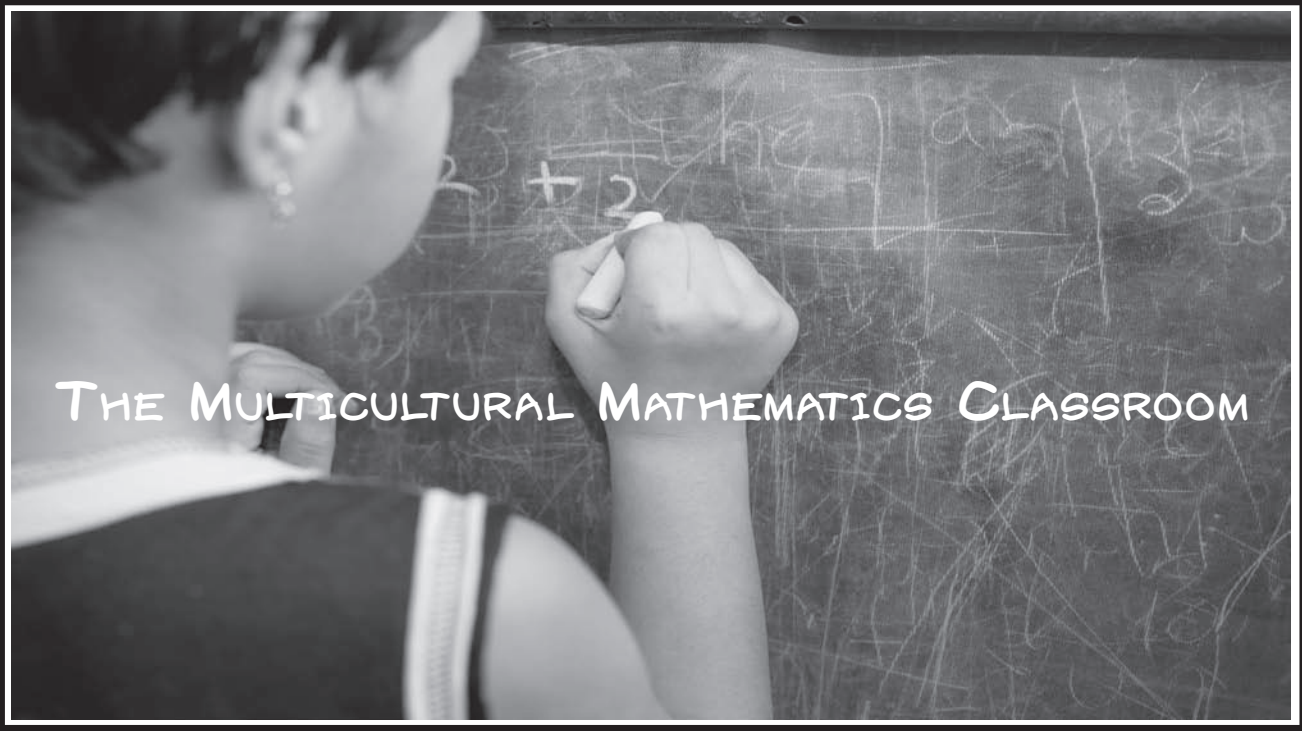

The Multicultural Mathematics Classroom

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Introduction

The National Council of Teachers of Mathematics (NCTM, 2000) has created a set of standards to reform mathematics teaching procedures to ensure that all students understand mathematics and learn to think mathmatically. The standards also require teachers to use strategies that allow all students to reason and communicate mathematically and develop students' abilities to solve problems.

In a multicultural classroom, these standards should—and can—be achieved. In such a setting, teachers must incorporate teaching strategies that allow students to develop their mathematics abilities in ways that are cognizant of the backgrounds of their students. Teachers must ensure that their teaching practices encourage all students to succeed.

The purpose of this article is to describe research surrounding the concept of culturally-based pedagogy in the mathematics classroom. I will address the following two questions:

- What are some of the different beliefs that mathematics students bring to their multicultural classrooms?
- How can teachers make their teaching more inclusive of diverse students?

In addition, I will describe the use of cooperative learning and multiple representation forms as two possible strategies that mathematics educators can use to assure that their teaching meets the needs of a diverse student population.

