to create a context in which students develop the need to use piecewise linear functions. Small-group discussions were important in the implementation of the lesson, because the teacher intended for students to connect their prior knowledge about linear functions with their work on the problem to define and understand piecewise linear functions.

Karl and his friends set out for a camping trip at Yellow Park. They are all leaving from different locations, and it is very important that someone arrives there by 8 PM. Otherwise, they will lose their reservation and their $100 deposit. Karl and his friends have been sending messages to each other and to Ms. Linn, the administrator of the park. She is receiving these updates and she needs your help deciding if they will be able to make it on time. Use the clues to figure out the order in which Karl and his friends will arrive to the park and whether anyone will make it before 8:00.
Figure 1. The introduction to the Yellow Park problem.

Mr. Taylor used the "Going to Yellow Park" problem in his 3 Algebra II classes over two consecutive days. We video- and audio-recorded each class period. During the time when students worked in groups, we video-recorded two groups of students in each class period, selected according to which students had elected to participate in the study. During the first day, Mr. Taylor introduced the problem, and students worked on the problem in small groups for the majority of the class period. During the second day, Mr. Taylor led a whole class discussion of the problem. The period of small group work on the first day of the lesson provided the greatest opportunity for students to have extended conversations about the problem with their peers. For this reason, we focused our analysis on the small group discussions to gain evidence of students' reasoning and sense making habits.

Analysis

In our analysis we looked for moments during group work when students showed evidence of the reasoning and sense making habits outlined by NCTM. Since our focus is specifically on how classroom discourse can serve as a tool for engaging in mathematical practices, we sought examples of reasoning and sense making that were enacted through students' conversations with one another. The first author reviewed the classroom videos, transcribing key segments when students displayed evidence of reasoning and sense making. We looked for moments in students' dialogue when they displayed the specific habits identified by NCTM as constituting reasoning and sense making (2010). The second author reviewed those segments to clarify and confirm the reasoning and sense making behaviors that the first author had identified. We discussed the examples and resolved disagreements during those discussions. Although students spent a full class period working through the entire problem in their groups, here we highlight a few examples of students' discussions in groups with the goal of illustrating how classroom discourse can help students to have opportunities to participate in meaningful mathematics.

Students' Applying and Extending Prior Knowledge

Students in Mr. Taylor's class had to draw upon their prior knowledge about linear functions and the relationship between distance, time, and rate to analyze the problem and find out when each friend would arrive at the campsite. Here, we focus on students' work on Isabel's trip (Figure 2), because this part of the problem proved to be the most challenging for students in all of the periods. The problem provided an outlet for students to work together to extend their use of the algebraic formula, $r=d/t$. We expected that the solution for Isabel's trip would involve two main steps. First, students would have to use information about speed and time to determine the length of the first half of her trip; then students would have to use the information about speed and distance to determine the time required for the second half of her trip. The excerpt below shows a dialogue between two students as they worked on the solution to Isabel's trip. The pair had already determined the length of the first half of her trip, and were now trying to decide how long the second half of the trip would take.

<u>Isabel puts some postings in FB:</u>
Isabel Riley Beautiful day for a bike ride. 12 mph, keeping the cadence!
August 19 at 2:44pm Like

Isabel Riley No rain on my parade! A flat tire, but I'm prepared with my kit!
August 19 at 3:38pm Like

Isabel Riley Back on track! I have to go faster now! 15 mph.
August 19 at 4:14pm Like

Isabel Riley At the gas station. Pretty much half way through.
August 19 at 5:12pm Like

Isabel Riley 10 mph. The snacks slowed me down, but nothing will stop me!
August 19 at 5:27pm Like

Figure 2. Isabel's portion of the trip.

Jamie: We're trying to find time. So the time would equal rate divided by distance. And the rate is 10 miles per hour. But the distance is 25.3 miles. But that's a total of, like, minutes, like an hour and fifty-two minutes. But you can't do that because, uhh, these aren't the same. So you have to make rate minutes. But I can't figure out how to get the rate. So you have to figure out how many miles she went in a minute. Like, it's probably gonna be like, some weird decimal. So, I think you can do 10 divided by 60…[*Jamie types in calculator*]. That's not right.

