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Different Grade Students' Use and Interpretation of Literal Symbols

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Abstract

The aim of the study was to determine and compare 7th, 8th, and 9th grades students' level of use and interpret the literal symbols. In addition, students' responses to questions that require use of different roles of literal symbol were examined to identify the errors. For this purpose, Chelsea Diagnostics Algebra test developed by The Concepts in Secondary Mathematics and Science (CSMS) research project was applied to total 407 students from different grade. The results of the study could be summarized in three headings : i) The majority of the 7th and 8th grade students had difficulty in using and interpreting literal symbols as generalized number, unknown and variable whereas 9th grade students had some problems particularly in understanding variable role of the literal symbols; ii) The performance of students' using and interpreting literal symbols do not increase monotonously with grade level and age; iii) Although the students' tendency of assign number value for letters, ignoring letters or considering letters as abbreviation of objects varies at different grade levels, this inclination decreased towards upper grades. However, it was revealed that when the complexity level of questions raised, upper grade level students displayed the mentioned behaviors.

Key Words

Variable, Unknown, Generalized Numbers, Use and Interpretation of Literal Symbol, Student Understanding.

Algebra, one of the oldest fields of the mathematics with about 4000 year history, was born out efforts to find general methods to solve equations. Algebra is the mother tongue of Mathematics and has specialties (Usiskin, 1997). This language provides opportunities for generalization, using algorithms and operations to solve problems, working out relations between quantities and investigating abstract terms such as; group, ring, vector spaces (Baki, 2006; Driscoll, 1999; Tall et al., 2000; Usiskin, 1999).

Literal symbols as "a, b, x, t…" are one of the most important elements of this tongue. Using literal symbols has a key role on teaching fundamental algebraic concepts and issues. In addition to this interpreting and using literal symbols is a base for all advanced mathematics subjects to be built on it (Dominguez, 2001; MacGregor & Stacey, 1997; Schoenfeld & Arcavi, 1999). Plenty of researches stated that students have been having great difficulties in using and interpreting literal symbols (Arzarello, Bazzini, & Chiappini, 1993; Dominguez; Kieran, 1992; Kinzel, 2000; Luo, 2004; MacGregor & Stacey; Philipp, 1999; Rosnick, 1999; Schoenfeld & Arcavi, 1999; Sfard & Linchevski, 1994; Stacey & MacGregor, 1997; Tall & Thomas, 1991). These difficulties caused the errors interpreting algebraic expression, algebraic operations and problem-solving process (Küchemann, 1978; Sfard & Linchevski, 1994; Stacey & MacGregor, 2000). In addition to past research indicates that students' abilities to use literal symbols especially variables as varying quantities have an impact on their success in calculus (Gray, Loud, & Sokolowski, 2009; Jacobs, 2002). For

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example, Jacobs found that secondary school advanced calculus students who had difficulty interpreting variables as covarying quantities were also likely to have difficulty understanding the calculus concepts of limit and derivative. One of the primary sources of these problems is that the literal symbols have a variety of roles in mathematics (Driscoll, 1999; Philipp; Schoenfeld & Arcavi). They are used to as unknowns (e.g., x in 3x-1=25), as generalized number (e.g., a,b in a.b=b.a), as varying quantities (e.g., x, y in y=sin(x)), as label (e.g., 3f=1y, where f represent "feet" and y represent "yards") and a constant (e.g., pi, e, c), among others (Philipp; Usiskin, 1999). So, it is important that students realize different roles that these literal symbols are playing. In the present article we focus on it.

Students' Interpretations of Literal Symbols

In late 1970's, The Concepts in Secondary Mathematics and Science (CSMS) research project carried out in the United Kingdom to develop levels of understanding in mathematics and to emerge incidence of students' errors by British students of age 12 to 15. For this purposes, the CSMS team prepared test papers ten different topics include in algebra. The test which was developed for algebra by CSMS project is called Chelsea Diagnostics Algebra test. Some results belong to this research project published as a book called "Children's Understanding of Mathematics: 11-16". According to results of this research, six different ways of interpreting and using the literal symbols were identified (Küchemann, 1978, 1981, 1998). These are described briefly in the following;

Letter Evaluated: Students generally assign a random number to the letter immediately. For example, to find the numerical value of the letter *a* in the equation, *a+5=8*, methods of trial and error can be used. It is not necessary to handle *a* as an unknown.

