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Identification of Number Sense Strategies used by Pre-service Elementary Teachers

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Abstract

The purpose of this study was to identify the use of number sense strategies by pre-service teachers studying at the department of elementary education. Compared to the previous one; new mathematics curriculum places more emphasis on various strategies such as estimation strategies, computational estimation strategies, rounding and mental computation, which are among the fundamental components of the number sense. The study was undertaken within the framework of the question "What strategies do the pre-service teachers, who will implement this curriculum, use while solving problems requiring number sense?". 133 pre-service teachers from the Elementary Education Department in a State University in the province of Istanbul participated in this study. "Number Sense Test" about five different number sense components was used as a testing instrument. Findings were analyzed with qualitative and quantitative methods. At the end of the research, pre-service teachers' number sense was found to be very low. When solution methods were analyzed, it was found that pre-service teachers preferred using "rule based methods" instead of "number sense" in each of the components. This finding was consistent with prior studies that show pre-service teachers' preference of written methods to number sense methods. With regard to research findings, suggestions were offered for future studies by emphasizing the necessity for measures to increase pre-service teachers' knowledge on number sense as well as its use.

Key Words

Number Sense-Based Strategies, Partial Number Sense-Based Strategies, Estimation, Pre-service Elementary Teachers.

When a student is asked to do the multiplication of 4.5 x 1.2, s/he gives the answer "54.0" by multiplying decimal numbers and by paying attention to the decimal digit (Reys et al., 1991, p. 3). When a thirteen-year-old student was asked to estimate the addition of $\frac{12}{3}$ and $\frac{7}{8}$ in America, among the choices of 1, 2, 19, 21 and "I don't know", 50% of the students chose the alternatives of 19 or 21 (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980). When 8th-grade students were asked the question "How many fractions are there between $\frac{2}{5}$ and $\frac{3}{5}$?" 46% of these students answered as "There is no fraction" (McIntosh, Reys, & Reys, 1992). When the question "If 750 is divided by 0.98, is the result larger, smaller or equal to 750?" was posed to the students in Turkey, 74% of the 4th-grade students and 70% of the 5th-grade students could not give the correct answer. Most of the students who provided the correct answer did so only after performing mathematical operations. Generally, the students thought that the answer should be smaller than 750 by making a generalization about the fact that division makes numbers smaller (Şengül & Gürel, 2003). There are

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many examples to the lack of number sense such as the ones stated above. Answers provided to those kinds of questions present student levels to estimate and understand the effects of mathematical operations concerning numbers and number sense.

What is Number Sense?

According to Reys et al. (1999) number sense refers to a person's general understanding of numbers and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful and efficient strategies for managing numerical situations.

As Schneider and Thompson (2000) state a student who has a good number sense is successful in flexible thinking about numbers, understanding their meanings and the relationships among them. Development of number sense is important in mathematics education. The National Council of Teachers of Mathematics (NCTM), in their *Principles and Standards for School Mathematics*, notes that number sense is one of the foundational ideas in mathematics in that students (1) Understand numbers, ways of representing numbers, relationships among numbers, and number system; (2) Understand meanings of operations and how they related to one another; (3) Compute fluently and make reasonable estimates (NCTM, 2000, p. 32).

Number Sense Components

Number sense is a complex process that includes many different compenents of numbers, operations and their reletionships and it has generated much research and discussions among mathematics educators, cognitive psychologists, researchers, teachers and mathematics curricula developers (Greeno, 1991; Hope, 1989; Howden, 1989; Markovits & Sowder, 1994; McIntosh et al., 1992, 1997; NCTM, 1989, 2000; Reys, 1994; Reys & Yang, 1998; Sowder, 1992a, 1992b; Yang, 2002a, 2002b). As a result, different psychological perspectives have been provided (Case & Sowder, 1990); theoretical frameworks of number sense have been proposed (Greeno, 1991 McIntosh et al., 1992); characteristics of number sense have been described (Howden, 1989; Reys, 1994) and essential components of number sense have been enumerated (Sowder, 1992a; Yang, Hsu, & Huang, 2004).

