An Initial Investigation into the Mathematical Background of Those Who Pass the CSET for Mathematics

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Introduction

California has long suffered from a shortage of credentialed single-subject mathematics teachers. Historically, its colleges and universities' credential programs have not graduated enough mathematics teachers to meet the demand, which has prompted a robust recruitment program of mathematics teachers from other states. In addition, some teachers from other disciplines earned a supplemental authorization, which allowed them to teach junior high and lower-level high school mathematics classes (California Commission on Teacher Credentialing, 2012b). The remaining shortfall was met by awarding emergency permits or credential waivers. An emergency permit allowed non-credentialed teachers to teach until they either earned the appropriate credential or another credentialed teacher could be found. Meant as a short-term solution, some teachers taught for extended periods of time with such permits (California State University Institute for Education Reform, 1996). A credential waiver was awarded when all other avenues for finding a teacher had been exhausted. In 2000-2001, 1885 emergency permits and 290 credential waivers were awarded to mathematics teachers (Burke, 2002).

In 2001, the federal government enacted the Elementary and Sec-

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ondary Education Act (ESEA), commonly known as *No Child Left Behind* (NCLB). Among other educational reforms, this law mandated that all children be taught by "highly qualified" teachers by the end of the 2005-2006 academic year. According to the law, "To be deemed highly qualified, teachers must have: (1) a bachelor's degree, (2) full state certification or licensure, and (3) prove that they know each subject they teach" (U.S. Department of Education, 2004, para. 10). This placed California in a difficult position. In 2003, perhaps in response to the mandates of NCLB, a new subject-matter competency exam, the California Subject Exam for Teachers (CSET), was instituted, and the foundational credential in mathematics was established. The CSET is administered by Evaluation Systems Group of Pearson (formerly National Evaluation Systems, Inc.).

California has long used a subject-matter exam to establish subject-matter competency of teachers. As an alternative to the exams, the California Commission on Teacher Credentialing approves waiver programs, a series of courses that, when taken successfully, allow the student to waive the state exam requirement. Generally, these waiver programs for single-subject mathematics subject-matter competency are equivalent to an undergraduate degree in mathematics. These degree programs require approximately 14 mathematics courses. Typically, a sequence of calculus courses are the first courses counted toward the major, with subsequent courses' requiring calculus as a prerequisite. General education mathematics courses are, for the most part, not counted toward the major.

The CSET for mathematics has three components, designed so that the first two subtests would not require calculus, which leaves the third exam to cover that content. Passing the first two exams establishes subject-matter competency at the foundational level. Interestingly, the topics covered by these two exams are many of the topics traditionally covered in classes for which calculus is a prerequisite (California Commission on Teacher Credentialing, 2012c).

A teacher with a foundational mathematics credential is authorized to teach all single-subject mathematics classes up through and including Algebra 2, which leaves the Pre-Calculus/Math Analysis, AP Calculus, and AP Statistics courses for those with a full mathematics credential. A majority of high school students do not take a mathematics course beyond Algebra 2, and completing Algebra 2 satisfies the entrance requirement for the California State University (CSU) system.

In 2005, both the University of California (UC) and CSU systems initiated programs to increase the production of single subject mathematics and science teachers (Schevitz, 2005). In 2002-2003, approximately 900 full and 0 foundational single-subject math credentials were awarded in California. By 2010-2011, the number of full credentials remained essentially unchanged at 888, while the number of foundational credentials awarded outpaced the number of full credentials, at 958 (California Commission on Teacher Credentialing, Professional Services Division, 2012). Table 1 shows similar trends in CSU mathematics teacher credential production (California State University Chancellor's Office, 2011).

