Changing Beliefs about Teaching in Large Undergraduate Mathematics Classes

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Many lecturers use teacher-centred styles of teaching in large undergraduate mathematics classes, often believing in the effectiveness of such pedagogy. Changing these beliefs about how mathematics should be taught is not a simple process and many academic staff are reluctant to change their ways of lecturing due to tradition and ease. This study describes the journey of a mathematician as he accepted the challenge to ask students to work interactively on well thought out questions in large lectures. The mathematician's espoused and enacted beliefs about lecturing were confronted through a cyclical process of developing questions, testing them in lectures, and refining them in collaboration with a research group. As he went through the process of testing and reflecting on his teaching practice, the gap between his espoused and enacted beliefs decreased as they became more aligned. The study demonstrates that the process of collaborative reflection with a team of educators can be a useful strategy for effecting change in lecturers' beliefs.

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The Context: Large Lectures

The traditional undergraduate mathematics lecture is typically an oral presentation to large numbers of students, as lectures have a long history of being practical and economical (Bergsten, 2007). These lectures are routinely content-driven and delivered from behind a podium to students sitting in rows listening to the lecturer describe mathematical ideas and techniques, while taking notes as appropriate (Beswick, 2005).

The lecture as a mode of teaching, however, attracts much criticism (Bressoud, 2011; Phillips, 2005; Pritchard, 2010) and research suggests that undergraduate students often become passive listeners in mathematics lectures, finding it difficult to understand the lecture content and maintain attention throughout (Hourigan & O'Donoghue, 2007). How, then, can students develop mathematical judgment and the confidence to approach mathematical problems, if they struggle to engage in large lectures? Making a lecture-led environment more student-centred is not an easy task and research in this area has tended to concentrate on methods of information transfer or on assessment procedures, although some methods of improving student interaction have been tried (d'Inverno, Davis, & White, 2003).

More recently, research in tertiary mathematics education suggests that this process can become easier when mathematicians and mathematics educators reflect on their teaching practice collaboratively using appropriate theoretical tools (Nardi, 2007). Paterson, Thomas and Taylor (2011) describe an example of a positive collaboration between mathematicians and mathematics educators who analysed video-clips of each other's lecturing. In another example, Hannah, Stewart and Thomas (2011) report how a mathematician reflected on his teaching decisions in a linear algebra course by sharing journal entries with two mathematics educators. In both cases, such forms of collaborative reflection can encourage lecturers to reflect on their teaching and make changes.

In an earlier paper, we discussed the social norms and didactical contracts that influence students' passive behaviour and expectations during large lectures (Yoon, Kensington-Miller, Sneddon, & Bartholomew, 2011). The benefits and feasibility of incorporating small group work in large lectures and how this might be implemented to engender conceptual understanding was highlighted. This current paper continues to advance this theme and investigates the espoused beliefs that lecturers have about teaching mathematics in large lectures versus their enacted beliefs; what they say they do, compared with what they actually do.

In this paper, we describe the journey of one mathematician, called Chris (not his real name). Chris volunteered to examine his beliefs about lecturing in collaboration with a research group of mathematics educators and lecturers (after Bouchamma & Michaud, 2011). He was keen to participate in the research project as he had an interest in innovation in his teaching, and was open to trying something new. At the time, Chris was teaching a large undergraduate mathematics course with a mathematics educator when she suggested developing the project. He accepted the challenge from the research group of asking his students in this course to work interactively on a mathematical question, once during each lecture. This seemingly simple challenge opened up a minefield of issues: What kinds of mathematical questions are appropriate? When and how should he ask them? How would students react to this change of lecture norms, from being passive observers to working in small groups? Over a three-year period, Chris worked with the research group to develop, test, and refine strategies for asking effective questions in lectures. As he tested out practical strategies and reflected on these, Chris simultaneously examined and evaluated his evolving beliefs about the role of lectures and lecturers in students' mathematical learning.

The article begins by reviewing literature on teacher beliefs and teaching practice, different approaches to lecturing, and how teacher beliefs can change. Our conceptual framework is presented with a model that demonstrates how espoused beliefs can be evaluated against enacted beliefs. Next, we describe the participants involved in our study and the methods used to collect and analyse the data. The story of Chris follows, detailing his journey, with examples of how he tested and revised his prior beliefs, in collaboration with the research group. We end with a discussion on how these findings might inform professional development in tertiary mathematics teaching.