Based on a review of the number sense literature, this study focused on number sense to include:

- Understanding of the meaning and size of numbers: This skill is associated with the ability to recognize the relative size of numbers. For example, when a student is asked to compare $\frac{2}{5}$ with, $\frac{1}{2}$ knowledge of how to do this is the indicator of this skill (Behr, Wachsmuth, Post, & Lesh, 1984; Cramer, Post, & delMas, 2002).
- Understanding the meaning and effect of operations: This component is related to the ability to recognize how the result will change when operations or numbers are changed in calculations (Graber & Tirosh, 1990; Greer, 1987; McIntosh et al., 1992; Tirosh, 2000).
- Understanding and use of equivalent expressions: It is the ability to know the equivalent numbers and using them when necessary. For example, being able to answer the question, "Which product of m number gives the same result when the m number is divided by 0.25?"
- Flexible computing and counting strategies for mental computation**:** Individual problem solving without resorting to written calculations and estimations in order to investigate the appropriateness of the result emphasizes the ability to do mental calculations (McIntosh et al., 1992; Sowder, 1992a).
- Measurement benchmarks*:* This skill is comprised of the ability to determine and use reference points that can vary according to situations (McIntosh et al., 1992).

In the last 20 years, studies on the improvement of number sense have been carried out with increasing interest. With respect to the increased importance of number sense, it is a significant responsibility to improve students' number sense in mathematics education (Alajmi & Reys, 2007; Anghileri, 2000; Australian Education Council [AEC], 1991; Cockcroft, 1982; Japanese Ministry of Education, 1989; Kilpatrick, Swafford, & Findell, 2001; Mullis, Martin, Gonzalez, & Chrostowski, 2004; NCTM, 1989, 2000; National Research Council, 1989, 2002; Reys et al., 1999; Toluk-Uçar, 2009; Umay, Akkuş, & Paksu, 2006).

Both international studies and the studies carried out in Turkey in the field demonstrate that the primary school students' number senses are low (Ball, 1990a, 1990b, 1996; Ball & McDiarmid, 1990; Bell, 1974; Bobis, 2004; Case & Sowder, 1990; Charles & Lester, 1984; Harç, 2010; Kayhan Altay, 2010; Markovits & Sowder, 1994; McIntosh, Reys, Reys, Bana, & Farrel, 1997; Reys et al., 1999; Reys & Yang, 1998; Sulak, 2008; Van den Heuvel-Paanhuizen, 1996, 2001; Verschaffel, Greer, & DeCorte, 2007; Yang, 2005).

Purpose

The purpose of this study was to indicate which strategies were preferred by pre-service students studying at the department of elementary education while solving problems that require the use of number sense.

Method

Research Design

Current study employed case study methodology, one of the qualitative research designs that thoroughly investigates and analyses one or several specific cases.

Study Group

Sample of the study was made up of 133 senior pre-service teachers from the department of the elementary education in a state university located in Istanbul. The participants attended BM(1) [Basic Mathematics I -2 credit hours] and BM(2) [Basic Mathematics II -2 credit hours] courses in the first and second terms respectively and completed TM(1) [Teaching Mathematics I -3 credit hours] and TM(2) [Teaching Mathematics II -3 credit hours] courses in the fifth and sixth terms.

Data Collection Tools

Testing instrument for data collection was selected and adapted from the literature based on the problems discussed. The number sense test (NST) included five number sense components: (1) Understanding the meaning and size of numbers; (2) Understanding the meaning and effect of operations; (3) Understanding and use of equivalent expressions; (4) Flexible computing and counting strategies for mental computation; and (5) Measurement benchmarks. These five components of number sense were also stated in Reys et al.'s study (1999). The questions were selected from the studies conducted by Reys et al. (1999), Yang, Reys, and Reys (2009), Tsao (2005) [Table 1].

Procedures

Each participant was given the number sense test in which each page included one item and ample space was provided to allow students to record the reasons for their answers. The test continued for 60 min. Before the test, the researcher read the rules to follow during the test. Specifically, the following directions were given: (i) Participants were told to estimate or mentally compute and not to carry out a written algorithm to find an exact answer on each item; (ii) Participants were asked to write the answer to each question and then briefly explain how they arrived at their answer; (iii) Participants were told that the time on each item was controlled (3 mins), and they should not move on to the next page without permission. Controlling the time ensured that all students would have an opportunity to respond to each question. The researcher monitored the test to control the pace and also to validate that the directions, such as not executing a written algorithm, were followed.

