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Mathematical Investigations for Supporting Pre-service Primary Teachers Repeating a Mathematics Education Course

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Abstract: Preparing to become an effective primary school mathematics teacher is a challenging and complex task; and is influenced by one's past experiences, personal knowledge of, and beliefs and attitudes towards mathematics. This paper examines the experiences of a small group of pre-service teachers who did not pass their first year mathematics education course. As part of their second attempt at this course for a Bachelor of Teaching (primary) degree the pre-service teachers engaged in a mathematical investigation. Data suggests that undertaking an open-ended mathematical investigation facilitated positive shifts with regard to the pre-service teachers' knowledge of, and beliefs and attitudes towards, mathematics. Participation in such investigations appears to have potential for supporting pre-service teachers who have initially experienced difficulties in their journey to becoming effective mathematics educators.

Introduction

There is a growing body of research stating that teachers' knowledge of mathematics and pedagogical content knowledge are vital aspects for effective mathematics teaching and children's learning (Anthony & Walshaw, 2007; Ball, Hill & Bass, 2005). In New Zealand there has been a concerted push over recent years to improve children's learning in mathematics (Ministry of Education, 2008; Young-Loveridge, Mills and Bicknell, 2012), and teachers are regarded as key figures in changing the way this subject is taught and learned in schools (Ministry of Education, 2008). The type of teacher knowledge (Pedagogical Content Knowledge) needed is critical, with recent mathematics education reforms focussing on teachers' (and children's) conceptual understandings of mathematics rather than the procedural skills that have been a feature of much mathematical learning in the past (Boaler, 2002, 2008; Young-Loveridge, 2010). In New Zealand, having appropriate content and pedagogical content knowledge is a requirement for graduating teachers (New Zealand Teacher's Council, 2007).

Pre-service teachers bring to teacher education many years of mathematical experiences. It is known these experiences shape their beliefs and attitudes towards mathematics, and have the potential to shape future teaching (Guillaume & Kirtman, 2005; Malara & Zan, 2008; Young-Loveridge, Mills and Bicknell, 2012). For some people mathematics holds negative connotations and memories (Leder & Grootenboer, 2005). Grootenboer (2001) states, "you do not need to look too far to find someone who will admit to hating mathematics" (p. 17). Such mathematical dislike, associated anxiety, and concerns about content knowledge, have been specifically linked to primary teachers and prospective

primary teachers (Grootenboer, 2001; Liljedahl, 2005).

Recent research posits that emotions impact on pre-service teachers' learning of mathematics (Dogan, 2012; Rule and Harrell, 2010; Liljedahl, 2005; Loughran, 2006) and are regarded as key to supporting changes in beliefs and attitudes. For example, positive experiences with their attendant emotions will eventually produce positive beliefs and attitudes towards mathematics (Liljedahl). Liljedahl explains that learners may need to experience sustained and repeated successes in their learning of mathematics in order to achieve a change in beliefs and attitudes. In contrast however, beliefs and attitudes can be drastically changed when a single AHA! experience occurs (Liljedahl). An AHA! experience being that moment when there is a flash of insight and a problem is suddenly solved. Vygotsky has also written about learning being an emotional as well as cognitive endeavour. The recently available English translation of Vygotsky's unfinished manuscript about the role of affect, suggests that negative emotions such as fear and anxiety diminish the zone of proximal development (Holbrook and John-Steiner, 2008).

Dogan's (2012) findings suggest that a mathematical learning environment that includes collaboration, inquiry-based projects and support with mathematics content helps to reshape pre-service teachers' emotions such as anxiety. Similarly, Rule and Harrell (2010) report that after a mathematics methods course that emphasised co-operative group work, an inquiry or problem-solving approach and an emphasis on mathematical discourse amongst other features, pre-service teachers' positive emotions about mathematics were significantly increased. Emotions are not only a critical aspect of mathematics learning but also a critical aspect of achieving change in professional practice. It is only when teachers are at ease with an innovation that they can adapt and adjust new practice to particular students and contexts (Timperley, Wilson, Barrar and Fung, 2007). Miller-Reilly (2006) found that focussing on understanding a second-chance learner's fear of mathematics was an important starting point in achieving a positive affective change in the student's beliefs and attitudes towards mathematics.

It would follow that pre-service teacher education needs to provide opportunities for pre-service teachers to consider their mathematical knowledge (Zevenbergen, 2004), and identify and examine their beliefs and attitudes towards mathematics (Dogan, 2012; Young-Loveridge, Mills and Bicknell, 2012). Teacher education must also include opportunities for pre-service teachers to examine their conceptions of teaching, including what and how to teach (da Ponte and Chapman, 2008). Learning and assessment tasks need to be carefully chosen, and the design of these is regarded to be a crucial aspect of supporting the development of a sound understanding about learning and teaching (Fenwick, Humphrey, Quinn and Endicott, 2013). Responding to all of these demands means the task of the pre-service mathematics educator is a multi-faceted and complex one (da Ponte & Chapman, 2008; Loughran, 2006). Supporting pre-service teachers to develop appropriate content knowledge and (possibly) changing attitudes and feelings about mathematics, is certainly seen to pose significant challenges (Young-Loveridge, 2010).