Beliefs and Teaching Practice

Everyone holds beliefs. These are the personal judgments, intentions, expectations, or values that people make about situations; or more simply, the lenses through which humans view the world (Goos, 1999; Philipp, 2007). All teachers hold beliefs about their work, their students, their subject matter, and their roles and responsibilities (Goos, 1999). Beliefs are so powerful they will persevere against contradictions caused by reason, time, schooling or experience and will filter what is seen, and in return what is seen will affect beliefs (Pajares, 1992; Philipp, 2007).

Teacher beliefs about how mathematics should be taught commonly fall into two categories (Jaworski, 1992; Nisbet & Warren, 2000; Perry, Wong, & Howard, 2006; Tracey, Perry, & Howard, 1998). The first is the teacher-centred style, where the teacher believes that knowledge is delivered or transmitted from the teacher to the student, and that frequent testing of students is necessary to check on progress. In this style, there is little recognition of the value of student errors as part of the learning process, and the teacher is expected to play an authoritative role. According to Bressoud (2011), this teacher-centred style is often used by mathematics lecturers, who typically expect students to do mathematics after the lecture while going over their lecture notes. However, most students, Bressoud says, do not know how to engage in mathematics on their own, nor do they know how to overcome this deficit.

The second is the student-centred style, where the teacher believes students play an active, central role in constructing their own knowledge. The teacher believes in a supportive climate in the classroom with discussion and exploration of problems related to the outside world. A shift to this style requires deep changes in beliefs associated with increased reflection and autonomy on the part of the mathematics teacher (Ernest, 1994). Although knowledge of mathematics is important, it is not enough to account for the differences in style between mathematics teachers. Two teachers may have similar mathematical knowledge, but one may teach with a problem solving orientation or student-centred style, whereas the other may have a more teacher-centred style or transmissive approach.

As teachers develop new understandings of mathematics and the learning and teaching of it, their position may shift (Jaworski, 1992; Phillips, 2005). However, faced with the constraints of actual classroom teaching, teachers may position themselves differently from their beliefs. In other words, although teachers espouse certain beliefs of theoretical principles, they may not implement them in their practice. In order to change practice, new beliefs must be created because old beliefs act as filters and can redefine what has been seen (Pajares, 1992; Philipp, 2007). This link between beliefs and practice is not linearly causal but rather it is dialectically related and, according to Beswick (2005), context is an important factor. In her work with secondary school mathematics teachers, Beswick argues that the ability level and grade level of the class, and possibly curriculum pressures, can influence the way teachers enact their beliefs.

The literature identifies three necessary conditions for teacher beliefs to change. First, teachers must acknowledge their current practice is problematic (Ernest, 1994; Thompson, 1992). Next, they must have an opportunity to trial new practices (Philipp, 2007) and their judgments be trusted as they expose learners to new and innovative situations which involve vulnerability (Davies, 2012). Finally, they need to reflect on their existing mathematical beliefs and knowledge (Philipp, 2007).

However, modifying long-held, deeply rooted beliefs and conceptions about mathematics and the teaching and learning of it, is a long-term process (Thompson, 1992). A number of writers believe the third approach, reflection, is the key to changing beliefs (Fennema & Franke, 1992; Goos, 1999; Larrivee, 2000; Pehkonen & Torner, 1999). They consider that through reflection, teachers gain an awareness of their implied assumptions, beliefs, and views, and become aware of viable alternatives. Although a teacher has the tools to reason, judge, weigh alternatives, reflect, and finally to act; it is only through reflecting on their experiences that change will come about. If then change occurs, their belief system may also undergo change and be restructured.

Examples of Lecturers' Changing Beliefs

The dominant mode of teaching in most university subjects consists of lectures, with the view to controlling knowledge, where content is transmitted from the lecturer to the learner (Phillips, 2005). Yet, as long ago as 1972, Bligh (1972) pointed out that the lecture method was unsuitable for stimulating thought or changing attitudes among students. Nevertheless, the lecture mode has continued to endure not only because of economics, but also because current research shows that students prefer lectures even while admitting they do not expect to learn much from this kind of delivery (Yoon et al., 2011). We now review some examples of research on mathematics lecturers' reflecting on their practice, and how this reflection impacts their practice.