Table 1.
Number Sense Test

1. Is $\frac{3}{8}$ or $\frac{7}{13}$ closer to $\frac{1}{2}$?Without finding an exact answer, please use estimation to decide. Explain why you have chosen this answer
2. Rank the following numbers in descending order. Explain why you have chosen this answer. a) 0.74x8.6 b) 0.74 + 8.6 c) 0.74:8.6 d) 0.74 – 8.6
3. Which number gives the same result when it is multiplied by m-number instead of dividing m-number to 0.25? Explain why you have chosen this answer. a) $\frac{1}{2}$ b) 2 c) $\frac{25}{100}$ d) 4
4. Ayşe used the calculator to compute 0.4975 x 9428.8 = 4690828, she forgot to write the decimal point. Without finding an exact answer, please use estimation to decide which of the following shows the correct location of the decimal point. Explain why you have given this answer. a) 46.90828 b) 469.0828 c) 4690.828 d) 46908.28
5. Ali walked 0.4828 km, Ayşe walked $\frac{13}{38}$ km, Murat walked $\frac{8}{15}$ km, Zeynep walked $\frac{17}{16}$ km, Deniz walked 0.966 km, and Betül walked $\frac{7}{29}$ km. Without finding an exact answer, please order the distance they walked from the farthest to the nearest. Explain why you have chosen this answer

Analysis of Data

Answers of the participants were examined by using qualitative and quantitative analyses (Bilgin, 2006). Participant' answers and explanations were separately analyzed both by the researcher and the two subject field experts and were classified into four categories, namely; n*umber sense based, partial number sense based, rule based* and *no explanation or unclear answers* The researcher and the two experts agreed on the evaluation by using the formula stated in Miles and Huberman (1994): "The Percentage of Agreement = [Agreement/ (Agreement + Disagreement)] x100". Percentage of the agreement was found to be 92 and it was concluded that the categories were consistent. Remaining contested responses were reexamined and discussed by both raters until a mutual agreement was reacher regarding the complete categorization. Later, the answers were analyzed quantitatively in terms of percentage and frequency.

Results

Answers and explanations of pre-service teachers to one of the questions for each component of the number sense test and their categories are presented below.

77.44% (103 participants out of 133) of pre-service teachers correctly answered the questions related to first component of the number sense. Thirty participants preferred to make comparisons between denominator and numerators while explaining their answers. Answers of pre-service teachers can be exemplified as follows:

These answers were categorized under "number sense based" methods.

1) $\frac{7}{3}$ *is closer. If the numerator is half or half of the denominator, then it is closer to* $\frac{1}{2}$ [16 participants].
2) $\frac{7}{3}$ *is closer, because* $\frac{1}{2}$ *means half.* $\frac{7}{3}$ *is a little smaller than the half* [13 participants].
3) $\frac{3}{8}$ *is smaller than its half and* $\frac{7}{3}$ *is larger than its half; therefore,* $\frac{7}{3}$ *is closer* [8 participants].

Twenty participants answered the question or rules by using paper and pencil algorithms based on operations. According to this algorithm, the solutions were accepted as "rule based methods

Thirteen participants who gave their answers in this context formed a "part to whole" relationship. The following explanations can be given as examples.

1) *In the fraction of* $\frac{7}{13}$*, since the denominator is divided into more parts and more than half of its parts are taken, this fraction is closer to* $\frac{1}{2}$. [7 participants].
2) $\frac{7}{13}$ *is closer. When the part to whole relationship is taken into account, instead of 3 out of 8 parts, 7 out of 13 parts are closer* [6 participants].

Others preferred finding the common denominator by equalizing the denominators. For example, it was stated as, " $\frac{7}{13}$ *is closer. I could not be sure of my answer. This is my estimation without performing an operation.*"

One of the participants explained the answer as stated below.

"$\frac{1}{2} - \frac{3}{8} = \frac{1}{8}\ \frac{7}{13} - \frac{3}{8} = \frac{56\text{-}39}{104} = \frac{17}{104}$ *then* $\frac{7}{13}$ *is closer because the difference between them is*

(4) (8) (13) smaller."