Determining what constitutes subject matter competency is complex, with the kind of mathematical knowledge necessary to be an effective teacher of mathematics being far from clear. There has been robust research into the mathematical knowledge necessary for teachers at the elementary level. Ma (1999) found that, while Chinese elementary teachers have fewer years of formal education than do U.S. elementary teachers, they have a better understanding of the mathematics relevant to the teacher of elementary-level mathematics. Ball, Hill, and Bass (2005) proposed that there is specialized mathematical knowledge for teaching. Using a Content Knowledge for Teaching Mathematics (CKT-M) assessment, Hill, Rowan, and Ball (2005) found two important results. First, their study of first- and third-grade teachers showed that teachers with lower levels of content knowledge had students who performed worse on mathematical assessments than did those students with teachers who possessed a higher level of this knowledge. Second, they found a minimal correlation between the number of mathematics and mathematics methods classes taken and the level of content knowledge for teachers.

McCrory, Floden, Ferrini-Mundy, Reckase, and Senk (2012) are working on an assessment of the content knowledge needed by secondary mathematics teachers, the Knowledge of Algebra for Teaching (KAT), which is still in development. Other research indicates that, at the secondary level, there is a positive correlation between the number of mathematics courses taken and the effectiveness of the teacher. Monk (1994) found that the more mathematics courses taken by a secondary mathematics teacher, the better his or her students fared on assessments of their mathematical knowledge, although only for the first four to six courses. Goldhaber and Brewer (1997, 2000) found that the students of mathematics teachers with a bachelor's or master's in mathematics performed better on mathematics assessments than did students of teachers without this content background.

Table 1
CSU Mathematics Teacher Credential Production from 2002-03 to 2009-10

Level	02-03	03-04	04-05	05-06	06-07	07-08	08-09	09-10
Full	349	447	405	402	525	479	452	382
Found.	0	28	119	170	258	307	321	336

Given the lack of a research-tested assessment of the mathematical knowledge needed by secondary mathematics teachers and the research that indicates that there may be a correlation between the number of mathematics courses taken and the effectiveness of the teacher, it appears useful to gather data on the mathematics course work taken by those who have passed the CSET for Mathematics. Because over half of the new mathematics credentials awarded in California in 2010-2011 were at the foundational level, the concern over the mathematical background of teachers who receive a foundational-level credential is of importance.

Method

Participants. Colleagues at CSU campuses were asked to collect data on the mathematical background of students accepted into their campus's fall 2010 single-subject mathematics credential programs. Of the 22 CSU campuses with credential programs, 13 provided complete enough data on the students to be included in the study. Alternate pathways to a single-subject credential, such as CalStateTeach, were not included in this study. Of the students from these 13 campuses, there were a small number of students for whom the data collected was incomplete or difficult to analyze, as described below. These few students were removed from the data set. In the end, data from 187 students were included in the data set.

Materials and Procedures. The author analyzed the subtest descriptions for the two subject-matter tests used to establish subject-matter competency at the foundational level provided by the California Commission on Teacher Credentialing. This analysis was used to identify coursework that might lead to mastery of the identified mathematics. The author and selected colleagues also reviewed the sample test items provided by the California Commission on Teacher Credentialing to identify the level of mathematical knowledge needed to answer these questions. Finally, we attempted to identify the mathematics courses successfully completed by students accepted into CSU single-subject mathematics credential programs for the fall 2010 semester. Students who apply to CSU credential programs must have completed an undergraduate degree program. Usually a transcript of this undergraduate program is submitted as part of the credential program application. These transcripts are used to determine the mathematical background of the students accepted into the credential programs.

Procedure. The author used the CSU Chico degree requirements for a bachelor's of science degree in mathematics, with a concentration in mathematics education, to analyze the coursework that would address the mathematics described in the California Commission on Teacher Credentialing summary of the first two CSET subtests. CSU mathematics degree programs have a majority of course requirements in common, which allows the mathematics contained in the Chico degree to serve as a reasonable template for other CSU mathematics programs. A faculty member in the in the CSU, Chico Department of Mathematics and Statistics for ten years, the author has taught, or is familiar with, the content of the courses under consideration. For each of the first two subtests, the sample test items were analyzed independently by the author and either a veteran high school mathematics teacher or, in the case of the probability and statistics questions, a CSU Chico statistics professor. The high school teacher who analyzed Subtest 1: Algebra and Number Theory, has over 25 years of teaching experience, many of them in teaching Math Analysis/Pre-Calculus and Advanced Placement Calculus. The high school teacher who analyzed the geometry portion of Subtest 2: Geometry, Probability, and Statistics, has over 20 years of teaching experience, many of them in teaching Geometry. The CSU Chico statistics professor who analyzed the probability and statistics questions is familiar with general education and upper division statistics courses offered at CSU Chico. After the independent analyses, the two reviewers met to discuss their analyses. There was a high degree of agreement, and, when there was a difference of opinion, the two views were discussed until there was a consensus.