One approach by Paterson et al. (2011) was based on Schoenfeld's (2010) theory of Resources, Orientations and Goals (ROG) that teachers bring to their classes. Schoenfeld's theory of teaching-in-context attempts to answer how and why teachers make "in-the-moment" choices throughout a lesson. However, Paterson et al.'s work was with undergraduate lecturers and "unlike most schoolteachers, lecturers are both research mathematicians and teachers [and bring] differing, at times conflicting, orientations into play" (2011, p. 986). In their study, the lecturer was both a teacher and a research mathematician, but the model of professional development Paterson et al. used involved a mixed community of mathematicians and mathematics educators voluntarily working together. When dissonance occurred between what the mathematician's stated ROG was and their practice, this provided the basis for discussion and cross-fertilisation of ideas.

Another approach is to examine the different roles a lecturer may exhibit between being a mathematician and being a teacher, especially if they are in conflict (Barton, 2011). Barton maintains that any framework for undergraduate mathematics will involve "the responsibility for learning, the discipline of mathematics, and the tyranny of examples" (p. 965). He asks the questions: "How can we most efficiently enculturate students?" and "How else might we encourage mathematical behaviour as well as mathematical understanding in the context of the university learning environment?" (p. 970). One answer Barton believes is to model doing mathematics by watching "a mathematician in action: doubting, questioning, being unsure, making mistakes, and persisting" (p. 971). Another answer, he suggests, is to design undergraduate delivery that escapes the tyranny of examples and instead fosters student independence.

A third approach is the 'Knowledge Quartet', a framework employed by Rowland (2009), to focus on teaching in undergraduate mathematics and how beliefs can influence what the lecturer does in the lecturing. His framework consists of four dimensions, each of which can be affected by beliefs: foundation (the application of subject knowledge); transformation (the presentation of ideas); connection (the sequencing of the material for instruction); and contingency (the ability to 'think on one's feet' in response to unanticipated and unplanned events). Rowland's study describes a lecture in Real Analysis for second- and third-year undergraduate mathematics students whereby students are involved in working on exercises, conjecturing, and interacting with the lecturer. By building a common culture with lecturers having similar beliefs, Rowland suggests that over time students will subscribe to this socio-mathematical norm of lecturing style.

Each of these three examples highlights instances where mathematics lecturers have examined their beliefs about presenting mathematics in lectures. The process of modelling and reflecting by the lecturer, interacting with students, and having discussions within a community of practice, are approaches that involve examining belief systems.

Conceptual Framework

Our conceptual framework (summarised in Figure 1) acknowledges that a lecturer's beliefs can be exhibited in two ways. Lecturers can describe their beliefs on a theoretical level (espoused beliefs), and they can demonstrate their beliefs through their practice (enacted beliefs). The beliefs exhibited in these two ways may not always be consistent: a lecturer may espouse to colleagues a set of student-centred beliefs when talking about their lecturing, but enact a different set of beliefs consistent with a teacher-centred approach in practice.

For example, one study describes a teacher, Joanna, who was a strong advocate of hands-on learning and professed that teachers should look for resources outside of textbooks (Philipp, 2007). Yet, in her classroom she was observed to be a strong authority, presenting teacher-directed instruction with some teacher-student dialogue, but no student-student dialogue. Joanna rigidly followed her mathematics textbook, and her students worked quietly on problems from the textbook. Although Joanna espoused non-traditional beliefs, her practice was categorised as traditional, which she justified by time constraints, scarcity of resources, concerns over standardised tests, and students' behaviour.

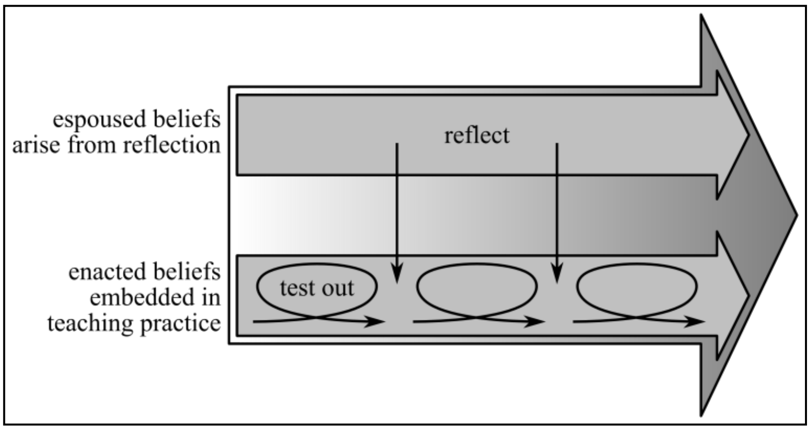

Figure 1: Our conceptual framework for change in lecturers' beliefs.