Ethan: Wait, what'd you have? Or what'd you—what'd you do?

Jamie: I did 10 divided by 60, and get .16 miles per minute. And then she…

Ethan: That's for the last 25 miles, right? Yeah.
Jamie: It's for, cuz she's going right now. She's going…
Ethan: Okay.
Jamie: And the distance she needs to go is 25.3 miles.
Ethan: Okay. So 25.3 divided by…[Jamie types in calculator] .1666…151.8?
Jamie: One hundred and fifty one…hours, minutes.
Ethan: Is that point 8?
Jamie: Yeah it's point 8; so a hundred and fifty two minutes.

The pair of students above dealt with two issues regarding the solution to Isabel's trip. First, they discussed the relationship between the speed that she was traveling and the amount of time that it would take her to travel. The students recalled the relationship that rate is the quotient of distance over time. Jamie used this information and pointed out to Ethan that they could use it to compute time. Because Jamie's calculations were correct, it seems that Jamie misspoke when she said that "time equals rate divided by distance." The critical aspect of the students' dialogue, in terms of their reasoning and sense making skills, came when they extended this prior knowledge about distance to accommodate their work on the task. Ethan extended the pair's prior knowledge when he said, "that's for the last 25 miles, right?" Ethan commented that, while the concept of rate applied, it was in a new context, which required a new treatment of the concept. This step in the students' reasoning was significant, because Ethan and Jamie showed the capacity to extend their knowledge to a new context. Not only did they recall the relationship between distance, rate, and time, but Ethan and Jamie also began to adapt this formula to a new context, specifically a context in which the rate was different at different times.

Also while working on the problem, the students reconciled a discrepancy between thinking of Isabel's speed in miles per hour versus miles per minute. Either unit of rate would have been a viable solution strategy to the problem. Initially, Jamie seemed uncomfortable with converting miles per hour into miles per minute. She did not want to have to work with, "some weird decimal." However, she chose to convert the rate into miles per minute in order to maintain consistency with her earlier measurements which she had done in minutes. In recalling and applying the formula for distance, Jamie performed the calculation purposefully. Specifically, she anticipated the results of her computation, which helped her to realize that she would need to establish common units. Finding common units before performing the computation was evidence that Jamie was purposeful in applying the formula for distance. Activities such as anticipating the results of a calculation and making careful use of procedures are especially important for reasoning and sense making in algebra (NCTM, 2010). Jamie used her prior knowledge in a productive and thoughtful way by thinking carefully about how the distance formula applied to the problem at hand. In their work together, Jamie and Ethan applied their previously learned knowledge about the relationship between distance, rate, and time to analyze the givens of the problem. They developed a solution based on the evidence provided and their knowledge about linear relationships. Jamie constructed an initial solution strategy, which did not hold up to her estimate of what the solution should be. When Ethan probed her to explain her thinking further, Jamie was able to go back and formulate a new strategy, which led to the solution of the problem.

Making Use of Multiple Representations

Karl's trip (Figure 3), which provided both an email and a graph that Karl sent, gave students an opportunity to make connections between the different representations of the problem in order to solve it. Many groups struggled to make these connections, and the excerpt below provides an example of how the teacher was able to scaffold students' prior knowledge about linear graphs, slope, and distance-versus-time graphs, to make sense of the information provided in the problem. This pair of students had already determined that it would take Karl 1.5 hours to drive to the campsite, but were having difficulties deciding what time he actually left. In the excerpt, italics indicate gestures that the teacher and students performed while talking to each other (see Figure 3 for reference).