To gain some insight into the mathematical background of people who pass the CSET exam for single-subject mathematics subject-matter competency, we looked at the undergraduate transcripts for students accepted into CSU credential programs in fall 2010. Of the 22 CSU campuses with credential programs, 13 provided enough data for analysis. Colleagues at each campus coordinated the data collection at their sites.

Results

Subset I: Algebra and Number Theory. The introduction to this exam, contained in the test guidelines, states that "candidates demonstrate an understanding of the foundations of the algebra contained in the Mathematics Content Standards for California Public Schools" and "to ensure a rigorous view of algebra and its underlying structures, candidates have a deep conceptual knowledge" (California Commission on Teacher Credentialing 2002a, para. 1). A similar statement is made about number theory and number sense.

These statements are followed by a list of 12 algebra topics and five number theory topics. Two of the 17 topics appear to be those that would be sufficiently addressed in a Pre-Calculus/College Algebra course, with another two addressed by a combination of a Pre-Calculus/College Algebra and a Calculus course. Pre-Calculus/College Algebra, while offered for credit at many universities, covers material similar to that which is taught in high school Pre-Calculus courses. The remaining 13 topics appear to require the other mathematics courses that are required of mathematics majors. These courses, Elementary Linear Algebra, Number Theory, Introduction to Proofs, Modern Algebra, and Advanced Calculus, traditionally have at least one semester of calculus as a prerequisite, and most have substantially more. For instance, at CSU Chico, Introduction to Proofs is required prior to all but the Linear Algebra course. While there are some general education mathematics courses that touch upon a limited number of the listed topics, they do so only at a superficial level.

To provide an idea of how mastery of these topics is determined, it is instructive to analyze the sample CSET questions provided by the CCTC (California Commission on Teacher Credentialing, 2012a). While these sample questions are not intended as a representative sample of the questions from the exam, they can provide us with a sense of how the topics are interpreted by the testing agency. There are 27 sample multiple-choice questions and four sample written-response questions. The author and an experienced high school mathematics teacher, with many years of teaching Pre-Calculus and AP Calculus, independently evaluated each of the problems. We then compared our results and resolved any disagreements that arose. Our goal was to identify the lowest level course for which a student successful in that course had a reasonable chance to correctly answer the question. We considered a successful student to be one with a solid B or higher grade, and we assumed that the student studied the relevant topics prior to taking the exam.

Some problems were challenging to classify because they covered content that was clearly from one course but did so in a sophisticated way. For instance, the following problem was classified as a Pre-Calculus problem even though the content is at least partially covered in an Algebra 2 course. We felt the way in which the question was phrased required a sophistication that is hard to quantify:

7. If $f(x)$ is a fourth-degree polynomial with real coefficients such that

$\frac{f(x)}{(x-3)} = q(x) + \frac{8 \quad f(x)}{(x-3)(x-3)} = q(x) + \frac{8}{(x-3)}$, which of the

following statements about $f(x)f(x)$ must be true?

A. $f(x)f(x)$has a zero at $x = 3$.

B. The graph of $y = f(x)$has a local minimum at (–3, 8).

C. $f(x)f(x)$ has two real roots and two complex roots.

D. The graph of $y = f(x)y = f(x)$ contains the point (3, 8).

Of the 27 multiple-choice problems, we felt that four had an element of sophistication that made course assignment difficult. In the end, all four were assigned to a Pre-Calculus course.

The results of our analysis are summarized in Table 2, which presents the number of sample questions that could be answered by a student successful in the given course. We included the number and percentage of questions that cover topics that are first substantially introduced in a typical calculus sequence, as the exam is designed to not require the completion of calculus courses.