In our study, we were concerned with changing lecturers' beliefs in a way that ultimately impacts on their practice; in other words, their espoused beliefs become aligned with their enacted beliefs. It is generally quite straightforward to convince a lecturer of the importance of student-centred teaching at a theoretical, espoused level (Ernest, 1994). However, it is harder to change their beliefs at a more fundamental, practical and enacted level so that they also change their practice. We consider it necessary to address the beliefs that lecturers exhibit at both levels, espoused and enacted, when trying to effect lasting lecturer change. In our approach, we would challenge lecturers to implement practical teaching strategies that are embedded with teaching philosophies that differ from the beliefs they currently espouse and enact in practice.

In this paper we describe one example of the results of a challenge given to a lecturer of large classes of up to 350 students: to ask students to work interactively on a mathematical question once per lecture. This seemingly simple teaching strategy was embedded with many issues that needed to be resolved about the nature of mathematical learning, the social norms of large lectures, and the role of the lecturer in students' learning. In accepting the challenge, the lecturer was compelled to become aware of, evaluate, and revise many of his beliefs about lecturing.

The large overriding arrowhead in Figure 1 indicates that as lecturers engage in the process of testing and reflecting, we hypothesise that they will be persuaded to reconcile potential discord between the beliefs they espouse and those they enact. We further hypothesise that if a lecturer becomes convinced of the value of the implemented strategy, the lecturer's espoused and enacted beliefs will evolve and become closely aligned; however, if the lecturer rejects the new strategy, the lecturer will revert back to his or her old practices, but will be forced to acknowledge the philosophy of teaching with which they are aligned. In either case, we conjecture that the process encourages lecturers to evaluate the consistency of their beliefs about lecturing.

Method

The data in this paper came from a research project that investigated the social norms of lecturer-posed questions in large undergraduate mathematics lectures. Five mathematics lecturers who were also mathematics educators worked together over a three-year period to design and test techniques for implementing questions in large lectures. Multiple forms of data were collected to assess the effectiveness of these techniques, including interviews, questionnaire responses and journals from students; interviews and written reflections from two of the lecturers on constructing new understandings in the process; and journal notes from members of the research team who observed lectures.

Participants and Setting

We focus here on Chris, a pure mathematician with 13 years of experience in undergraduate lecturing. Chris was involved in the project as both researcher and lecturer; as together with the rest of the research team he designed questions and questioning techniques, some of which he then tested in his lectures. Before the project began, Chris had been widely regarded anecdotally as an excellent lecturer by staff and he received very positive student evaluations of his teaching consistently. He had previously implemented innovative teaching strategies such as Team Based Learning (Paterson & Sneddon, 2011) and tablet PC recorded lectures (Yoon & Sneddon, 2011).

Chris implemented the questioning techniques in a large first year calculus and linear algebra course at a New Zealand university. This course catered predominantly for students not majoring in mathematics or related disciplines, and it had an ethos of delivering a skill set to students who will use mathematics in other participant areas: business and economics, statistics, computer science, and the physical sciences. Approximately 800 students enrolled in the course each semester (fewer in the summer semesters), and lectures were delivered in multiple streams with 100-350 students in each stream. During regular semesters, a team of up to eight lecturers would deliver the same content in three or four lecture streams.

Each lecturer followed a common lecture schedule, and taught from pre-published series of lecture slides that most students purchased. The lecturing team considered it important to teach in this consistent manner in order to prepare the cohort, as a whole, equally for common assignments, tutorials, and tests.

Student interviews at the end of the semester, from the wider study, indicate that the students in this course valued and endorsed a passive lecturing style, even while acknowledging that it does little to help them learn within the lecture environment (Yoon et al., 2011).

Data Collection

Chris implemented the questioning techniques in the calculus and linear algebra topics on three occasions—in 2009, 2010 and 2011—and kept journals reflecting on the process. He was interviewed four times, either during the semester in which he taught the course or immediately after the course finished. Interviews were conducted with other members of the research team present, and were audio-taped and transcribed. The interviews were semi-structured, and the research team asked Chris to reflect on the effectiveness of the questions and questioning techniques he implemented as well as his teaching goals and beliefs.