The Multicultural Mathematics Classroom

Today's North American classroom populations are increasingly diverse (Adams, 2008; Chen & Li, 2008; Dalin & Rust, 1996; Ernst-Slavit & Slavit, 2007; Wiest, 2001). There are students from different backgrounds, races, languages, ethnicities, and social groups in all classrooms, including mathematics classrooms. In these multicultural classroom, teachers should still strive to meet the standards set by NCTM (2000).

In Canada, for instance, many cities boast multicultural classrooms. In Toronto, approximately half of the elementary students in the public schools do not speak English at home. Toronto's students speak over 80 different languages. The values of the Toronto District School Board (TDSB) reflect this multicultural demographic. In additional to valuing

> . . . each and every student, we value the uniqueness and diversity of our students and our community. (Toronto District School Board, n.d.)

Similarly, in Vancouver approximately 60% of the students speak a language other than English at home. Within this K-12 context, over 125 languages have been identified in the schools and 25% of the students are designated as English as a Second Language learners. Similar statistics across Canada and the United States show that multicultural classrooms are a reality that teachers must address.

As these classroom dynamics contionue to change, so too should the practices of mathematics educators. Many veteran teachers may use practices that have been successful in a homogeneous classroom, however, those practices may not be appropriate for today's more diverse classroom community. By making efforts to expand their knowledge base and familiarize themselves as much as possible with the backgrounds and cultures of the learners in their classroom, teachers will be able to tune in to the strengths, weaknesses, and needs of their diverse learners (Adams, 2008; Bonner, 2009).

English Language Learners

In the multicultural classroom, students come from many different backgrounds, experiences, and ethnicities. In

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addition to learning in a different context from which they are accustomed, some of these students are also English language learners (ELLs), since their native and home language is not English. As a result of poor performance in traditional mathematics assessments, teachers often believe ELLs to be weak students (Gutierrez, 2002).

In fact, a study by Richardson and Wilkinson (2005) showed that these students do not perform as well on assessments not because of a poor academic background, but because of cultural and linguistic barriers. Teachers should be aware that ELLs are adjusting to a new culture, including new teaching strategies and assessment techniques that are regularly used in North American classrooms.

Challenges

As a result of being an ELL in a North American context, these students face a myriad of challenges. It is important to reiterate that while these students battle to learn a new language, they may be strong mathematics thinkers and doers.

One challenge for ELLs is the vast English vocabulary that they have to learn. Students need to learn the social language as well as the academic language of their new culture. Subtle nuances and the crossover between some words in the English language make such skills difficult for ELLs to master (Dale & Cuevas, 1992). One example in mathematics of such a situation includes the words 'column,' 'half,' and 'times,' as each of these words has both a mathematical and non-mathematical meaning.

ELLs may also find it challenging to grasp a concept when different words or symbols are used to express the same idea. For example, the symbols, 'x' and '*' and the use of round brackets are employed interchangeably to denote multiplication. Teachers can alleviate the confusion by using just one symbol consistently until the students have fully mastered the concept, and then introduce variations (Ernst-Slavit & Slavit, 2007). A visual teaching strategy such as a word wall where all options are visible to students to help them make reference to and become accustomed to mathematical vocabulary usage may also be helpful as students struggle to become familiar with the various symbols and notations.

Another common challenge for ELLs is the method of reading and writing mathematics compared to how they read and write narrative texts in their native language and in the English language. Some languages read from left to right, others from right to left. Some read top to bottom. Confusion may arise in the students' interpretations of how to read and write mathematics due to how their native language is organized and their ongoing attempts to navigate through learning the English language (Ernst-Slavit & Slavit, 2007).

Strengths

ELLs may not always be at a disadvantage in the mathematics classroom. There are often certain properties of an ELL's native language that may in fact boost the students' mathematical ability (Clarkson, 2007; Han & Ginsburg, 2001; Miura, Okamoto, Kim, Change, Steere, & Fayol, 1994; Moschovich, 2002).

Japanese, Korean, and Chinese students may be at an advantage with certain mathematical concepts due to the inherent base-ten system in their native languages (Miura et al., 1994). As these Asian languages count using a structured system organized around place-value, students from such backgrounds may have an easier time grasping concepts involving place-value (for example, two digit addition and subtraction) in comparison to many non-Asian language speakers who do not have this extra context of place-value.