While the classification of individual problems might be debated, according to our analysis, 14 of the 27 multiple-choice questions might be answered by someone who was successful in high school mathematics, an additional two might be answered by a student who has taken several calculus courses, and the remaining 11 should require a student to have taken mathematics classes for which calculus is traditionally a prerequisite. As for the free-response questions, one of the four might be answered by a successful high school student, and the other three should require post-calculus mathematics courses.

Table 2
Analysis of Subset Exam 1 Sample Problems

Subject Area/Class in Which Student Might be Expected to be Able to Solve This Type of Problem	Multiple Choice Questions (n)	Free Response Questions (n)
HS Algebra 1/Algebra 2	8	0
Pre-Calculus	6	1
Calculus I	0	0
Calculus II, III/Linear Algebra (vectors)	2	0
Linear Algebra	3	1
Modern Algebra	3	1
Other (e.g., Introduction to Proofs, Number Theory, Combinatorics)	5	1
Total	27	4
Calculus supported	2	0
% Calculus supported	7.5%	0%

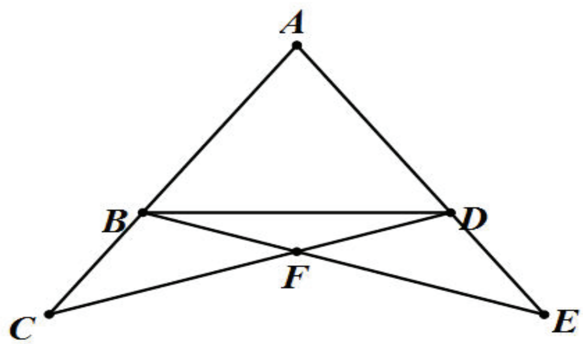
Subset II: Geometry, Probability and Statistics. The introduction, in the study guide to the Subset II exam, states that candidates should "demonstrate an understanding of the foundations of the geometry contained in the Mathematics Content Standards for California Public Schools," "demonstrate an understanding of axiomatic systems and different forms of logical arguments," and "understand, apply, and prove theorems relating to a variety of topics in two- and three-dimensional geometry, including coordinate, synthetic, non-Euclidean, and transformational geometry." With reference to statistics and probability, candidates should demonstrate an "understanding of the statistics and probability distributions for advanced placement statistics contained in the Mathematics Content Standards for California Public Schools" and "a deep conceptual knowledge" (California Commission on Teacher Credentialing 2002b, para. 1, 6).

This general description is followed by a list of 11 geometry topics, five probability topics, and five statistics topics. Some of the geometry topics may be partially or superficially addressed in a number of courses offered at CSU Chico; however, none of the topics would be fully addressed outside the College Geometry course. This course requires two semesters of calculus and the Introduction to Proofs course as prerequisites. It is difficult to identify specific courses for the probability and statistics topics; not only is there a wide variety of mathematics courses that cover selected topics, there also are courses from other majors that include statistics content. For this reason, we simply move on to an analysis of the relevant released questions.

The results of the analysis of the geometry questions, which was done by an experienced high school mathematics teacher who had taught geometry for many years, and the author, are presented in Table 3. The analysis of the statistics and probability questions was done by the author with the aid of CSU Chico statistics professors familiar with the content taught in the CSU Chico general education statistics as well as the content taught in statistics courses for mathematics and statistics majors. There were 20 sample multiple-choice questions and three written-response questions that covered geometry and related topics, and seven multiple-choice and one written-response question that cover statistics, probability, and related topics.

Each problem was assigned to a course in which we felt it reasonable to expect a student successful in the course to correctly answer the question. In some instances, we felt that a question addressed a topic from high school geometry but did so in a challenging enough way that we felt that exposure in a subsequent course should be included, so we assigned the question to both courses (e.g., High School Geometry/Pre-

Calculus). Even with these hybrid labels, we found that there were three problems that we considered substantially difficult, regardless of the courses taken. Two of these were included in the High School Geometry and one in the Pre-Calculus categories. As an example of our assignments, the following written-response question was rated as a problem that would be reasonable for a student who had successfully completed a high school geometry class to answer:



In the diagram above, B and D are points on segment $\overline{AC}$ and $\overline{AE}$, repectiveley. $AB = AD$ and $BC = DE$. Prove that $BF = DF$.