Karl sends an email
Date: Thur 23 August 18:30:03
To: Isabel
CC: Ms. Linn
Subject: On my way

Hi Isabel,

You know that I have issues with organization…Don't laugh Isabel! I've been keeping a graph of my trip to the camp, but the graph isn't finished (you can finish it for me). I left from my house, which is 97.5 miles away from the campground. It is so annoying that I forgot so much stuff—the tent, the sleeping bags, and also, the flashlight! So even though I left at 6:00, I had to go back a couple of times. SORRY! I'm trying my best to be on time…but I can't afford another speeding ticket! I'm driving 65 mph. I'll see you ASAP.

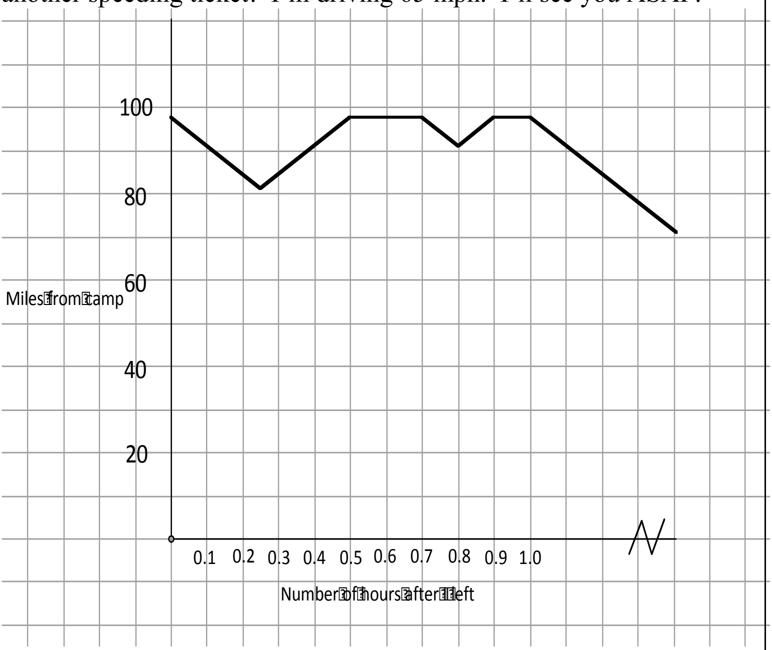

Figure 3. Karl's portion of the trip.

Mr. Taylor: What's it tell us up here he did, and how would it

correlate to that graph?
Ashley: Oh, he had to go back.
Mr. Taylor: He had to go back so where is he? [*Ashley points to the first 'V' in the graph.*] There. Then what's he doing there [*pointing to the first line segment with slope 0*]?
Ashley: Leaving again.
Mr. Taylor: How much does his distance change there?
Ashley: Umm…
Mr. Taylor: What numbers are right there for that [*pointing to the start of the segment of slope 0*]?
Ashley: 97.5
Mr. Taylor: What numbers are right there for that [*pointing to the end of the segment with slope 0*]?
Ashley: 97.5
Mr. Taylor: So what's he doing?
Steven: Oh so he would start there again.
Ashley: He's finding his stuff.
Mr. Taylor: He's finding his stuff. And then what's he do?
Ashley: He's still there. And then he leaves again. And then he has to go back. And then he's finding stuff, and then he's on his way. Okay.
Mr. Taylor: Okay, so, what's this over here [*pointing to the y-axis*]?
Ashley: Time? Oh, the *y*-axis…distance.
Mr. Taylor: Distance. Okay, so this down here [*pointing to the x-axis*]?
Ashley: It's time.
Mr. Taylor: And that's where he started, at time zero. Now you have to figure out, okay, at time zero he left. What time did he leave?
Steven: So, technically he would have started here at the one-hour mark?
Mr. Taylor: Yes, that's it.