At the end of the project, Chris wrote a reflection on how he thought his goals, beliefs, knowledge, and identity, had changed throughout the project as a consequence of implementing the questions and questioning techniques in his lectures.

Data Analysis

The four interview transcripts and the written reflection were analysed in four stages. First, we used open coding (Corbin & Strauss, 2008) to identify and classify recurring concepts in the written reflection and transcripts, which were refined and categorised into a coding scheme. We then cycled back and forth between applying the coding scheme independently and meeting in groups to compare and revise our coding until consensus was reached. Five primary codes (beliefs, change, reflection, emotion/attitude, and teaching practice) emerged, with 20 sub-codes (see Table 1) as being tightly interconnected in describing how Chris' beliefs about lecturing changed over the three years. In the final stage, we used pattern coding (Miles & Huberman, 1994) to identify recurrent themes and stories that emerged from the data that were attached to these five primary codes. This was then confirmed by Chris as a true account of the events described in this paper.

Table 1
Sub-codes

Primary code	Sub-codes	Primary code	Sub-codes
Teaching Practice	Interaction Questions Strategies Goal Style	Change	Growth Habituation Planned Change

Emotion/ Attitude	Awareness Blame Difficulty Evaluation Justification	Reflection	Confidence Discomfort Fear Fun Pressured Surprise Tentative
Beliefs	No sub-codes		

Chris was not only the subject of this study but also an active member of the research team who participated fully in the data analysis. His dual role therefore introduces potential for bias in the results, as Chris may have been inclined to show himself in a positive light. This was mitigated by the involvement of the rest of the research team, who observed the lectures and kept journal notes. As well, they could dispassionately challenge Chris' perspective by alternating between independent coding and team consensus to check and corroborate findings. It was advantageous having Chris involved in the analysis, as he was able to clarify aspects where the transcripts were unclear or ambiguous.

Results

Chris' beliefs about lecturing at the beginning of the project

Before being involved with the project, Chris was a typical traditional mathematics lecturer. He was a confident mathematician and saw his role in lectures as putting forth knowledge. This involved a lot of talking on his part and his style could be described as being primarily transmissive.

> Early on in my teaching career, I would have been reluctant to spend more than a small amount of time making my lectures more interactive. I think I gave good lectures, but for the most part they were "sage on the stage" style.

His students would arrive, sit passively, scribble down essential notes, and leave without him knowing if they understood the concepts being taught. The classes were quiet as interaction was kept to a minimum and Chris felt that he must always be in charge. This was a routine that he was used to, and it had been established from his own undergraduate experience. He had not challenged his approach to delivering large lectures as he thought the current situation worked well and he was happy with the feedback he had received from student evaluations.

Any questions that Chris might have asked the class tended to be procedural and closed so as not to waste time. He gave us an example of a question he had used, saying it was typical of the closed and process-driven questions he frequently used:

> *Question:* Show that the function $f(x) = x^3 - 6x^2 + 20x - 24$ has no critical points and one point of inflection.

These types of questions reinforced the notion that mathematics is procedural and can be acquired as a skill set or tool box of routines to be applied. When students volunteered answers to questions posed in lectures, Chris saw it as his role to validate (and if necessary reword) student answers, and had not considered other possible avenues for validation. He stated:

> It's very natural to say "Okay, who knows what the answer is?", elicit that response from the class, get an answer you know, validate it, rephrase it in your words so you're happy with the answer that everybody copies down, and then go on with the lecture.

Chris remarked that he felt constantly under pressure to deliver the pre-published content in the prescribed time and referred to this often as the "tyranny of content":

> I had mostly felt the pressure to deliver the expected product. The phrase "tyranny of content" comes to mind; there is a pressure to deliver everything in the course material. This is especially true for this course, in which there are multiple streams and pre-prepared lectures. There is a pressure to deliver the same experience that other streams get. Skipping lecture slides to spend more time on something else diverges from the status quo.

At each lecture Chris would introduce new topics as they came with comprehensive PowerPoint slides to supplement his teaching. Content coverage was an overriding goal, and Chris explained that he would feel guilty if he had not delivered the "expected" content. The students would observe the slides Chris used and watch him execute examples so that they could learn to do these for themselves later on for homework. Chris believed that this was how large lectures should be run, and that his control over the time, content, and students reflected his effectiveness as a mathematics lecturer.