According to this analysis, of the 20 geometry multiple-choice questions, 13 would be reasonable for a successful High School Geometry student to answer, with another three accessible to a successful Pre-

Table 3
Analysis of Subset Exam 2 Geometry Sample Problems

Subject Area/Class in Which Student Might be Expected to be Able to Solve This Type of Problem	Multiple Choice Questions (n)	Free Response Questions (n)
HS Geometry	10	1
HS Geometry/Pre-Calculus	1	0
Pre-Calculus	2	1
HS Geometry/College Geometry	3	1
College Geometry	0	0
Non-Euclidean Geometry	0	0
Calculus 2, 3, Linear Algebra	2	0
Linear Algebra	2	0
Total	20	3
Calculus supported	2	0
% Calculus supported	10%	0%

Calculus student. While a College Geometry course would be of benefit in answering many of these questions, only three questions are listed as addressing topics in a way that such a course might be required. There were no questions about non-Euclidean Geometry.

In analyzing statistics, probability, and related topics, the questions were assigned to one of two CSU Chico general education courses, M105 Statistics or M108 Statistics of Business and Economics, or to statistics courses for mathematics and statistics majors that require at least a semester of calculus. Of the eight sample problems, only the free response was considered challenging for its assigned course, with the issue's being that the topics are not always covered in a general education statistics course but are accessible to students from such a class if they independently study the topic (Table 4).

Mathematics Classes Taken by Those who Pass the CSET. Colleagues at each of the CSU campuses that offer a credential program in single-subject mathematics were asked to analyze the mostly undergraduate transcripts submitted by the students accepted into the CSU credential programs in fall 2010. A total of 13 CSU campuses provided enough data for analysis. For each student accepted to the credential program, the following data were collected. If the student completed a CSET-approved subject matter waiver program, this was all that was noted. For those who earned their subject-matter competency via the CSET, the level at which they passed (foundational or full) was noted, and their transcripts, provided as part of their application for the credential program, were analyzed. If they had completed an undergraduate mathematics degree, this was all that was noted. If not, then each of the mathematics courses on the transcript was listed. Occasionally, these lists of courses were difficult to analyze. For instance, courses taken

Table 4
Analysis of Subset Exam 2 Statistics and Related Topics Sample Problems

Subject Area/Class in Which Student Might be Expected to be Able to Solve This Type of Problem	Multiple Choice Questions (n)	Free Response Questions (n)
General Education Statistics	4	1
Business Statistics/Math Major Statistics	2	0
Math Major Statistics	1	0
Total	7	1
Calculus supported	0	0
% Calculus supported	0%	0%

outside the United States were difficult to compare to courses offered in the United States. In these cases, the students were not included in the data set.

In the end, 62 students who passed the CSET at the foundational level, 36 students who passed at the full level, and 79 students who completed a waiver program were included in the data set. This was not intended as a representative sample, but given that it represents approximately half of the students who entered CSU single-subject mathematics credential programs in fall 2010, it is a reasonable data set from which to begin an analysis of the mathematical background of those who pass the CSET.

We categorized the mathematics classes into four groups. In the first group are general education mathematics courses, that is, mathematics courses for non-math-intensive majors. These courses generally have no mathematics prerequisites aside from students' establishing, through the entry-level mathematics requirement, that they have adequate mathematical knowledge of high school mathematics up through an intermediate algebra course (Algebra 2 at many high schools). Among the more mathematically demanding of these general education courses is a Pre-Calculus course, sometimes referred to as a College Algebra course. The content of this course is similar to the content of a High School Pre-Calculus or Math Analysis course. This is the course taken by high school students who have successfully passed an intermediate algebra course.