All of these answers were evaluated under the category of "rule based methods". Five of the participants answered the question correctly by determining the missing parts necessary for the given fraction to reach half.

"In my opinion, $\frac{7}{13}$ *is closer because in order to make* $\frac{1}{2}$*, you need to add 0.5 to 7 but as for fraction of* $\frac{3}{8}$ *based, you need to add 1 to make it* $\frac{1}{2}$" . Such answers were categorized as "partial number sense based" methods.

Twenty seven participants gave the correct answer but they did not give any explanation or their explanations could not be understood. On the other hand, 16.54% (22 participants) of the sample group gave incorrect answers. Six out of these twenty two participants preferred to compare the fractions by changing them to decimal numbers. Pre-service teachers who had difficulties in answering the fractions without performing an operation preferred to find the answer by changing the numbers into a more complex structure like decimal numbers. The pre-service teacher Selin's answer can be given as an example.

"$\frac{3}{8}$ *is closer to* $\frac{1}{2}$ *because* $\frac{3}{8}$ *is approximately equal to a value of 0.37 but* $\frac{1}{2}$ = 0.50 *and* $\frac{7}{13}$ = 0.18."

Her result is incorrect due to the miscalculation in the division. These answers were classified as "partial number sense based" method. Others generally based their answers on the rules that they have learned or on the paper-pencil algorithm to equalize the denominators without establishing the relationship of proximity to the half. While six of the pre-service teachers justified their answers by using part to whole relationship *"3/8 is closer because although both of them are close values, 3/8 is divided into fewer parts;"* the other four teachers justified their answers as follows:

"$\frac{3}{8}$ *is closer because when the difference between them is equal, the one with the smaller denominator and numerator becomes closer.*"

Two of the pre-service teachers gave the explanations below:

"$\frac{3}{8}$ *is closer. I equalized the denominators." or "* $\frac{3}{8}$ *is closer. I employed operations and this is the result."*

Incorrect answers stated above were classified under "rule based" methods. It is obvious that pre-service teachers gave wrong answers because they could not remember the rule: *"if the difference between a denominator and a numerator of a fraction is equal, then the one with the smaller numerator is larger."* None of the pre-service teachers arrived at wrong answers by using "number sense based" methods. Ten pre-service teachers could not not justify their answers or their answers were categorized as unclear explanations. The answer *"1/2 is closer because it is an easy question."* is an example of unclear explanations. Besides, eight participants did not answer the question by stating that "*it can be solved only by performing an operation*."

In the second question related to the component of understanding the effects of operations, pre-service teachers were expected to recognize that multiplication may not be an operation that makes numbers always larger and division may not be an operation that makes numbers always smaller. However, it is observed that pre-service teachers responded without thinking and their answers were based on their prior knowledge, *"the operation (the result) absolutely becomes larger if there is a multiplication, and if there is a division, no matter what the figure is, the number becomes smaller."* As a result, only 17.29 % (23 participants) could answer the question correctly. Among pre-service teachers, only two justified their answers as stated below and these answers were accepted as number sense.

1) *The choice 'a' is smaller than 8.6; the choice 'b' is larger than 8.6; the choice 'c' is close to 0, and the choice is 'd' negative. Then the result is b>a>c>d.*

2) *The choice 'd' has already resulted negatively, the smallest one is the choice 'c'; when 0.74 is divided by 8.6, the result is something like 0; in the choice 'a', if we multiply 8.6 by 1, it will be 8.6. If we multiply by 0.74, it will be smaller than 8.6, in the choice 'b', it will be larger than 8.6. b>a>c>d.*

Four pre-service teachers stated that they could not predict the solution; therefore, they preferred to find the common denominators by changing decimal numbers into fractions. Answers of pre-service teachers who used paper-pencil algorithms were categorized as "ruled based" methods. Among pre-service teachers whose answers were categorized as rule-based methods, Aylin preferred to perform the operations by using simple expressions. *"Instead of using the given expressions, I used simple expressions like 1/2 and 3/2 and did the calculation"*. She estimated the result after the operations. What the pre-service teacher did here is an operational prediction but since she got the result by using paper-pen algorithm at first, this explanation was included in the rule based algorithm within the scope of this study.