Changes in Practice Leading to Changes in Beliefs

After joining the research group, Chris was keen to introduce some different teaching strategies that might benefit his students during the lectures. Although Chris was enthusiastic for changes he was aware of the limited time available in lectures to devote to questions and wondered how this would play out. He told us:

> Stepping back from the position of control to give students time to talk about mathematics was not something that I was particularly comfortable with. When I did ask questions in lectures, they were often mundane: either "What is the next entry in the matrix?" kind of questions—almost rhetorical—or at most procedural questions which reinforced a recent example.

Despite having concerns, he willingly took on the challenge to ask students to work interactively on questions during his lectures.

The first phase of change for Chris was a period of setting goals, associated with some expressed trepidation: "You don't know where you're going to go ... It does move you out of your comfort zone the first few times". He could foresee implementation barriers for his questions, in terms of engaging traditionally passive students, the tyranny of content, and the difficulty in validating student answers. Chris commented:

> You start out as a new lecturer and kind of feel like it's your job to be down at the front putting forth knowledge to the students to absorb and so [if the students are] spending five minutes or ten minutes where they're talking to each other but you're not talking to them, how is that giving a lecture?

A significant step towards this change for Chris was the amount of talking he would do. Instead of talking for most of the lecture, he facilitated more class discussion times. This involved using open conceptual questions to get students engaged in thinking about and discussing the mathematics, first with their neighbours. Chris illustrated this change with an example he professed was one of his most effective questions:

> *Question:* A student is trying to find $\lim_{x\to\infty}\frac{f(x)}{x}$ and writes the following:
>
> $$\lim_{x\to\infty}\frac{f(x)}{x} = \lim_{x\to\infty}\frac{1}{x}f(x) = \lim_{x\to\infty}\frac{1}{x}\cdot\lim_{x\to\infty}f(x) = 0\cdot\lim_{x\to\infty}f(x) = 0$$
>
> Is this correct?

This was the first type of question where Chris asked the students to evaluate student work. In an interview with Chris, he explained:

> It was received quite well. There was a lot of discussion underway, and not a lot of agreement. The verbal follow-up question was: "*If* it is wrong, where is it wrong?" and then: "*It* is wrong—where is it wrong?" I had hoped to have a student give a counterexample: "What if $f(x) = x$?" but we did not get that deep into the question; or at least, no student was prepared to volunteer this much.

In the implementation of questions such as the one above, Chris used a framework of "Stop, think, and discuss", suggested by the research team. For example in a typical lecture, after the question was presented the lecture would stop and students would be instructed to work alone on the question and then to discuss it in small groups. Chris used this time to move around the lecture theatre and interact with the students taking note of some groups' answers. He then asked the class to feedback short verbal answers, or he summarised the common answers he had heard arise. This type of question encouraged students to discuss the underlying mathematics: "They're getting something out of it that they can't get out of the question by reading it in the textbook or doing it at home by themselves." Chris would often tell the research group "I want them to be discussing the maths" and that he now believed that this was important for the students' learning. As a bonus, he found that he also enjoyed it: "It's nice to be giving them the opportunity to come up with the ideas themselves to see some of the concepts coming out of the examples". Davies (2012) contends that when teachers see themselves as capable of making good educational decisions about their students' learning and having positive experiences, this will excite and motivate them further.

The students, Chris said, became accustomed to the "Stop, think, and discuss" sessions and engaged actively with the questions. Chris could see that this was the result of the type of questions he was asking: "I think really a lot of the stop and think kind of thing is coming down to the question design being in this more kind of open ended style". He began to take a more scientific approach to his teaching, whereby he set goals, tested them out, and observed the effects of his implementations. Chris noted that he now had more "confidence in saying I think my question is more useful than that example"; and he told students that "You don't learn by watching me do it". As Chris asked students to work on more questions and reflected on the process each time, he kept an account of what worked, what did not, and what changes he could make.

It was interesting for the research team to observe Chris' attitude to lectures changing, and that he no longer was labouring under the "tyranny of content". Chris believed that each lecture should focus on the underlying concept, and prioritised these such that "The goal is for them to understand the big ideas of the course". He espoused a new belief that procedural questions, which reinforce mathematical skills, are of limited use in lectures, and there was evidence that he was beginning to enact this belief in his teaching practice because he would skip some slides that he would normally have shown in the past.