The second set of courses are those in a calculus series, which are generally the first college-level mathematics courses taken by students who major in mathematics, engineering, or computer science. Many high schools offer an Advance Placement Calculus course, and students who pass the associated Advanced Placement Exam are allowed to claim credit for the first semester of college calculus. For this reason, we can view Calculus 1 as a transition course between high school mathematics and the mathematics of mathematically-rich college majors. Stated another way, Calculus 1 is the first course in waiver programs for single-subject mathematics subject-matter competency. Calculus 1 is the first course in a calculus sequence that is usually comprised of three courses, although an Introductory Differential Equations course can be viewed as a fourth Calculus course. While mathematically rich, these courses have a somewhat narrow mathematical focus and often emphasize computational aspects of mathematics over the conceptual aspects.

The third set of courses is those for which at least a semester of calculus is required. These are generally mathematics courses that count toward a mathematics major, although some are also taken by other majors in which mathematical knowledge is important. Proofs, and the deductive reasoning that these require, play a more prominent role in these courses; therefore, these courses are considered conceptually richer than are the calculus courses. This group of courses contains many of the courses that traditionally cover the topics on which the first two CSET subset exams are based: linear algebra, introduction to proofs, number theory, college geometry, modern algebra, and statistics. Many campuses also offer a mathematics course that focuses on mathematics for secondary teachers, but these courses generally also require prerequisite mathematics major courses.

A final set of courses is the set of mathematics courses for prospective elementary teachers. While these courses tend to promote an understanding of mathematical concepts, the focus is on the mathematics of the elementary grades and, thus, should have a limited impact on the mathematics tested through the CSET. As it turned out, very few of the teacher candidates in the data set had taken these types of mathematics courses, so they have little impact on the data analysis. We thus included these courses in the set of those that did not require a calculus prerequisite. As with any categorization, there are courses that did not clearly fall into any of these categories, but these were few, and we used our discretion in evaluating their mathematics content. Note that, in the results of the analysis that follows, the groupings are often nested, with each including those from the previous grouping.

For the 36 who passed all three CSETs:

- 3 (8%) had no mathematics course beyond first-semester calculus
- 7 (19%) had no mathematics courses beyond 2 semesters of calculus
- 9 (25%) had no mathematics courses beyond four semesters of calculus (to possibly include differential equations)
- Of these 36, 14 (39%) had a major in mathematics.

For the 62 who passed only the first two CSETs:

- 13 (21%) had no calculus course on their transcript
- 26 (42%) had not taken a course beyond a first semester of calculus
- 37 (60%) had, at most, a series of calculus courses (to possibly include differential equations), that is, none of the math major courses that cover the content indicated for the two exams

Below are examples of typical students from the sample set who had no mathematics course beyond first semester calculus.

- Student A: Business Administration major who took a single math course, Statistics of Business and Economy

- Student B: Social Science Major who took two mathematics courses, Calculus 1 and General Education Statistics
- Student C: Physical Education Major who took one mathematics course, College Algebra

Given that many of the topics covered by the exams are traditionally taught in mathematics courses for which calculus is a prerequisite, we considered the percentage of those passing CSET Exams I and II who have taken these mathematics courses and found the following:

- 41 (66%) of those who passed at the foundational level had not taken any of the following courses: linear algebra, modern algebra, number theory, or introduction to proofs
- 54 (87%) of those who passed at the foundational level had not taken an undergraduate geometry course
- 32 (52%) had not taken any college level statistics course
- 48 (77%) had not taken statistics other than a single general education statistics course

Discussion

Given that the data set was not randomly chosen and that only mathematics courses that appear on the undergraduate transcripts submitted with the credential program application were considered, it is important that the limitations be taken into account when considering the conclusions. Nevertheless, these data are the first collected on the mathematical background of a significant portion of a new generation of secondary mathematics teachers. The data indicate that we may be credentialing a large number of people as "highly qualified" secondary mathematics teachers who have little formal mathematical training. Our analysis of the released sample problems showed that the exams that are being used to establish subject-matter competency appear to contain only a small number of questions that require college-level mathematics courses. This perspective is supported by the data collected, as a majority of those who have passed the exams have taken few college-level mathematics courses, and even fewer have taken mathematics courses with the mathematical rigor of courses designed for mathematics majors.