Letter Not Used: Here, the letter can be ignored or not interpret. Students acknowledge letters' existence but without giving it a meaning. For example "*If a+b=43, a+b+2=?*" At this level, students can essentially ignore the expression *a+b* and focus on the operation "*+2*". The question can be correctly answered by using a matching technique without explicitly attending to *a* and *b*.

Letter as an Object: Letter can be viewed as an object in its own right. An inappropriate use of the letters as objects (or labels) is when the letters are used to represent objects, rather than numbers of objects. We think about below question;

"Blue pencils cost 5 pence each and red pencils cost 6 pence each. I buy some blue and some red pencils and altogether it costs me 90 pence. If *b* is the number of blue pencils bought, and if *r* is the number of red pencils bought, what can you write down about *b* or *r*?" This level of interpretation is evidenced by a response such as "*b+r=90*", "*6b+10r=90*" to this question. To solve this question, the letters have to be regarded as specific unknown.

Letter as a specific unknown: In the previous three categories, students avoid having to operate on a specific unknown. The present category students use and interpret the letters as genuine unknown even though the idea of a specific unknown number is still a rather primitive notion. The following question requires the letter to be interpreted as a specific unknown;

"Multiply *n+5* by *4*". The operation has to be applied to both element of algebraic expression *n+5*. But many student produced answers like without operating to the algebraic expression as a whole.

Letter as a generalized number: The letter is interpreted as a generalized number, differing from the specific unknown in the last category in that the letter is seen to be able to take on several values. Below question seems to require the letters to be seen as generalized number;

"What can you say about *c* if *c+d=10* and *c* is less than *d*?". For example, an appropriate response to the question shown below would be *c<5*.

Letter as a Variable: Küchemann (1998) used the word, variable, for letters representing varying quantities. In other words the letter is seen as representing a range on unspecified values, and a systematic relationship is seen to exist between two such sets of value. The following question can be an example;

"Which is larger, *2n* or *n+2*? Explain.". Such a task seems to require considering several values for the letter and attending to the expressed relationship.

In this study the word variable is referred in a way that Küchemann (1998) defined. As a result of this project, four hierarchical levels of understanding were identified based on the six ways students interpret and use letters. Students categorized at Level 1 or 2, two lower levels, appear to evaluate the letter, ignore the letter, or use the letter as an object. According to Küchemann (1998), the main difference between Level 1 and Level 2 is that students classified Level 2 can solve more complex problem. At Level 3, students can use a letter as a specific unknown. Students classified Level 4 highest level can interpret and use letters as specific unknown, also, as generalized numbers and as varying quantities. They can solve problems which have a more complex structure and require more difficult problem solving methods than those which would assigned Level 3 understanding. Küchemann suggested that these four levels correspond to the Piagetian stages of below late concrete, late concrete, early formal and late formal respectively. According to Küchemann (1998), there is a relationship between students' levels of understanding of literal symbols and Piagetian stages of cognitive development.

When the previous studies related to using and interpreting literal symbols were reviewed misconceptions of students can be summarized as; assigning a specific value to a literal symbol [e.g., 2n is always bigger than n+2 because 2.5 is 10 and 2+5 is 7 (Gray et al., 2009; Küchemann, 1998; Knuth, Alibali, McNeil, Weinberg, & Stephens, 2005)], interpreting a literal symbol as a shorthand label for an object [e.g., *a* stands for apple (Küchemann, 1998; Knuth et al.; MacGregor & Stacey, 1997; Rosnick, 1999)], viewing a literal symbols as a place holder for a missing digit [(e.g., if n=5 ise 3n=35 (McNeil et al., 2010)], and assigning a value to a literal symbol that correspond to its position in the alphabet [e.g., a=1, b=2, c=3 (MacGregor & Stacey)].

When the relating literature was gone through it is seen that a lot of researches carried out in different countries about children's understandings literal symbols using the Chelsea Diagnostics Algebra test or using the result of CSMS project (Bateman, 1997; Gray, Loud, & Sokolowski, 2007; Gray et al., 2009; Klanderman, 1996; Lin, 1994; MacGregor & Stacey, 1997; Sokolowski, 1997; Wyllie, 1996). For example Lin applied this test with students from Hong Kong, Wyllie with American and Bateman with Canadian students. In this research, Chelsea Diagnostics Algebra test was applied to students in Turkey which has different educational system.