The second phase of change for Chris was a period of goal setting, and putting these goals into practice. Chris set out to change the implementation of his questions in the lectures to be a more integral part of the course, and embraced the importance of being a facilitator in lectures, allowing time for students to discuss the mathematics at hand. He often expressed surprise at the effectiveness of this change, saying "It was quite unexpected that there would be that level of engagement and discussion". Because of this unanticipated success, Chris was surprised that some barriers to student engagement persisted. However, he knew the process was "still a learning experience, coming up with the question that gets them to engage with the mathematics beyond just monkey see, monkey do".

The final phase of change for Chris involved a period of reflection and growing confidence. Critical evaluation of his teaching practice became a regular focus of discussions within the community of practice. The group could see a growing confidence in Chris's beliefs about the nature of effective teaching in large lectures, and the implementation methods he used. The fear he had often expressed at the start about the prospect of asking students to work on questions during lectures was mostly gone. The change in Chris' beliefs had been gradual and ongoing and he became aware that, for his students:

> They've got an opportunity to think about what you've just taught them, what you've been talking about at least, to see whether they've learnt anything. So I think that it kind of, yeah, it gets their brain working on a different level. It gets them thinking about the material rather than just sitting and passively listening to it.

As Chris continued to reflect on his lectures, he felt more confident about the changes in his beliefs about teaching. He commented: "I feel more with each semester that this is the right thing to be doing for students in this course".

An Espoused Belief that was not Enacted in Practice

Although many of Chris' beliefs about teaching changed over time, his involvement within the research group revealed that some of his beliefs did not. This supports Bouchamma and Michaud's (2011) finding that one advantage of a community of practice was that it provided a setting for participants to acknowledge not only successful practices but also less successful ones.

One noticeable example was the way Chris responded to students' answers to questions in lectures. At the outset, he reported that his current response was to validate students' correct answers. Chris indicated that he was unhappy with this practice, as it reinforced an image that he did not agree with—that in mathematics, there is only one correct answer, and the lecturer is the arbiter of mathematical correctness. He made the following comment with an ironic tone of voice to convey his disagreement:

> There is the mythical correct answer and it only exists in my head, and although the student can write something down I have to validate it.

Chris frequently espoused his belief that the students should be actively engaged in evaluating the mathematical correctness of each other's answers to questions posed by the lecturer. In practice, however, Chris continued to validate students' answers in class by stepping into the role of content expert to confirm a student answer or to reword a students' answer to match the one he expected. Chris was aware that his actions and intentions were at odds with each other: "I'm still falling into the trap of confirming their answer". Despite this awareness, Chris and the research team failed to come up with practical strategies for testing out Chris' emerging beliefs about validating student answers. It is likely that this failure to test out these espoused beliefs led to the continuing dissonance between what he said he believed and what he did in practice.

This example provides a useful contrast to the many instances where Chris' beliefs changed, which were described in the previous section. Whereas Chris changed his beliefs by testing them in practice, reflecting on them and revising them, his beliefs about validating student answers remained untested, and therefore did not change in practice. This result is consistent with our argument that when beliefs about practice are deeply entrenched and persevere against contradictions (Pajares, 1992; Philipp, 2007) modifying or changing them requires continually testing them in practice, reflecting, and then revising. Without this approach, our research suggests it is unlikely that beliefs will change.

Reflection on Chris' Changing Beliefs

Chris initially joined the research group without thinking that his beliefs about teaching would change as a result. Indeed, none of the members of the research group anticipated that Chris' teaching beliefs would change during the course of the project. The focus was on a more practical teaching issue: how to ask questions in large lectures in a way that draws students out of their seemingly passive behaviour and engages them in the mathematical content during the lecture. Initially, our research was centred on student behaviour, and the design of questions—not the lecturer's teaching beliefs. Our goal was to make small changes in the way the lecturer asked questions in lectures.

As Chris engaged in the cycle of implementing and reflecting on new questions and questioning styles, many of his beliefs about the role of questions changed. He designed and adopted the method of introducing questions to students with a slide that said "Stop, think, and discuss", and when he wanted students to end their discussions he applied the technique of flicking the lights on and off. Chris came to value asking conceptual questions in lectures in addition to procedural ones, whereas previously he had asked procedural questions almost exclusively. He also came to believe that students should be actively involved in trying to answer the questions, rather than passively watching a demonstration on the board. Furthermore, he came to believe that students would benefit from discussing their answers in small groups instead of answering the lecturer individually.