Only seventeen pre-service teachers gave correct answers but they did not provide any explanations. More than half of the pre-service teachers, 68.42% (91 participant out of 133), gave incorrect answers. Highest rate of incorrect answers was obtained for this question.

Four of the ten pre-service teachers who provided incorrect answers justified their answers as; *"If we take 0.74 as a focal point, it will be d<a<b<c."* While this answer was accepted as number sense based method, the other six teachers' explanations - *" 'd' is negative; 'b' is close to 1; 'a' is larger than 1; 'c' will be inverted; therefore, it is a>b>c>d"* - were coded as partial number sense based method. Here pre-service teachers felt the need to use "division algorithm at fractions".

Forty respondents answering incorrectly justified their answers as follows:

1) *When we multiply, we get the largest number. In the operations of divisions, we get the smallest number. After the operations of multiplications, we obtain the second largest number from additions and we get the smallest number from subtractions.*

2) *In the operations with the same numbers, the largest result is obtained from multiplication, then from addition, and then from division respectively. Since the first number is smaller than the second number, the smallest one is obtained in subtraction.*

These incorrect explanations are accepted as rule based method. Among those who gave incorrect answers, forty one respondents did not provide any explanations. Nineteen pre-service teachers did not answer the question by adding *"I do not know. I am not good at fractions and decimal numbers. I could not find the solution without performing an operation."*

To the question associated with the third component of number sense, *"understanding the equivalence of numbers"*, 74.44% (99 participants) of the pre-service teachers gave the correct answer. The following explanation of the pre-service teacher, Umit, was accepted as number sense based method. *"0.25 means a quarter that is 1/4. In the whole, there are four quarters. In m, there are 4m quarters."*

Thirteen pre-service teachers who answered this question with number sense stated the following explanations:

> *"0.25 means one fourth. Therefore, it does not matter whether we divide it by 0.25 or multiply by 4. The result will be the same in both operations." or "When a whole is divided into quarters, the result you get will be equal to the one when it is multiplied by four."*

The other fifty three pre-service teachers answering correctly explained their answers as stated below:

1) *0.25 is equal to 1/4. Dividing a rational number by another number means inverting and then multiplying it. Therefore, we multiply it by four.*
2) *While dividing, 0.25 is equal to* $\frac{1}{4}$, *the first number is taken the same and the second number is inverted and multiplied. Therefore, it is multiplied by 4.*
3) $0.25 = \frac{25}{100} = \frac{1}{4}$ *if we divide x number to* $\frac{1}{4}$ *then it will be* $\frac{x}{\frac{1}{4}} = \frac{4x}{1} = 4.x$

These explanations were accepted as rule based method. If the actual case is taken into account, it can be said that most of the pre-service teachers relied on paper and pencil algorithms. The other thirty three pre-service teachers did not provide any explanations.

24.06% (32 out of 133) of the pre-service teachers gave incorrect answers to this question. Two of them gave the following explanations in their answers.

1) *"Dividin., Since 0.25 is equal to* $\frac{25}{100}$, *the choice 'c' is correct.*
2) *Dividing a number by a decimal number means multiplying it by that number. The choice'd' is correct.*
3) *Since expending /simplifying a fraction does not change the value of the fraction, the fraction of* $\frac{25}{100}$ *is equal to* $\frac{1}{4}$ *x 0.25 means that you can find '25' four times in 100, as a result it being divided by 4."*

This answer was classified as number sense based method. Although, the other twenty one pre-service teachers knew the equivalence of the fraction, that is $0.25 = \frac{25}{100} = \frac{1}{4}$, they had errors on the effects of the operation. For example, they stated the following explanations:

1) *Since 0.25 is equal to* $\frac{25}{100}$, *the choice 'c' is correct.*
2) *Dividing a number by a decimal number means multiplying it by that number. The choice'd' is correct.*
3) *Since expending /simplifying a fraction does not change the value of the fraction, the fraction of* $\frac{25}{100}$ *is equal to* $\frac{1}{4}$.