Current View of Algebra Education in Turkey

The ages and grade levels at which algebra is introduced differs from country to country (Erbaş, 2005). So, we should mention about the teaching and learning of algebra in Turkish context. Generally, although the content of school algebra in Turkey is not much more different from the other countries, the teaching and learning of algebra is more traditional. However, as a result of the reform movement which was put into practice in mathematics education in Turkish schools a couple of years ago the national K-5, K-8, K-12 mathematics curricula have gone through major changes in terms of content and instructional strategies with more student-centered teaching, use of manipulatives, and utilization of technology, particularly calculators (Erbaş). Mathematics curricula in Turkey contain algebra as a subject starting from sixth grade. Although mathematics programs for primary schools (1-5 grades) in Turkey do not include algebra as a subject, there are some expectations related to algebra. It requires the earlier introduction of algebraic concepts. For example, the students are expected to use different items, such as letters, numbers and shapes to create pattern, to find the rule for this pattern and to explain the rule. Formally, introduction to algebra takes place in secondary schools (6-8 grades). Algebra in secondary schools contains the following topics in a spiral structure; pattern and relations, algebraic expression, equality and equation. Also, mathematics curriculum for 8th grade includes inequality. If 6th, 7th and 8th grade mathematics curricula in Turkey is examined it can be seen following expectations related to algebra; (Milli Eğitim Bakanlığı [MEB], 2009, 2011).

- The students will be able to model a number pattern and express the relationship in this pattern with the letter.
- The students will be able to understand the concept of identity, equation, inequality and variable and explain the difference among these concepts.
- The students will be able to solve a system of linear equation and a system of linear inequality using algebraic and graphical methods.

9th grade mathematics program includes algebra topics such as sets, relation, function and numbers (natural numbers, integers, modular arithmetic, rational numbers, real numbers, absolute value, exponential numbers, and radical expressions) predominantly. So, 9th grade expectations require a good way to use algebraic letter as a variable as well as unknown and generalized number.

When we review Turkish literature about algebra learning we come across researches trying to determine student misconceptions and difficulties in algebra (Akgün & Özdemir, 2006; Akkan, Çakıroğlu, & Güven, 2008, 2009; Akkaya, 2006; Akkaya & Durmuş, 2006; Baki, 1998a; Çelik, 2007; Dede, Yalın, & Argün, 2002; Erbaş, 2005; Erbaş & Ersoy, 2002a, 2002b, 2002c; Ersoy & Erbaş, 2005; Soylu, 2008; Şandır, Ubuz, & Argün, 2007; Yaman, Toluk, & Olkun, 2003; Yenilmez & Avcı, 2009), evaluating students in terms of procedural and conceptual knowledge in algebra (Baki, 1998b;

Baki & Kartal, 2004). Most of these studies were implemented with 9th grade students (Baki & Kartal, 2004; Erbaş; Erbaş & Ersoy, 2002a, 2002b, 2002c; Şandır et al.). On the other hand there were also studies applied at 6th grade (Akkaya; Yenilmez & Avcı), 7th grade (Soylu) and 8th grade level (Akgün, 2007; Dede et al.; Ersoy & Erbaş) There were also some, less frequent, other studies comparatively presenting algebraic understanding of students from different grade levels. For example; Baki (1998a) attempted to state errors and misconceptions of 8th and 11th grade students while performing algebraic operations; Yaman et al. tried to state understanding of students from various grades (2nd, 3rd, 4th, 5th and 6th grade) about the equality concept. Akkan et al. (2008) aimed to determine probably error and misconceptions of 6th and 7th grade students in understanding literal symbols, using notations, applying rules and generalizing. On the other hand, Akkan et al. (2009) focused on competences of setting equations based on verbal explanation and making up a problem according to given equation with 6th and 7th grade students. However, no studies found directly concerning 7th, 8th and 9th grade students' understanding about different use of literal symbols, which is a significantly meaningful in terms of transition from arithmetic to algebra.

Goals of the Study

The aim of the study was to determine and compare 7th, 8th, and 9th grades students' level of use and interpret the literal symbols. In addition, students' responses to questions that require use of different roles of literal symbol were examined to identify the errors.

Method

Participants

The sample studied consisted of 407 different grade students in Trabzon, Turkey. In this study, 7th (120), 8th (144) and 9th (143) grade were selected taking into account the ages of students in international studies. To ensure the strength of the sample to represent the universe, the number of students in each grade level was considered above 100 (Karasar, 2011).

In the present study, 18 students (one student from 7th, 9 students from 8th, 8 students from 9th grade) weren't included in data analysis because of their unfitness to any of the levels. So, the findings were presented based on the data which were obtained from 389 students.