A natural part of any design process is to reflect critically on what works. Consequently, whenever Chris reported on a questioning technique that worked, the entire research team would help him reflect on why he had made these changes. Such reflection on what we thought were rather small teaching changes opened up some very deep reflections about his teaching beliefs. For example, Chris used the "Stop, think, and discuss" slide to signal to students that the lecture momentum would change significantly, but also in a way that he valued—active engagement with the ideas rather than passive observation of the material. Further, he introduced flicking the lights because he was afraid he might lack the power or authority to stop such a large class from discussing.

This tight focus on beliefs about questions tapped into many of Chris' deeper beliefs about the role of lectures and lecturing in general. Chris acknowledged that he had previously lectured under the "tyranny of content", by trying to cover as much content as possible with less emphasis on student understanding. As Chris reflected on the role of questions, he came to believe it was important for students to understand the mathematics presented in lectures, rather than merely copying down notes to study later, and he adjusted his lecturing accordingly by focusing on big ideas, with less emphasis on procedural skills. Over time, it became noticeable that Chris' identity as a lecturer was changing from being the "font of all knowledge" or the "expert in the classroom" to being a facilitator who engaged the students in mathematical thinking.

Our focus on the role of questions in lectures proved a remarkably powerful yet simple tool for accessing Chris' deeper beliefs about teaching in general. Through reflection, Chris was forced to consider his beliefs about what kind of mathematical learning should take place in lectures, and the lecturer's role in facilitating such learning. Readers wishing to effect similar kinds of change in teaching beliefs with other lecturers would be well advised to target their efforts by focusing on beliefs about the role of questions, or another small aspect of teaching. We hypothesise that the narrow and specific focus on beliefs about the role of questions enabled Chris to reflect more deeply about his teaching beliefs than he would if we had simply asked him to reflect on his beliefs about teaching more generally.

When Chris reflected on these changes, the research team would join him by reflecting on their teaching and referring to theoretical concepts in the literature (such as didactical contract, sociomathematical norms) that helped to make sense of these experiences. Chris also reflected on his beliefs while analysing data gathered from the research project. In one memorable session spent analysing student interview responses, Chris threw up his hands and exclaimed, "They [students] don't even *expect* to learn during lectures! What am I doing?!" These changes in Chris' teaching beliefs were brought about by his involvement in the research culture—not only as a reflective practitioner but also as a full research partner, using literature and theoretical constructs to make sense of real data.

Thus, although Chris primarily changed his beliefs by testing them out in practice, the involvement in a research team was also influential in his development. The team provided insightful observations and suggestions to develop his teaching practice and make it more effective. This process of reflecting on the implementation of the questions was particularly effective in bringing about some of the changes in his beliefs, which he would often talk about with the group in research meetings. Although it is certainly possible for a lecturer to work through this process alone, working within a group of educators provides support not only for extending the reflection but also for accelerating the process.

Some Final Comments

The findings in this study collectively advocate the benefit of other lecturers similarly engaging with mathematics educators, testing and reflecting on modifications to their teaching practice. As such, our results are applicable as a facet of professional development. Discussions within a team of educators can encourage a lecturer to engage scientifically with his or her teaching beliefs. In promoting this, we note that lecturers are likely to be apprehensive when implementing new teaching practices, so it is imperative that the team within which to reflect is supportive and a significant period of time to implement a programme of planned change is given.

The value of using the framework provided in this study, when designing future interventions, provides a robust pathway for reflecting and testing out lecturers' beliefs about teaching practice. Although Chris was clearly interested in innovation and was a willing volunteer to participate, we acknowledge that engaging more reluctant lecturers who see no need for change is a challenge. Being dissatisfied with current practice is an important requirement for change to be a possibility and our recommendation therefore is to encourage the formation of small groups, composed of mathematicians and mathematics educators, to engage in conversations about their own teaching practices. We further suggest to start small with one espoused belief about teaching practice and to repeat the testing and reflecting of this belief, as we did through the role of questions in large lectures. In this way consonance is more likely to be achieved between espoused and enacted beliefs.

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