It is likely that these challenges are compounded for pre-service teachers who have not initially been successful in their mathematics methods or mathematics education courses. Little has been written about the experiences of pre-service teachers failing their mathematics education courses, although some research has been conducted on the impact on mentor teachers when pre-service teachers struggle or fail their internship (see Siebert, Clark, Kilbridge and Peterson, 2006). This current study begins to address this gap by investigating the impact of an intervention designed to support a small group who failed their first year mathematics education course. The intervention consisted of requiring the pre-service teachers to undertake a mathematical investigation as part of their second attempt at passing the mathematics education course. Mathematical investigations have been successfully used in other teacher education settings (see Bailey, 2007; Hawera, 2006) and it was hoped that this particular group of pre-service teachers would also benefit.

Mathematical investigations are open-ended tasks that enable a variety of mathematical avenues to be explored. They often require a significant period of time to undertake, and there is an embedded emphasis on the development of a conceptual mathematical understanding (Frankcom, 2009). Such understanding can be aligned with Skemp's (1991) notion of relational mathematics which is described as "knowing both what to do and why" (p. 5). What it means to 'do mathematics' and the social norms that characterise an investigative classroom are different to those of a more traditional mathematics classroom. Learners are expected to be creators of mathematical knowledge and mathematics is seen as a 'human activity' (Freudenthal, 1991) with an underlying goal of 'mathematising'. Learners are expected to justify their answers and methods and also support, communicate with and help others in their learning (Boaler, 2002, 2008). Choice and autonomy characterise the investigative classroom. Often, a variety of projects (investigations) are offered. In this study two open-ended investigative tasks were available with each task "provid(ing) different access points for different students and enabl(ing) students to work on them at different mathematical levels" (Boaler, 2002, p. 57). Once learners have been introduced to some starting questions or themes there is considerable flexibility about what mathematics may be encountered and explored.

The Context of this Study

This research occurred in the context of a Bachelor of Teaching (primary) degree. This three-year degree includes two compulsory mathematics education courses, a 'half' course (total expected workload 75 hours) at first year level and a 'full' course (total expected workload 200 hours) at second year level. Pre-service teachers who do not successfully meet the requirements for the first year course may be given an opportunity to repeat the course. The cohort in this group is normally small, with approximately 10-15 pre-service teachers.

It has been the researcher's experience, as the lecturer taking this 'repeat' course, that these pre-service teachers often face a number of challenges as they learn about the learning and teaching of mathematics. These often include self-reported concerns about their understandings of mathematics, and sometimes an intense dislike for the subject. In order to respond to these challenges and provide support, the course while having the same overall aims, has had different assignments to the usual first year course. The first of these assignments incorporates an open-ended mathematical investigation.

The pre-service teachers in this study chose one of two investigations. One of the investigations focussed on tessellations and the other on measurement formulae. The tessellations investigation started with asking the pre-service teachers to analyse a picture of the Dutch artist, Escher, asking what they noticed about the picture (an irregular tessellation). The investigation then asked the pre-service teachers to investigate tessellations. A series of three guiding questions were offered as starting points, including, "explore:

- shapes that tessellate?;
- why some shapes tessellate and others do not?; and
- how to create your own tessellation of an irregular shape (a diagram was provided as a supporting example)".

The pre-service teachers were advised that they could follow their own ideas in the investigation, as long as they were mathematically based. The measurement investigation started with exploring the formula for the circumference of a circle, $C=\pi d$. A task was given, asking students to physically count out footsteps for the radius of a circle and compare this to the circumference, measured in the same footsteps, for a number of different sized circles. This was followed by a question asking what the relationship between the radius and circumference might be, and what links they could then make to the formula, $C=\pi d$. The

investigation next required the pre-service teachers to explore the formula for finding the area and/or volume of another shape of their choice.

The investigations were started in week one, and continued until week six of the course. Approximately one-two hours were provided for the pre-service teachers to work on their chosen investigation during class each week. It was also expected that they would spend up to three hours of their own time working on the investigation and/or other tasks set in class, eg. reading selected relevant journal articles or book chapters.