Chinese students may also have an additional advantage in that the Chinese language has been created in a very organized way. There are different characters to represent each number, one through ten, and past this point numbers inherently use the base-10 system. For example, the

number 74 is represented by three characters: seven, ten, and four, and is read as 'seven tens, four' (7x10 is 70, plus 4, is 74). Through their understanding of the Chinese language, these students have developed a cognitive ability to organize ideas that can help them in tackling mathematical problems (Han & Ginsburg, 2001).

Although Chinese ELLs may struggle with deciphering the English language related to mathematics in a North American environment, their cultural background may indeed help them with their cognitive ability to find success in the domain of mathematics.

Building on the Native Language

Clarkson (2007) found that ELLs who were strong in mathematics in their native language have a higher success rate in the new cultural context compared to students who were weak in their native language. In his study of Australian Vietnamese students, Clarkson found that mathematics scores improved when students completed the questions in Vietnamese. He hypothesized that students already proficient in mathematics would be hampered only by the new language of instruction, and that this disadvantage would decrease with exposure to the new language.

One strategy for allowing these students to develop their language skills at a faster rate is to expose the students to concepts in which they are already proficient in their native language (Cummins, 1981). Thus, students do not need to be concerned about understanding the mathematical concept and can concentrate on building their English language repertoire. It follows that an ELL has more difficulty completing a word problem compared to being presented with an analogous problem in a numerical format (Bernardo, 2002).

By allowing ELLs to work in their native language, teachers can reduce the stress on these students so that they only need to concentrate on developing their mathematical understanding and, once these skills are solidified, then transfer them into an English context to develop their English language abilities (Gutstein, Lipman, Hernandez, & de los Reyes, 1997; Moschkovich, 2000). Gutierrez (2002) found that Latino/a students were able to thrive in a classroom where they were allowed to communicate in either Spanish or English. These students were found to select one language or the other depending on their comfort with the topic. The students sometimes used phrases that were a combination of the two languages. ELLs will benefit from strategies that allow them to develop both their English and mathematics language skills jointly (Winsor, 2007).

These research studies show that cultural differences can affect a student's experience and performance in the mathematics classroom in both positive and negative way.s Teachers must not automatically assume that ELLs will struggle learning mathematics, and the use of appropriate teaching strategies can highlight their strengths as well as support the students in overcoming their challenges.

Diverse Cultural Experiences

Students bring to the classroom a unique set of previous experiences that often stem from their background. While it is true that students from the same community will often have different personal experiences, those from different cultural and language backgrounds will similarly have a set of experiences framed from their native context. Thus a multicultural classroom requires teachers to use strategies that allow students to draw on their previous experiences to strengthen their learning and create a classroom environment that minimizes any stress resulting from any discrepancy in cultural norms.

Previously I defined a multicultural classroom as one in which there are students from a variety of backgrounds, races, languages, ethnicities, and social groups. Each student, as a result of these descriptors, brings a set of beliefs to the classroom, which categorize as culture. I will next highlight some of the characteristics that students may bring to the classroom as a result of their culture, and discuss how these characteristics may affect a student's ability in the mathematics classroom.

I am aware that the characteristics that I highlight here are only a small portion of the identities that shape learners from diverse cultures. However, for the purposes of this article, I have focused on a variety of learning styles that may be present in a multicultural classroom.

Parental Expectations

Yang and Cobb (1995) report that the cultural differences between students in Taiwan and the United States contribute to a variance in mathematical competency for elementary school students. These researchers noted that in Taiwan both parents and teachers expected children to learn concepts about place-value in Kindergarten. Even if students struggled to learn the concepts, as a community the concepts were still taught to students at that grade level and parents would take an active role in ensuring that students mastered the concepts.

Conversely, similar concepts were pushed back to Grade Two in some schools in the United States after parents and teachers observing students struggling. As American parents and students saw place-value to be a challenging concept to the young students, they delayed the learning of this material to a later grade when they thought the students would be better able to understand the concept. Parents in the U.S. were not as aggressive at pushing their students academically, and were not as actively involved in their students' learning.