Making connections between the narrative and the graph that Karl provided was crucial to finding the solution for his trip. Ashley and Steven needed to link Karl's message and the graph in order to finish their work on the problem. Mr. Taylor provided scaffolds through the concepts and features of graphs that the students had already studied. In talking through the problem, the students established that (a) the change between positive and negative slope corresponded to Karl turning around and going back home; (b) the flat line segment corresponded to Karl being at his house; and (c) the labels on the axes provided crucial information about when Karl left his house for the final time. Ashley and Steven used their knowledge about distance versus time graphs to make a connection between the narrative and the graph that Karl provided. Making connections between different representations of a problem is a critical aspect of reasoning and sense making in all domains (NCTM, 2009). Specifically in algebra, multiple representations allow students to explore the concept of function through a variety of perspectives (NCTM, 2010). Given that the Yellow Park problem served as an introduction to piecewise linear functions, Ashley and Steven's careful examination of the graph supported them in thinking carefully about how a function may be represented graphically. By connecting the graph to the narrative that Karl provided, Ashley and Steven identified the critical features of the graph (e.g., segments of increasing or decreasing, the meaning of the x- and y-axes). Through that work, they came up with a correct solution for Karl's trip. Ashley and Steven used an important reasoning skill in algebra—making connections between different representations—to solve the problem.

Using Non-Mathematical Knowledge to Reflect on the Solution to the Problem

In another period, a different group of students established a connection with their prior experience with driving and used that experience to make sense of their computational work. Karl's email provided the information that his house was 97.5 miles away from the campground, and he was driving 65 miles per hour. The task required students to determine how long it would take Karl to get to the campground once he left his house for the final time. This particular group of students performed the computation of dividing 97.5 by 65 to get the quotient 1.5. Thereafter, the students debated whether the 1.5 represented miles, minutes, or hours,

Renae:	So, we have the 1.5, but…
Audrey:	We don't know what to do.
Renae:	Would that be, it took him an hour and a half to get there?
Casey:	I don't know.
Renae:	Wait a minute. How far away is Burlywood?
Audrey:	I don't know. It's like 30 minutes to Burlywood. But I don't know how many miles. Like, 27 or something.
Renae:	I'd say it's close to 30. About 30 miles to Burlywood. So, like, there and back, it would be an hour. So take that and then do the 97.5 miles away he is from the campground. That'd be about right. An hour and a half? Does that make sense?
Audrey:	I don't know.
Renae:	You know what I'm saying? Like, take it as if you're going to Burlywood, and that'd be like an hour. And then…
Casey:	Right, that's what you'd think about.
Renae:	Ninety sev—right. If you like, sit here and think about it, that'd be about right. An hour and a half.

This group used the graph to determine Karl's departure time and correctly determined that Karl would arrive at the campground at 8:30. To do so, Audrey, Casey, and Renae drew on their non-mathematical prior knowledge about speed, distance, and time to determine whether their solution to the problem made sense. They interpreted the value of 1.5 based on their experiences with driving in a car. It would not have made sense for 1.5 to represent the miles or the number of minutes that Karl had travelled. It did make sense that it would take Karl an hour and a half to travel 97.5 miles, because in their experiences driving back and forth between Burlywood, that is approximately how long it would take. An important aspect of reasoning and sense making is the ability to reflect on the solution to a problem (NCTM, 2009). In this case, Audrey, Casey, and Renae used their real-world experiences to reflect on the solution they came up with. In an ideal situation, students would have recalled that distance divided by rate equals time, and would have used that information to make sense of the problem. However, in the absence of that information, this group of students was still able to apply their reasoning skills. They performed reasoned solving (NCTM, 2010, p. 2), in which their prior experiences helped them to make sense of their computations and consider the reasonableness of their solution in the context of the problem.

Mr. Taylor's Role in Promoting Reasoning and Sense Making

The work of the teacher in setting up and structuring a lesson cannot be disregarded when students work in cooperative groups. Mr. Taylor could not be everywhere at once while students were working in groups. However, he made decisions in his teaching that worked to support students' productive conversations and collaboration. First, Mr. Taylor assigned students to work in groups of 4 or 5 on the problem, thus allowing students to share (and to contrast) different strategies. In setting up the problem, Mr. Taylor drew students' attention to some of the mathematical aspects of the problem that he wanted them to attend to. Mr. Taylor told his students, for example, to "Watch your units. Be careful of units." And he also told his students, "One of the things that we're trying to look at is graphs…and the graphs will help you solve these problems." By giving students cues to pay attention to certain things before they began working on the problem, Mr. Taylor supported his students in being mindful about using previously learned formulas and making use of the representations that were available to them. However, Mr. Taylor did not prescribe the strategies that students were expected to use to solve the problem, instead allowing the situation to be problematic (Hiebert & Wearne, 2003) and allowing students to be actively involved in making sense of the context and of the mathematical content on their own (Horn, 2012).