For this assignment, there was a requirement to keep a personal record (hand-written or electronic) of ongoing, regular entries detailing their investigative work. It was expected that the record would include explanations, with specific examples, of their mathematical thinking; the solution(s) reached for their chosen mathematical investigation with a focus on the particular avenues that they explored; a summary of the mathematics concepts learned or reconsidered, with explanations and examples that showed their understanding of each concept; and reflective comments about the investigative process. At the end of the investigation the pre-service teachers wrote a summary statement reflecting on: what it means to them to be a learner of mathematics; their own mathematical knowledge; their insights about their beliefs and attitudes towards mathematics; and associated possible implications for their future teaching. It was also expected they would consider their further development needs as a mathematics educator. Specific links were required to entries from their personal record and at least three published readings.

A conception of mathematics as an endeavour that is social, constructive and experiential underpinned the use of investigations and teaching that took place. With this conception mathematics is viewed as “a constructive and creative enterprise” (Mason, 2008, p. 4) and is created in communication with others (Barton, 2008). These ideas ascribe to a mathematics that is not separate to one’s self, nor is it finite, but a creation that is “never finished, never completed” (Barton, 2008, p. 144). The need to instill confidence in learners by offering caring support is recognised as an integral component of the learning process (Holbrook and John-Steiner, 2008). An investigative approach was regarded as one that could afford these opportunities.

Data Gathering and Analysis:

Within a constructivist or interpretive paradigm whereby multiple realities are acknowledged, and understandings are regarded as being co-created (Denzin and Lincoln, 2000a; Borko, Liston and Whitcomb, 2007) three sets of data were collected to support analysis of the extent to which a mathematical investigation supported the pre-service teachers to develop a conceptual understanding of some mathematics ideas, and to positively develop their beliefs and attitudes towards mathematics. Reflecting the need for the use of multiple methods (triangulation) to gain an in-depth understanding of the phenomena being studied (Denzin and Lincoln, 2000a) data included a group interview, copies of the mathematical investigation assignments, and the researcher’s (also the lecturer of these pre-service teachers) informal, written semi-structured observations recorded during the five weeks.

After the pre-service teachers had finished the course (including both assignments) they were invited to be part of this project. They were sent a letter outlining the research, asking if they would be willing to participate in one or more of three ways. These included: participating in an informal, audio-taped group interview (of up to approximately 6 or 7 participants) where some questions would be asked about their experiences of undertaking a mathematical investigation; sharing their stories in a written format; and/or offering their work from the mathematical investigation assignment for photocopying. A group interview,

which can include openness, emotional engagement and trusting relationships (Denzin and Lincoln, 2000b) was chosen as a research method known to be useful for gathering data on attitudes, opinions and values, and empowering participants to share their thinking in their own words (Cohen, Manion and Morrison, 2007). This cohort had developed strong systems of support and friendship during the time of the course, and a group interview was envisaged as congruent with what had become an established way of working within this small class, and with the researcher.

Written, informed consent indicating a willingness to be involved was obtained from each pre-service teacher who wished to be included. Permission to conduct this research had been previously sought and given by the university’s ethics committee. It was envisaged that conducting this research after the completion of the paper minimised potential harmful ‘power’ relationships, as grades had already been finalised, and the researcher would not be teaching these pre-service teachers during their second compulsory mathematics education paper.

Six pre-service teachers (out of the eleven who completed this course) volunteered to participate in the informal group interview. Of those six, five also offered their first assignment for the research, with three being eventually received (a stolen laptop, and a pre-service teacher’s family commitments prevented the other two assignments being obtained). No-one chose to record their stories in a written format. For the purposes of confidentiality, pseudonyms have been assigned. These pre-service teachers are Ellen, Sian, Anna, Holly, McKenzie and Carol.

Pre-service teacher (pseudonym)	Group interview participation	Assignment received for research	Choice of investigation
Ellen	Yes	Yes	Tessellations
Sian	Yes	Yes	Measurement
Anna	Yes	Yes	Measurement
Holly	Yes	No	Tessellations
McKenzie	Yes	No	Tessellations
Carol	Withdrew	No	Measurement

Table 1: Pre-service teacher involvement in research

During the one-hour audio-taped group interview a series of questions were asked to guide the conversation as needed. These included:

In your mathematics education paper this year you did a mathematical investigation for one of your assignments.

- *What do remember about your investigation?*
- *What impact did the investigation have on you as:*
 - *a learner of mathematics?*
 - *as a pre-service teacher?*
- *Did your beliefs and attitudes about mathematics change as a result of doing the investigation?*
 - *Can you tell me more about this?*
 - *What do you think will be the impact of these changes when you begin teaching?*
- *Would you recommend this assignment be used for future groups doing the ‘Learning and teaching mathematics’ paper?*
 - *What changes would you suggest?*

An emergent analytical approach (Strauss and Corbin, 1994; Borko, Liston and Whitcomb, 2007) was employed to analyse the group interview and assignments. The audio-recording was listened to several times and notes, including direct quotes, were documented.