Answers of these twenty one pre-service teachers who gave these explanations were evaluated under the category of rule based method. Seven pre-service teachers who gave incorrect answers did not provide any explanations. Two pre-service teachers just answered as *"I do not know"* and did not give any reasons.

The fourth question related with flexible computing and counting strategies for mental computation shows that pre-service teachers had difficulties in this area. Only 11.28% (15 persons out 133) of pre-service teachers could provide the correct answer. Four pre-service teachers used reference points to justify their answers. For example;

> *"0.4975 means half, half of 9428.8 is approximately close to the choice 'c'*
>
> or
>
> *"In my opinion, 0.49 can be thought to be 0.5. Then, it will be the half of the value of 9428. I placed the comma accordingly."*

These explanations were recorded as number based method. Three pre-service teachers decided about the location of the comma by applying the rule below:

> *"The factor has four decimal digits and the product has one decimal digit. For this reason, it has 4+1=5 decimal digits. However, since as a result of the multiplication of 75x8, there will be two zeros, the comma should be shifted three places further. Therefore, the result is 4690.828."*

Pre-service teachers found the correct solution by both relying on a rule and associating this rule with the thought of what may be logical. Therefore, these answers were accepted as partial number sense based method.

One pre-service teacher provided the explanation stated below which was accepted as the rule based method.

"In the first number there is a comma after the four digits, in the second number it is placed after the first digit. Because of the difference in the digits (4-1 = 3), the place of comma should be before three digits".

Four pre-service teachers gave correct answers but did not provide any explanations.

58,65% of the pre-service teachers gave incorrect answers. One-eighth of the participants (17 respondents) used number sense based methods. The answers related to this category were:

1) *Accepting the number of 0.4975 as half means to divide the other number into two. For this reason, the answer should be choice 'a'* [4 participants].

2) *When I think that 0.4975 is equal to 0.5 and that 9428 is equal to 9000, the choice 'b' seems correct. The result is 469.0828* [3 participants].

3) *Since, I round up 0.4975 to $\frac{1}{2}$, it should be something like the half of the 9428.8. The closest choice is 'd'* [10 participants].

Although pre-service teachers were asked to estimate the result without doing an operation, 38 pre-service teachers gave incorrect answers by choosing this method. Their answers were as follows:

1) *The comma is placed by starting from the last digit in the left, according to the sum of the total digits seen in the result. In multiplications, the number of digits after commas are summed up* [20 participants].

2) *I have learnt such a rule in mathematics. After doing a standard operation, the place of the comma should be shifted according to the number of digits after commas* [10 participants].

3) *The total numbers of the product after the comma must be equal to the total numbers of the factors stated after the comma* [7 participants].

4) *8x4=40, if we also take this '0' into account, the comma will be after 4 digits not 5 digits. Therefore, the result is 469.0828* [1 participant].

These answers were recorded as rule based method as they were based on an operation or on a rule learnt before. While 23 pre-service teachers gave incorrect answers without providing an explanation, 40 pre-service teachers did not answer this question and stated that *"it requires the use of an operation"*.

In the fifth question related to measurement benchmarks component of number sense, it is required to order fractions and decimal numbers by paying attention to reference points of 1, $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$. 25.6% of the pre-service teachers (34 participants out of 133) listed the values correctly. Only 6 pre-service teachers who provided the correct answer used reference points in their explanations. Pre-service teacher, Müge, who answered according to the reference points, gave such an explanation:

"Zeynep walked to the furthest point, farther than 1 km; the second should be Deniz because she walked to a distance close to 1 km. Ali should be the third because he walked half a mile Ayşe is the fourth, she walked approximately $\frac{1}{3}$ km and Betül is the fifth, she walked approximately $\frac{1}{4}$ km."

Such explanations were recorded as number sense based method. Five pre-service teachers answered the question by paying attention to reference points of 1, 0.2, 0.3, and 0.5 instead of those of the fractions $\frac{1}{2}$ and $\frac{1}{4}$ by using paper and pencil algorithm. Although pre-service teachers used paper and pencil algorithms while explaining their answers, the answers were categorized as partial number sense based method since they knew the decimal numbers' equivalence of the fractions. For example,

"Zeynep walked the furthest because it is a compound fraction. That is, she walked 1 more km. Therefore, Deniz>Ali. Betül walked something like 0.2 km. Ayşe walked something like 0.3 km. Murat walked something like 0.55 km.The result Zeynep>Deniz>Murat>Ali>Ayşe>Betül."