Instrument

CSMS team developed, refined and validated a test called Chelsea Diagnostic Algebra Test using thousands of English secondary students (Hart, Brown, Kerslake, Küchemann, & Ruddock, 1998). It is seen to be a valid and reliable instrument for the determining students' levels of use and interpretation algebraic letter (Brown, Hart, & Küchemann, 1985; Çıkla, 2004; Gray et al., 2009; Sokolowski, 1997). So, in this research to assess students' levels of use and interpretation algebraic letter was used Chelsea Diagnostic Algebra Test. This test classifies students' usage the letter into four hierarchical levels. The test which included 51 items originally was subjected to a statistical analysis by researchers (Küchemann, 1998). Thirty of the 51 items were used to determine students' levels of usage algebraic letters.

Chelsea Diagnostic Algebra Test was translated and adapted into Turkish by Çıkla (2004). For this purpose, minor changes in the context of some of the test items were made. Reliability measure as based on KR-20 coefficient was found to be 0,93.

Results

The findings of the study were presented under two headings as the findings related to students' level of using and interpreting literal symbols and the findings related to students' answers to test items.

The Findings Related to Students' Level of Using and Interpreting Literal Symbols

When the level of the students' using and interpreting literal symbols was examined, it was found that the number of students at Level 0, 1 and 2 decreased as the grade level increased. On the contrary, number of students falling Level 3 and Level 4 increased as the grade level increases (except for transition form 7th and 8th grade at Level 4). The obtained data showed that 7th and 8th graders were not quite successful at using and interpreting the unknown, the generalized number and especially variable role of the letter symbols. The proof of these was very low percents at Level 4 (only 4.2% of 7th grader and 1.5% of 8th grader were at this level). When it came to 9th graders (12.6% of 9th graders are Level 4), there was an improvement which didn't meet the expectations.

The Findings Related to Students' Answers to Test Items

In this section the percentage of the correct answers given by the students from different grade levels to test items were presented. In addition, the responses given by the students towards the items of different level were analyzed in detail.

When the student answers for test items were investigated, it was observed that the percentages of correct responses for Level 1 and 2 questions at each grade were more than 50 percent (except one situation- 8th grade 9c items). At Level 3, although the majority of the percentage of correct responses in 7th grade and the half of the percentage of correct responses in 8th grade is under 50 percent, all of the percentage of correct responses in 9th grade is over 50 percent. When the percentages of the correct responses of items in Level 4 were examined, the percentage of the correct responses in each grade was very low especially for items 3 (only 0.8% from 7th, 0% from 8th, and 3.7% from 9th), 17a (3.4% from 7th, 0.7% from 8th, and 8.9% from 9th) and 22 (3.4%from 7th, 0.7%from 8th, and 13.3% from 9th).

Some items which were at Level 2 or higher in the test were analyzed throughoutly. The correct responses of the students for each item were presented with the percentile of the wrong answers and no answer. Some of the items analyzed in this section were:

Level 2/ İtem 13d: What is the simplest form of the 2a +5 b + a?

Level 3/Item 13b: What is the simplest form of 2a+5b?

Level 3/Item 16: What can you say about c if c+d=10 and c is less than d?

Level 4/Item 3: Which is larger, 2n or n+2? Explain.

Level 4/ Item 17a: Hakan's basic wage is 20 TL per day. He is also paid another 7 TL for each hour of overtime that he works. If s stands for the number of hours of overtime that he works, and if k stands for her total weekly wage, write down an equation connecting s and k.

The analysis of Item 3 at Level 4, which was determined as the hardest question for all classroom levels, was presented here. The rest of the analyses were given in the full text.

"Level 4/Item 3: Which is larger, 2n or n+2? Explain". This item required recognizing that the relative size of two algebraic expressions (2n and n+2) was dependent on the value of *n*. Most of the students (53.8% from 7th, 54.8% from 8th and 48.1% from 9th) answered the question as "2n is a larger number". One of the main reasons why the students answered question so was that they thought that the multiple of *n* by 2 should be larger than *n* added by 2. So, it could be said that majority of the students interpreted *n* as a specific value, not as a variable. The students who gave the answer "n+2" or "the same" produced their answers by assigning one value to *n*. While the percentage of students who answered this way at seventh grade was 21%, at eighth grade was 15.6 %, at ninth grade was 5.2%. The students who answered the question as "depends on the value of *n*" usually assigned more than one values to *n*. If they had used the trial and error method systemically they could have reached to correct solution. In addition majority of these students gave the positive integer values to *n*.