Times were noted at intervals to enable multiple re-checking. Each assignment was read and summary notes were made encapsulating the 'essence' of the writing in the personal record including the summary statement that had been completed at the end of the assignment. If an idea reoccurred the page numbers were listed against the idea. Open coding with constant comparison (Cohen, Manion and Morrison, 2007) was then used to identify emerging themes. Comments that appeared to be similar in content and/or intent were coded, using colour. If a comment was relevant to more than one theme both colours were used to indicate the overlap. The initial analysis looked for responses that were directly related to the questions. As the reading and re-reading progressed additional themes were noted, and the notes re-checked to look for evidence of the emerging theme.

The third set of data took the form of informal, semi-structured observations written by the researcher (lecturer) during the five weeks the pre-service teachers were engaged in the investigative process. Observations of the pre-service teachers' learning and thoughts about various issues that were arising during the teaching of this class were recorded. This writing was later read multiple times looking for connections with the data collected from the group interview and assignments.

Criteria for validity within a constructivist or interpretivist research paradigm include trustworthiness, authenticity, credibility, transferability, dependability and confirmability (Lincoln, Lynham and Guba, 2011). While notions of validity are contested and varied, the question of whether findings are sufficiently authentic to the point of taking action on implications is critical (Lincoln, Lynham and Guba). Although postmodernists state that no method or interpretation can deliver an ultimate truth, rigor in methods and interpretation must be considered (Lincoln, Lynham and Guba). It is therefore recognised that the findings that follow are one version of a number of possibilities. Care has been taken to fairly represent the perspectives of all research participants. In an effort to ensure this, the findings including quotes were shared with the preservice teachers, enabling opportunity for clarification and feedback. It is also acknowledged "the way in which we know is most assuredly tied up with both what we know and our relationships with our research participants" (Lincoln, Lynham and Guba, 2011, p. 123). Thus the knowledge and beliefs of the researcher, and the relationships that were an integral component between this group and the researcher also influence the interpretation of the data.

Findings

Five key themes emerged from the analysis. These included: stories of unhappy past mathematical experiences; wanting to teach mathematics differently; the pre-service teachers coming to understand the importance of personally developing a sound, conceptually-based understanding of mathematics; changing beliefs and attitudes about mathematics; and the centrality of 'emotion' in the pre-service teachers' experiences of learning mathematics. Because of a limited word count, a selection of quotes has been included to illustrate the data. These are representative of the themes discussed.

Stories of Unhappy Past Mathematical Experiences

All five pre-service teachers participating in this research had stories of unhappy past mathematical learning experiences. Ellen spoke of developing a negative attitude towards mathematics during her high school education, of not knowing what she was doing, of hating being "*put on the spot*" and the fear of feeling stupid in front of her peers. McKenzie described mathematics as "*just numbers*", and "*ooo, uck*". She remembered having to do

mathematics twice a day, recalled a teacher who appeared to think “*she was useless*”, and so McKenzie too, had decided that she was “*useless*” at mathematics. Sian and Anna had similar stories to tell. Holly was still finding it difficult to move beyond her negative experiences, and closely associated mathematics with her teachers. She stated during the group interview “*you (referring to the researcher as her lecturer) are mathematics*”. She explained how this belief immediately created distance between herself and her teacher.

Pre-service Teachers Wanting to Teach Mathematics Differently

Ellen, Anna and Sian all wrote in their assignments about wanting to teach differently compared to how they had been taught as school students. McKenzie and Holly echoed this sentiment within the group interview. Ellen wrote during week one of the investigation, “*I know what not to do as a teacher, but need to figure what the right thing to do is*” (Ellen, personal record, p. 1). In the analysis of the three assignments there were some beginnings in two of the personal records about how mathematics might be taught differently, or possibly incorporating an investigative approach in their subsequent teaching. Sian had initially considered text-books to be the best vehicle for teaching mathematics. There was a subsequent shift in her thinking. She wrote, “*a mathematical classroom consisting of class discussions and practical activities could be the most effective way of teaching mathematics*” (Sian, summary statement, p. 1). Sian also referred to teaching children different strategies for solving problems. The focus on the ‘teacher’ doing the ‘teaching’ in this context appears to conflict with statements made during the group interview where the importance of ‘self-discovery’, an integral component of the investigative approach, was highlighted.

Ellen also referred to children using different strategies (eg. for adding numbers) but did not necessarily refer to herself as the ‘teacher’ of these. She wrote, “*I want to empower students with different strategies which will enable students to work with bigger numbers...*” (Ellen, personal record, p. 10). Here, there is a possible suggestion of the children being supported to create their own strategies.