Mr. Taylor made it clear to students that they needed to develop their own strategies for solving the Yellow Park problem and to collaborate within their groups to come up with these strategies. In his 3rd period class Mr. Taylor told his students, "You're gonna have to get kind of creative in terms of how you want to analyze it." In the next class Mr. Taylor reiterated this idea, telling his students, "How you decide to solve your problems—you guys need to discuss that amongst yourselves." Research on cooperative learning suggests that when a lesson involves higher order thinking, instructions that routinize interactions may actually hinder the productivity of a group (Cohen, 1994). Therefore, by allowing students to make their own decisions in terms of how to work on the problem, Mr. Taylor enabled his students to have the autonomy necessary for mutual exchange and collaboration.

Similarly, Mr. Taylor expected his students to ask each other questions and provide explanations when they were working on the problems. In the 3rd period class one student, Eileen, told the rest of her group that Karl would arrive at the campground at 9:00. Although this solution was incorrect, none of the members of her group questioned the solution. At this point, Mr. Taylor stepped in and encouraged the rest of the group to press Eileen to explain her reasoning. When the group members responded that they simply trusted Eileen, Mr. Taylor told the students, "You can't let the power rest with one person." In this way, Mr. Taylor refrained from stepping in with his own questions and scaffolds, but at the same time he pushed the group to formulate their own questions so that Eileen could explore her reasoning and correct her error. A great deal of emphasis has been placed on the value of encouraging students to provide detailed explanations of their thinking (see Cohen, 1994; Webb, 1983, 1991 for comprehensive reviews). In encouraging his students to ask questions of each other, Mr. Taylor created opportunities for students to benefit from explaining their reasoning strategies and procedures. Finally, although Mr. Taylor did not micromanage group work during the lesson, he did offer help when students asked for it. As we see in the second vignette above, Mr. Taylor asked a series of scaffolding questions: How would it correlate to the graph? What's he doing there? How much does his distance change? Mr. Taylor scaffolded students in developing specific reasoning skills, such as making connections between the different representations of the problem. The classroom environment that Mr. Taylor created may have broadened students' opportunities to develop specific habits of mind in reasoning and sense making in algebra (NCTM, 2010).

Conclusion

Prior research in mathematics education has shown that group work can provide opportunities for students to engage in classroom discourse, to develop a shared understanding of mathematical ideas, to increase students' accountability, and to increase their achievement in mathematics (Boaler & Staples, 2008; Esmonde, 2009; Zahner & Moschkovich, 2010). The three examples above demonstrate opportunities for students to develop shared reasoning skills through group discussions about a problem in algebra. We found that students have the capacity to provoke each other to extend previous knowledge to new concepts. We also noticed that the teacher has the potential during group work to invite students to consider new perspectives and develop their reasoning skills. And finally, students may use shared prior life experiences in order to reflect on a solution to a problem. The focus on reasoning and sense making is in accordance with current policies guiding the teaching and learning of algebra (NGAC, 2010; NCTM, 2009, 2010). Moreover, creating contexts for authentic reasoning and sense making skills and habits to be used in high school algebra can allow the study of algebra to be less about abstract symbol manipulation and more about seeing algebra as a reasoned way to engage in the world (Chazan, 2000). The group discussions provided a setting in which the students and teacher were able to work together to develop reasoning skills while studying traditional topics in algebra. Classroom conversations allowed for students to participate in the reasoning and sense making norms of the mathematical community, and at the same time they provided a window into the students' reasoning processes.

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[1] We use pseudonyms for individuals and institution.

[2] Burlywood is a pseudonym for a nearby city.