The most glaring example of both of these issues is the geometry portion of Subtest II. Of the 23 geometry-related questions, 65% do not appear to require anything more than a solid high school mathematics education. When considering the mathematics courses taken by those passing Subtest II, we see that 85% took no college-level geometry courses.

It would be difficult to argue that these future mathematics teachers have been able to "demonstrate an understanding of axiomatic systems and different forms of logical arguments," and "understand, apply, and prove theorems relating to a variety of topics in two- and three-dimensional geometry, including coordinate, synthetic, non-Euclidean, and transformational geometry" (California Commission on Teacher Credentialing 2002b, para. 1).

Of equal concern is the lack of evidence that those who pass Subtest I have demonstrated a "rigorous view of algebra and its underlying structures" and a "deep conceptual knowledge" (California Commission on Teacher Credentialing 2002a, para. 1) or an equivalent understanding of number theory. Approximately 45% of the sample questions appear to require only a solid high school mathematics background. While 42% of the sample questions seem to require mathematics major courses in linear algebra, number theory, introduction to proofs, or modern algebra, only 34% of those who pass at the foundational level (Subtest I and Subtest II) have even one of these courses listed on their undergraduate transcripts.

We could better weigh these concerns if we determined which credential students took mathematics course work that does not appear on their undergraduate transcripts. Perhaps a more critical direction for further research would be to determine whether the lack of mathematics course work, taken by these credential students, translates into a lack of mathematical knowledge for teaching and, thus, into a lack of effectiveness in the classroom. While the research already cited indicates that a lack of mathematics course work has a negative impact on a single-subject mathematics teacher's effectiveness, further research in this area is warranted. In addition, what constitutes necessary mathematical knowledge for teaching secondary mathematics has not been established. Nor is it clear that traditional mathematics major course work is the best way to prepare a teacher to teach secondary mathematics.

Even if completing a mathematics major is an effective way of obtaining the necessary mathematical knowledge for teaching, the number of mathematics majors who choose to go into teaching falls well short of the number of single-subject mathematics teachers needed. Therefore, identifying the best way to provide non-mathematics majors with the mathematical knowledge necessary for teaching is of critical importance. We note that efforts to develop mathematics teachers with substantial and relevant subject-matter competency continue. For example, CSU Chico created "Project Mathematics and Teaching on the Horizon," an enrichment program for students interested in becoming secondary mathematics teachers, and, in 2010, UC Berkeley implemented Cal TEACH, a program to attract STEM students into teacher preparation programs (Newton, Jang, Nunnes & Stone, 2010; "Project M.A.T.H.," 2011).

The results of this study, at the very least, argue for a reevaluation of the process by which we establish subject-matter competency in California. It raises the concern that we are credentialing secondary mathematics teachers who do not have the subject-matter knowledge to teach secondary mathematics. It is a call for a more careful study of what mathematics background is needed by our next generation of mathematics teachers.

It is also a cautionary tale of how legislative attempts to "fix" education, such as NCLB, can put those responsible for credentialing teachers into a bind. Declaring that all teachers must be "highly qualified" without also identifying a realistic plan for implementing such a directive forces responsible agencies to make policy decisions that run counter to the declared goal.

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Appendix

CSET Content Domains

ALGEBRA

0001 Algebraic Structures (SMR 1.1)

a. Know why the real and complex numbers are each a field and that particular rings (e.g., integers, polynomial rings, matrix rings) are not fields

b. Apply basic properties of real and complex numbers in constructing mathematical arguments (e.g., if $a < b$ and $c < 0$, then $ac < bc$)

c. Know that rational numbers and real numbers can be ordered and that complex numbers cannot be ordered but that any polynomial equation with real coefficients can be solved in the complex field

0002 Polynomial Equations and Inequalities (SMR 1.2)

a. Know why graphs of linear inequalities are half planes and be able to apply this fact (e.g., linear programming)

b. Prove and use the following:

- The Rational Root Theorem for polynomials with integer coefficients
- The Factor Theorem
- The Conjugate Roots Theorem for polynomial equations with real coefficients
- The Quadratic Formula for real and complex quadratic polynomials
- The Binomial Theorem

c. Analyze and solve polynomial equations with real coefficients using the Fundamental Theorem of Algebra

0003 Functions (SMR 1.3)

a. Analyze and prove general properties of functions (i.e., domain and range, one-to-one, onto, inverses, composition, and differences between relations and functions)

b. Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems)

c. Analyze properties of exponential and logarithmic functions in a variety of ways (e.g., graphing, solving problems)

0004 Linear Algebra (SMR 1.4)

a. Understand and apply the geometric interpretation and basic operations of vectors in two and three dimensions, including their scalar multiples and scalar (dot) and cross products

b. Prove the basic properties of vectors (e.g., perpendicular vectors have zero dot products)

c. Understand and apply the basic properties and operations of matrices and determinants (e.g., to determine the solvability of linear systems of equations)

NUMBER THEORY

0005 Natural Numbers (SMR 3.1)

a. Prove and use basic properties of natural numbers (e.g., properties of divisibility)

b. Use the Principle of Mathematical Induction to prove results in number theory

c. Know and apply the Euclidean Algorithm

d. Apply the Fundamental Theorem of Arithmetic (e.g., find the greatest common factor and the least common multiple, show that every fraction is equivalent to a unique fraction where the numerator and denominator are relatively prime, prove that the square root of any number, not a perfect square number, is irrational)

GEOMETRY (SMR Domain 2)

01 Parallelism (SMR 2.1)

a. Know the Parallel Postulate and its implications and justify its equivalents (e.g., the Alternate Interior Angle Theorem, the angle sum of every triangle is 180 degrees)

b. Know that variants of the Parallel Postulate produce non-Euclidean geometries (e.g., spherical, hyperbolic)

0002 Plane Euclidean Geometry (SMR 2.2)

a. Prove theorems and solve problems that involve similarity and congruence

b. Understand, apply, and justify properties of triangles (e.g., the Exterior Angle Theorem, concurrence theorems, trigonometric ratios, Triangle Inequality, Law of Sines, Law of Cosines, the Pythagorean Theorem and its converse)

c. Understand, apply, and justify properties of polygons and circles from an advanced standpoint (e.g., derive the area formulas for regular polygons and circles from the area of a triangle)

d. Justify and perform the classical constructions (e.g., angle bisector, perpendicular bisector, replicating shapes, regular n-gons for n equal to 3, 4, 5, 6, and 8)

e. Use techniques in coordinate geometry to prove geometric theorems

0003 Three-Dimensional Geometry (SMR 2.3)

a. Demonstrate an understanding of parallelism and perpendicularity of lines and planes in three dimensions

b. Understand, apply, and justify properties of three-dimensional objects from an advanced standpoint (e.g., derive the volume and surface area formulas for prisms, pyramids, cones, cylinders, and spheres)

0004 Transformational Geometry (SMR 2.4)

a. Demonstrate an understanding of the basic properties of isometries in two- and three-dimensional space (e.g., rotation, translation, reflection)

b. Understand and prove the basic properties of dilations (e.g., similarity transformations, change of scale)

PROBABILITY AND STATISTICS (SMR Domain 4)

0005 Probability (SMR 4.1)

a. Prove and apply basic principles of permutations and combinations

b. Illustrate finite probability using a variety of examples and models (e.g., the fundamental counting principles)

c. Use and explain the concept of conditional probability

d. Interpret the probability of an outcome

e. Use normal, binomial, and exponential distributions to solve and interpret probability problems

0006 Statistics (SMR 4.2)

a. Compute and interpret the mean, median, and mode of both discrete and continuous distributions

b. Compute and interpret quartiles, range, variance, and standard deviation of both discrete and continuous distributions

c. Select and evaluate sampling methods appropriate to a task (e.g., random, systematic, cluster, convenience sampling) and display the results

d. Know the method of least squares and apply it to linear regression and correlation

e. Know and apply the chi-square test