Ellen envisaged a ‘correct’ way of teaching mathematics. She referred, in several places in her personal record, to wanting to teach “correctly” (eg. Ellen, summary statement, p. 3).

Anna, like Ellen and Sian, also wrote about wanting to teach differently. She initially recorded that she wanted to “*make mathematics more accessible, meaningful, and relevant for future students*” (Anna, summary statement, p. 2). Anna went on to develop some deep insights about how she might teach ‘differently’. In the following excerpt from her summary statement Anna seems to have envisaged how she might begin to teach mathematics in a way that was congruent with her own experience of mathematical investigations being a powerful means by which to learn mathematics.

Knoll (2008) suggests the appropriate use of silence in the classroom is vital if we are to encourage light bulb moments in students’ mathematical learning. She suggests the moments children are “groping in the dark” (p. 133) are an integral part of doing mathematics. It is the facilitation of this silent time when students are able to find their way and map out a path that requires inquisitiveness, investigation skills not to mention determination and perseverance. I would like to provide students with the opportunities to develop these skills and qualities through teaching mathematics. I can now value the use of silence in the classroom as this allows students to ponder, reflect, plan and try various possibilities while working towards outcomes or solutions. I can relate to this process as I did a lot of pondering and reflecting myself during the investigative assignment and now feel I have a much deeper understanding of the circumference formula...

(Anna, summary statement, p. 2).

In this excerpt the 'silence' referred to is not the silence of a more traditional classroom, rather it refers to the teacher being silent, "...when we don't tell the answer, when we resist the temptation to take over and tell the story ourselves" (Knoll, 2008, p. 25). While I cannot be certain about Anna's interpretation of who is being silent, there is, embedded within the extract a clear articulation of mathematics being created by children and being a process rather than 'answers'.

Pre-service Teachers' Mathematical Knowledge

All five pre-service teachers recognised a need to improve their own conceptual mathematical knowledge. This was considered as a need for themselves, and for the children they will teach in the future. Sian wrote, "... aim to get into the problem, pull it apart to understand the concept better, rather than aiming only to get the answer" (Sian, summary statement, p. 1). Ellen too, referred to developing a "deeper understanding" (Ellen, summary statement, p. 1). Anna was quite clear about the importance of developing a conceptual understanding, and had an emphasis on 'making meaning' and 'making connections'. Anna wrote, "...maths concepts shouldn't be explored as separate unrelated topics for a limited time of the year, but rather as a springboard to explore the way all maths concepts and topic areas are interweaved and interrelate" (Anna, personal record, p. 18).

An example of some of the mathematical understandings gained is evident in this explanation of the formula for the circumference of a circle. Anna wrote, "we can use this formula ($C = \pi d$) to work out the circumference of any circle, this has been proven... we've unpacked this by finding a relationship between the diameter of a circle and the circumference. This relationship is π ... So, in simple terms: the circumference of any circle can be found by wrapping/multiplying the diameter around 3.14 (π) times" (Anna, personal record, part b, p. 6). Sian and Ellen's writing, while revealing a developing understanding of the mathematical ideas being explored, were less clear. Ellen wrote, "For shapes to tessellate we need to use regular polygons that add to 360° . For example, interior angle of a pentagon, $108^\circ + 108^\circ + 108^\circ = 324^\circ$. This shape won't tessellate because the interior angles doesn't add to 360° . Whereas a hexagon will because interior angles of a hexagon are $120^\circ + 120^\circ + 120^\circ = 360^\circ$ " (Ellen, personal record, p. 53). While not clear in this statement, previous writing had alluded to Ellen's understanding of the relevant angles being those from three hexagons meeting at a point.

Needing a sound, conceptual mathematical knowledge was a topic that was present in the rationale given for the assignment, paper readings and discussions held in-class. Recognition of the need for this type of knowledge was also evident within the group interview, although was not named as 'conceptual' or 'relational' understanding. Stories that were told mentioned the 'permanence' of the learning and importance of self-discovery. Anna said "we've all remembered what we learned with this first assignment and it's still in my head a semester later" and Ellen said "I remember everything". Anna also mentioned that "learning is so much more valuable... you retain that new knowledge when you are able to discover it yourself".

While identifying the importance of the teacher's own mathematical knowledge, all of the pre-service teachers also recognised this as an ongoing need. Early on in the assignment Sian identified that she needed to "expand on my own maths knowledge to be able to teach..." (Sian, personal record, p. 1). This was still evident five weeks later when she wrote "to be an effective teacher in mathematics I need to build up on my own knowledge of mathematics" (Sian, summary statement, p. 5). Ellen wrote in a similar way saying, "For me to develop as a mathematics educator I need to make sure I understand other aspects of

maths such as measurement, geometry, algebra etc" (Ellen, summary statement, p. 3).