Modes of Communication

In Native American groups, there is a common practice of transmitting information through story-telling and visual representation (Wiest, 2001). African-American communities value creative and individual verbal expressiveness. Much like Native Americans, these students face challenges when in classrooms that value written modes of communication over story-telling (Shade, 1989).

Thus, for these students, a sensory-oriented teaching style will be more appropriate (Davison & Schindler, 1988). These students are more likely to thrive in an environment in which they were allowed to respond creatively in a mode of their own choosing.

Economic Variations

A study in three different Chinese communities showed that varied economic contexts within the same country can affect mathematics achievement (Kun, Naiqing, & Mingzhen, 2005). With a relatively stable economy, substantial cultural development, and an outlook that both men and women are able to achieve success in education, students in the city of Guiyang score well in mathematics and also have a positive attitude and self-confidence when learning mathematics.

In contrast, the Chinese counties of Luodian and Sandu have a lower economic status and there is a marked difference between expectations for men and women. In those areas the cultural view is that men are superior to women in an educational setting and teachers reinforce these biases. The researchers in this study assert that educators play a significant role in the development of the learner, and that if the teachers in the Luodian and Sandu communities attempted to neutralize the gender imbalance, a resulting increase in mathematics achievement and attitude

in favor of women would ensue. This is further evidence that teachers can use strategies that allow all members of the classroom to reach their potential regardless of each student's background.

Using Different Strategies

Although students may come into the classroom with notions formed from their individual and cultural expriences, teachers can be sensitive to these differences and incorporate into the classroom ideas and procedures which will allow students to adapt to a new classroom community and achieve academic success. By exposing students to different strategies, teachers may help students become accustomed to the practices preferred in any given school and community.

Nevertheless, while teachers may try to create their own mathematics classroom practices based on the needs and experiences of the students in the class, the pre-set curriculum and local, state, or national educational regulations will in some instances limit the types of classroom practices that teachers will be able to utilize.

Cultural Differences

In some Spanish-speaking nations, problem solving procedures and mathematical symbols are different from North American norms (Ernst-Slavit & Slavit, 2007). For example, in division problems, the position of the divisor and dividend are switched. Ernst-Slavit and Slavit (2007) assert that teachers should allow for students to voice their strategies for solving problems so that they, the teachers, can learn about the procedures that students have brought with them to the classroom. This approach will also help students clarify confusions and understand North American norms.

One simple example of a cultural difference in mathematics is found in measurement. Students who have grown up in a nation using a metric system may find it challenging to switch to an imperial system (or vice versa). Culture and mathematics are tightly interwoven and differences between a student's everyday approach to mathematics and the approach championed in school may confuse a learner who is trying to adjust to both a new social and cultural environment as well as a difference structure for learning mathematics.

Rote Memorization

While learning by rote memorization is still popular in China, current Western classrooms have turned away from this method of learning. This changing approach has much to do with cultural beliefs regarding how students learn best and what each culture values in its students. The traditional East Asian mathematics classroom highlights "content and the procedures or skills in dealing with the content" (Leung, 2001, p. 38). Therefore, the primary concern of the teacher is for the student to acquire the content. Rote learning is an accepted method for such learning in the Chinese context.

By contrast, in response to (and as evidenced by) the NCTM (2000) standards, the modern Western classroom puts more of an emphasis on *how* a student learns the content (Leung, 2001). Teachers are now incorporating more problem-solving and investigation activities to replace the traditional skill and drill exercises that were previously used in North America and are still prevalent in East Asian schools.

Another reason that repetitive practice and memorization is accepted in the Chinese community relates to the belief that students must study hard and persevere before reaping the benefits and enjoying their eventual success (Leung, 2001). The emotional experience of a mathematics student is different in Asian contexts. In the student-centered, modern, Western classroom, society views education to be a positive experience in which students should enjoy a pleasurable learning experience.

Home and School

The discrepancy between school practices and home or native practices can also contribute to poor student achievement (Banks, 1993; Bonner, 2009). A contradiction exists between the mathematics that the students have become accustomed to in their native context and the need to redefine these ideas and practices for their new environment can be confusing and can hinder student learning. Therefore, it is important for teachers to try to bridge the two and provide strategies that allow for students to understand and develop the skills needed to succeed in the North American school context.