Eleven pre-service teachers provided the floowing answers and recorded as correct and rule based method.

"Betül < $\frac{13}{38}$ < 0.4828< $\frac{8}{15}$ <0.966 < $\frac{17}{16}$. The larger the divisor is, the smaller the result will be." or

"I listed the fractions by equalizing their denominators".

Although the remaining twelve pre-service teachers provided correct answers, they did not provide any explanations to classify their answers. Therefore, they were evaluated under "unclear explanations" category. 42% (57 participants out of 133) of pre-service teachers used paper and pencil algorithms to decide on their answers. However, they gave incorrect answers because of the mistakes they made during calculations. Among the pre-service teachers, answers of 14 participants were accepted as partial number sense based method. The related examples are:

"The result is Betül<Ayşe<Ali<Murat<Zeynep<Deniz. Betül walked the shortest distance because the distance corresponds to 0.25 km. Deniz is the one who walked a distance closer to 1 km." or "We get approximately this result if we change the fractions into decimals."

The answers of 24 pre-service teachers were classified as rule based method. For example:

"The smaller the numerator than the denominator is, the smaller the value will get. Therefore, the result is Betül, Ali, Ayşe, Murat, Deniz, Zeynep." or "Because of the differences between denominators and numerators, the result is Deniz, Betül, Ayşe, Ali, Murat, and Zeynep."

Sixteen pre-service teachers gave incorrect answers but they did not provide any explanations. 31.58% of the pre-service teachers did not answer the question by explaining *"It is hard for me to solve."* or *"I am not good at fractions."*

Conclusion and Discussion

Results obtained in this study show surprising similarities with the strategies determined in the studies regarding students' number sense in Taiwan, Sweden, Kuwait, Australia and the United States. For example, Markowits and Sowder (1994) stated that while solving arithmetic problems in schools, very few students exhibited number sense and in the studies of Yang and Reys (2002) it is stated that the students from Taiwan had an inclination to use standard written algorithms to a great extent while explaining their answers. Reys et al. (1999) stated in their study regarding the number sense of the students of the U.S. Taiwan, Australia and Sweden that although performance levels of the students concerning number sense differentiated on the basis of the countries, students showed consistently low performance. In Turkey, similar findings were stated in the studies of Harç (2010) and Kayhan Altay (2010).

According to Ekenstam (1977) number sense covers the improvement of various relationships among mathematical concepts, knowledge and skills; therefore, it provides access to many concepts at the same time when necessary. Students who do not comprehend these relationships have to remember and learn various rules in order to cope with practical problems in everyday life. Besides, it is stated that over-emphasized standard written algorithms prevent students from both using number sense and from improving important thinking skills such as reasoning, estimating and interpreting (Burns, 1994; Calvert, 1999; Even, 1993; Even & Tirosh, 1995; Hiebert, 1999; Kamii, Lewis, & Livingston, 1993; Leutzinger, 1999; Ma, 1999; Mack, 1995; Milli Eğitim Bakanlığı [MEB], 2009; Reys & Yang, 1998; Schifter, 1999; Shulman, 1987; Wearne & Hiebert, 1988; Yang, 1997, 2002a, 2002b, 2005, 2007; Yang & Reys, 2002; Yang et al., 2009).

On the other hand, by instructing pre-service teachers on the current situation of the students' number sense, it will be possible to provide them with experiences related to how students' number sense abilities can be improved, how lesson plans can be prepared and what kind of activities should be used. Additionally, pre-service teachers can understand the necessity and importance of number sense and this will help them to improve students' number sense as well by providing pre-service teachers with proper training programs in which mental calculations and estimation skills can be improved.

For future studies, it will be useful to analyze the relationships among pre-service teachers's meta-cognitive levels and their abilities to use number sense; mathematical self-efficacy and the level of using number sense components, classroom teachers' level of mathematical attitudes and concerns and their abilities to use number sense components.

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