Changing Beliefs and Attitudes about Mathematics

During the group interview there was unanimous agreement about the value of an investigative approach in supporting changes in beliefs and attitudes about mathematics. McKenzie spoke about mathematics not being "*as scary now*" and all referred to increased confidence. With the exception of Holly (who acknowledged additional factors were impacting on her situation), there was a much more positive attitude about mathematics.

Sian, Anna and Ellen were all aware of the close connection between their beliefs and attitudes and the impact on their mathematical knowledge and teaching. Anna, early on in her investigation wrote, "*our attitudes and beliefs as learners of mathematics can affect our practice as teachers of maths*" (Anna, personal record, p. 11). Sian wrote, "*throughout this investigation, I have realised that the way we think impacts on our mathematical knowledge*" (Sian, summary statement, p. 1). She also inscribed "*I have realised that for me to become a better teacher, I need to first become a better learner in mathematics*" (Sian, summary statement, p. 2). Realising the connection between herself and her teaching appeared to also provide Sian with an incentive to not 'give up' when the investigation became rather daunting.

It was evident that mathematics was now thought of, by the pre-service teachers, as "*everywhere*" (Anna) and Holly agreed that mathematics was integrated into 'everything' including daily life. Mathematics had come to be viewed as a process and something that could take time rather than being focused on answering a question as quickly as possible. Ellen said that initially she thought it would take her about a week to complete the investigation, but by the end of the five weeks she recognised that there was still so much more she could learn about this one topic. Anna wrote of coming to see mathematics quite differently, writing "*I now think maths is not just about memorising algorithms, using methods to solve equations without understanding why, or learning mathematical facts by rote. I now know maths involves active exploration and investigation, real life experiences and practical application... and making meaning*" (Anna, summary statement, p. 1).

Another change recorded by Ellen involved the concept of uncertainty. She initially wrote of how she felt uncomfortable with not knowing an answer, but later wrote beside an excerpt from a journal article that, "*it's not all about getting the right answer. It's ok to be wrong; it's part of the learning process*" (Ellen, personal record, p. 54b). At the end of the five weeks she reflected, "*as a learner of mathematics... I feel more comfortable sitting in the dark not knowing the answer or what to say, whereas before I wouldn't be*" (Ellen, summary statement, p. 1).

The pre-service teachers told stories of considerable and ongoing struggles that underpinned some of these changes in belief and attitude. After one week working on the investigation Ellen wrote of her frustration at not "*getting anywhere*" (Ellen, personal record, p. 14) and three weeks later, wrote "*I'm actually over this, I've done so much work and still feel like I'm getting nowhere... I don't know how much more I need to do...*" (Ellen, personal record, p. 50). Sian spoke of the considerable time it took for her to "*make sense*" (Sian, personal record, p. 2). This was evident throughout her work. Confusion was also mentioned (Ellen, personal record, p. 13).

The researcher also witnessed 'struggle', particularly during the early weeks of working with this class, and was aware of her own struggles in her role as lecturer. She wrote in her notes, "*my teaching today was like pushing water uphill with a rake – completely ineffective. I'm struggling with getting about half of the students to engage beyond the 'glib' level – and I don't know how to move them. Some are making deep-er connections – how can I get the others to a similar level of reflection?*" (lecturer's notes, 09/03).

Despite the discomfort that some of the pre-service teachers were feeling there was also evidence of risk-taking and engagement with their work. Mathematical questions were being asked (eg. “*can all triangles tessellate?*” Ellen, personal record, p. 11); predictions being made and tested (eg. “*I predicted.... I want to change my prediction...*” . Sian, personal record, part b, p. 1); and following their own lines of inquiry. As previously mentioned, a desire to ‘really understand’ the mathematics being explored also emerged. Anna wrote, “*I am now keen to discover and explore new ways I can make meaning of foreign mathematical concepts, and enjoy doing so! My main aim is now gaining and developing a relational understanding*” (Anna, summary statement, p. 1). In a similar manner, Ellen wrote, “*I love it when I get the light bulb moment because it shows me I understand what I’m doing*” (Ellen, summary statement, p. 1).

The Centrality of ‘Emotion’ in Learning Mathematics

There was considerable emotion experienced throughout the process of engaging with the investigations. While the over-riding feeling present during the group interview (held approximately two months after the course had finished) was happy and positive, there was a definite mix of emotions logged in the personal records. Pre-service teachers wrote and spoke about their past negativity towards mathematics. Phrases used when reflecting on past experiences included being scared, gunned down, being frozen, and ashamed.