Although 9th grade students had the maximum correct percentage for this question, it really was very low (3.7% for 9th grade). Formally the introduction of the function, and therefore variables, take places at 9th grade in Turkish mathematics curriculum. But this question could be solved using inequalities and this topic was taught at 8th grade. It is very interesting that there were no students who answered this question correctly at 8th grade.

Discussion

Based on the findings, when the percentages of the correct answers given by the students from all grade as response to questions of different levels (Level 1, 2, 3, and 4) were compared, it was observed that there was a decline at Level 3 and a sharp fall at Level 4. When grade levels were concerned, it could be said that majority of the 7th and 8th grade students had difficulty in understanding and using literal symbols as generalized number, unknown and variable whereas 9th grade students had some problems particularly in understanding variable role of the literal symbols. In spite of the fact that 9th grade mathematics curriculum requires using different roles of literal symbols such as variable, unknown, generalized number, parameter, constant *etc.,* very few of the 9th grade students could reach Level 4 (12.6%). Lots of studies carried out with secondary school students showed that students have similar difficulties in understanding different use of literal symbols (Kinzel, 2000, 2001; Rosnick, 1982; Stacey & MacGregor, 2000).

When the understanding levels of the students were examined by grade, it was determined that there was no significant difference between 7th and 8th grade students. When the responses of the 7th and 8th grade students were examined for each individual question item, it was observed that 7th grade students have high percentage of correct answers than 8th graders at some items, most of which were at Level 3 or 4 (item 7b, 9a, 7c, 9b, 9c, 15a, 9d, 13b, 3, 13e, 17a, 20, 21, 22). This piece of finding indicates that the achievement of students' using and interpreting literal symbols cannot be explained only by age and cognitive development. Numerous factors (student pre-knowledge, curriculum, classroom applications etc.) can be alleged to contribute. One of these factors can be the U-shaped cognitive development as it is referred. According to this approach, student performances do not increase monotonously with age (Baylor, 2001; McNeil, 2007), students may exhibit rise and falls depending on the interaction between their own cognitive structure and patterns of the outside environment (out of student's mind) instead (Baylor; McNeil).

Mcneil et al. (2010) recommended not using mnemonic literal symbols (e.g., using *a* for the number of apples, and *p* for the number of pen) frequently in transition from arithmetic to algebra. Mcneil et al. argued that the understandings students possess (right or wrong) related to a concept can be activated depending on the content. Therefore, when a particular way of thinking about aconcept is well established–as it is in interpreting literal symbols as label, it can be activated in an easier way across a wide range of contexts. Incontrast, when a particular way of thinking about a concept has been just emerged -as it is in interpreting literal symbol as variable, it may be active or inactive depending on the content. From this point of view, students may need substantial contextual support to help them interpret letters as variables. Students' interpretation of letters as variables may be hindered in contexts that strengthen the interpretation of letters as labels (e.g., using a for the number of apples) and helped in contexts that do not strengthen the interpretation of letters as labels (e.g., using x for the number of apples) (Mcneil et al.).

Backed with the results coming from the data analysis, the students had tendency to assign number value for literal symbols and this inclination decreased towards upper grades. Previous studies showed that students have tendency to equal the right side of the given algebraic equation to a numerical value with a habit transformed from arithmetic (Gray et al., 2009; Jacobs, 2002; Kieran, 1992; Tall & Thomas, 1991). Similarly, students working with the set of natural numbers for a long time preferred to use the set of natural numbers as source for variable values instead of the set of real numbers. It can be said that 8th grade students recognized literal symbols much better than 7th graders do but they did not fully understand their roles in algebraic expression. Similarly Lucariello mentioned "interpreting literal symbols as shorthand labels for objects (e.g., *s* stands for students)" as the most frequent misconception in the study conducted with 450 students in grades 6-12 (most of them from 8th and 9th grade students) (see: McNeil et al., 2010, p. 626). On the other hand, it was determined that 9th grade students did not frequently use approaches like assigning values for letters, ignoring letters or considering letters as abbreviation of objects in the low level questions but they started to do as the level of the questions increased. According to McNeil and Alibali (2005) when students develop a correct understanding about a concept, they do not directly replace old, incorrect knowledge. Instead, the old knowledge structures continue to co-exist together with the new ones, and can get active when circumstances favor it. The data of the present study support this finding. The powerlessness of the students as in question items 16, 3, and 17a might cause such circumstances to occur.

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