As the investigations progressed however, there were mentions of learning to persevere and cope with frustration. This increased perseverance was noted in all three of the assignments that were analysed. In her summary statement, Ellen made a connection to a journal article where the notion of learners developing determination to ‘stick with a problem’ was discussed (Ollerton, 2009). In referring to developing her own perseverance, Ellen wrote, “*this has been a big accomplishment for me*”. (Ellen, summary statement, p. 2). Anna also spoke of increased perseverance, “*I have noticed I am much more willing to stick with and try new areas of learning in mathematics*” (Anna, summary statement, p. 1).

Interestingly, the learning to persevere and cope with frustration was mirrored by the researcher, as the pre-service teachers’ lecturer. Towards the middle of the investigative period the researcher wrote, “*I witnessed my usual reaction of frustration at them ‘giving up’ within about 5-10 min. Today I just observed those feelings, and kept working with pairs modeling the expectation that a strategy can be found. As I did that they re-engaged, and began to show persistence! I need to keep a check on my frustration and believe that ‘we will get there!’*” (lecturer’s notes, 22/03).

As the investigations progressed more positive feelings were mentioned. For example, Anna wrote of being empowered, and experiencing a boost of confidence (personal record, p. 30); and Ellen recorded that she was “*feeling more positive towards maths because I actually understand what I’m doing. I personally feel my attitude towards maths has changed*” (Ellen, investigation record, p. 24). McKenzie referred to maths not being “*as daunting now*”.

All of the pre-service teachers spoke of the relationships that developed within the group and with the researcher as their lecturer. McKenzie spoke about the “*close class this semester*” and for Sian, the difference for her learning was “*definitely the group*”. During the group interview Sian identified some of the strategies that she recognised the researcher (as lecturer) using earlier in the semester in an endeavour to establish a community of learners. She recalled how the class had been asked to do some of their work in pairs, and later requested groups working on the same investigation to gather together and share their thinking and learning. The researcher would agree with the pre-service teachers in recognising the community that developed, and remember the efforts made (at the class’s instigation) for an end-of-semester shared meal and group photograph.

The relationship established with the lecturer was also regarded as pivotal. Sian felt

that lecturer support and approachability were essential. Referring back to their school experiences the role of teacher was also spoken about. McKenzie categorically stated, “*it’s all down to the teacher*”. Ellen also spoke about teacher-student relationships, and in addition recognised the role of her own attitude within such relationship. The researcher had taught Ellen during her first attempt at this course, and both student and researcher acknowledged that a positive relationship had not developed during the first course. Reflecting back, Ellen remembered her negative attitude and how she did not want to approach the researcher for support. The researcher also remembered feeling somewhat reactive towards Ellen’s former negativity.

Discussion

These findings support research (see Grootenboer, 2001; Guillaume & Kirtman, 2005) that identifies the significant impact of past mathematical experiences as pre-service teachers journey towards becoming effective mathematics teachers. Consistent with the findings of Miller-Reilly (2006), this data points to the value of taking the time to acknowledge past mathematical experiences and their attendant emotions. Given the pressing nature of mathematics education initiatives such as the introduction of the New Zealand Curriculum Mathematics Standards for years 1-8 (Ministry of Education, 2009) it is possible however, that less time will be available to spend examining pre-service teachers’ beliefs and attitudes about mathematics. It would appear though that for those engaged in a second attempt with a mathematics education course, opportunities to share and examine past experiences are needed.

For those learners who have negative mathematical memories in their past, positive experiences are needed so new stories can develop and emerge (Guillaume and Kirtman, 2005; Liljedahl, 2005). For this small group of repeating pre-service teachers, engaging in an investigative approach provided an experience that appears to have fostered alternative and more positive conceptions of mathematics as a discipline. The need for pre-service teachers to move “beyond their personal experiences of studenting” (Nicol, 1997, cited in Loughran, 2006, p. 6) and consider alternative perspectives and develop arguments for their future teaching practices is evident in the stories shared. Mathematics came to be viewed as a sense-making process involving perseverance, with the pre-service teachers coming to consider themselves as creators of mathematical knowledge. The power of the AHA! moment (Liljedahl, 2006) was evident in two of these pre-service teachers’ assignments. The attendant writing surrounding these moments seems to support Liljedahl’s (2006) claim of the power that a single AHA! moment can have with regard to changing beliefs and attitudes.

It is quite common for pre-service teachers to express a desire to teach in a manner that is different to how they were taught (Bobis, Mulligan, Lowrie & Taplin, 1999). This was evident within this small group. Hunter (2010), referring to the work of Wilkins (2008) describes, “how teachers who have had negative experiences as mathematical learners may develop supportive voices for an idealised opposite to their own experiences” (Hunter, 2010, p. 399). However, enacting the desire to teach ‘differently’ is likely to be somewhat difficult, perhaps particularly so for pre-service teachers who have experienced failure in aspects of their teacher education. Contradictory statements made by some of these pre-service teachers about the learning and teaching of mathematics point to these difficulties. Such dissonance resonates with Kane’s (2007, p. 68) reference to the “robust nature of beginning teachers’ beliefs and preconceptions, often in the face of new and contradictory knowledge and experiences”.

Participating in a mathematical investigation appeared though, to enable one pre-service teacher in particular, to construct alternative conceptions of what mathematics teaching and learning might look like in her classroom. The tentative beginnings made by

others suggest that ongoing support would be needed to help them realise their desire to teach 'differently'. This is an alert to the challenges and complexities of supporting all pre-service teachers to make connections to their future teaching practice.

The requirement for pre-service teachers to have a sound, conceptual understanding of mathematics is not new. In past years however, it would seem from the informal, anecdotal observations of this researcher that the importance of a teacher's own knowledge has not always been realised by pre-service teachers. In contrast, the investigative approach afforded this small group an appreciation of the need to develop a sound and conceptual understanding of mathematics, and the connection between this and their future teaching.

It was interesting to note in the group interview that despite the pre-service teachers' awareness of the need to continue to build their own mathematical knowledge, only two of the group clearly identified that they would consider taking an optional third year mathematics education paper. It seemed that for three of the group 'old fears' were still a significant part of their emotional landscape. This is consistent with Young-Loveridge's (2010) thinking. She wrote, "it would be naïve to think that one or two years of professional development could miraculously change teachers' attitudes, feelings, beliefs, and values as well as their understanding of mathematics" (Young-Loveridge, 2010, p. 30). The researcher acknowledges that with very few optional papers available in the Bachelor of Teaching degree, the wish to pursue other curriculum or professional development papers will also impact on decisions that are made.

It would appear that collegial support and the role of the teacher educator are important facets to consider for those repeating a mathematics education paper. It is evident in the data from this study that the creation of a nurturing environment enabling the building of relationships between students is vital. It would also seem critical for pre-service teacher educators to provide caring support and emotional scaffolding, described by Holbrook and John-Steiner (2008) as including the gift of confidence and the creation of a safe zone for learning. Data suggests that when pre-service teachers behave negatively towards their lecturer it is likely they are re-acting from their past mathematical experiences. A willingness from lecturers to persevere with developing positive relationships, even in the face of considerable negativity and opposition, appears to be needed.

Data from the researcher suggests that supporting pre-service teachers to become engaged with doing mathematics and developing perseverance was not an easy process, and her notes record the frustrations and vulnerabilities experienced in this process. Loughran's (2006) contention that emotions, feelings and reactions are a valid and enmeshed aspect of learning and teaching about teaching resonate with this small study. The role of the teacher educator is indeed complex (Loughran, 2006).

A critical question to consider is whether the changes experienced by these five pre-service teachers will be sustained. As Zeichner & Tabachnik (1981, cited in Russell, 2007, p. 190) state, "our pedagogy of teacher education is crucial to what follows... for that is when we need to ask whether anything remains of what we try to teach in our pre-service programs". The notion of sustaining change appears to be further compounded by the apparently contextual nature of beliefs, and the likelihood "that the changes reported may be diminished by subsequent experiences" (Leder & Grootenboer, 2005, p. 4). An additional question to consider includes whether the experiences from these five pre-service teachers would be similar and/or different to the others in this class who did not participate in the research.

Conclusion

Using an investigative approach to support repeating pre-service teachers' thinking and learning about mathematics has effected some positive changes for this small cohort. It

appears that the time taken to examine past experiences, associated emotions, and consider beliefs and attitudes, was time well-spent. Some beliefs and attitudes about mathematics have changed, with mathematics now being viewed in a more positive manner. The power of an AHA! moment may have affected a more dramatic and significant change for two of this group. It is likely that ongoing positive mathematical experiences are needed for others to affect a more lasting change. While one pre-service teacher was able to envisage teaching practices congruent with her experiences in undertaking a mathematical investigation, others require further support to envisage teaching in a manner congruent with their changing beliefs and attitudes. All of these pre-service teachers have clearly identified the importance of developing a conceptual understanding of mathematical ideas, and the link between this and effective teaching. Collegial support, the development of a nurturing learning environment and a trusted student-teacher relationship were regarded as vital for supporting mathematical learning. The process of undertaking a mathematical investigation was not without struggle and negative emotion. Uncertainty and perseverance became an accepted part of the process for the pre-service teachers and lecturer alike.

Although the robust and tenacious nature of pre-service teachers' beliefs and preconceptions is well-known (Kane, 2007); and the challenge of bringing about deep, lasting change in teachers' understandings of teaching and learning mathematics should not be underestimated (Young-Loveridge, 2010) engaging repeating pre-service teachers in an open-ended investigation may be an important step in facilitating a move towards a future of effective mathematics teaching.

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