Although many students in a multicultural classroom may have the same general background, and in turn, have some common characteristics, each individual will have additional personal characteristics that make them unique. As such, the use of a variety of teaching strategies makes mathematics accessible to all students in a multicultural classroom.

Culturally-Based Pedagogy

To meet the standards set by NCTM (2000), teachers must incorporate a variety of teaching strategies that appeal to and consider all of the learners in their classrooms (Gay, 2000; Lipka, 1998; Nieto, 2003). Ladson-Billings (1994) uses the term culturally relevant pedagogy to denote a type of teaching that incorporates student culture in order to preserve it and overcome obstacles that may arise due to the weight of the dominant culture.

Historically, students who were not of the dominant culture have suffered because they were not given a fair chance to adapt to the norms of the classroom. Such students have been unable to perform comparably on assessments and have typically been labeled as incompetent or unable to achieve academic success (Malloy & Malloy, 1998). Most assessments have used strategies and modes of evaluation that were unfamiliar to the non-native students. Thus, non-native students could not perform to the level of the students of the dominant culture who had grown up being taught and assessed in such a manner. Too often these low-performing students were placed in lower academic tracks or special education (Bonner, 2009; Zola, 1993).

Teachers need to rethink these traditional approaches and provide their diverse classroom community with opportunities for deeper mathematics learning. As teachers and students strive to meet the standards set by NCTM (2000), it would be irresponsible for educators to disregard the needs of diverse learners. Students from all cultures deserve to be taught in a way that ensures that they will understand mathematics. Teachers can use teaching strategies that acculturate and enculturate the students in their classroom.

Acculturation and Enculturation

The process of *acculturation* asks students to adapt to dominant classroom norms (Malloy & Malloy, 1998). To do this, teachers need to socialize students into the norms and practices of the mathematics classroom (Cobb, Wood, & Yackel, 1993). While initially this may appear that teachers are forcing their students to change their culture to adapt to the norms of the school as set by the teacher, sometimes this reworking of cultural patterns may be necessary for the student to be successful in the school context.

Enculturation, on the other hand, occurs when teachers include a variety of cultures in their practices to reflect and serve their multicultural classroom (Malloy & Malloy, 1998). This approach to teaching is widely accepted in the field of multicultural education. The inclusion of students' cultural and linguistic back-

grounds can be a significant source for student motivation (Ernst-Slavit & Slavit, 2007). Likely students will feel a personal connection to the concepts being taught and will be more motivated to learn.

Communities of Practice

Wenger (1998) discusses several strengths of communities of practice. Students must be active participants in the classroom community for the group to reap benefits. If students do not feel a part of the classroom community, they will likely not be able to learn (Cobb & Yackel, 1996). Radford (1997) states, "the content of mathematical knowledge is properly and intimately defined by the culture in which it develops and in which it is subsumed" (p. 32).

Thus, students can create an appropriate mathematics culture in the classroom if given the opportunity. If teachers allow students to be active participants in the classroom, those students will mold the mathematics culture in ways best suited to their learning needs (Malloy & Malloy, 1998).

As a result of classrooms becoming more diverse, it is unreasonable for teachers to demand that all students adopt the norms set by traditional Eurocentric viewpoints (Malloy & Malloy, 1998). If teachers can use strategies that encourage acculturation and enculturation, teachers can remain true to the necessary constraints of the schooling institution and still provide an inclusive and rewarding learning experience for their students.

A Challenging Task

A culturally-based teaching approach may be challenging. Teachers need to be cognizant that their attempts should not result in generalizing or stereotyping any groups (Wiest, 2001). Teachers must instead relate their teaching to their students' lives in a non-superficial manner. In a lesson about finding the lowest common multiple (LCM), Bonner (2009) relates a case where an elementary school teacher used a non-superficial connection to the students' cultural reality. The teacher put the LCM into the context of a story where a big brother (the LCM) goes to pick up his little brother (a smaller factor) at school. The central idea to her lesson is that the factor and the LCM are related, just as the two brothers are.

In the students' neighborhood, older siblings in families often care for their younger siblings and the close connection within the family is valued by the students. By contrast, some examples of superficial connections include using ethnic names in a word problem, though beneath the surface the problem has no relationship with the ethnicity; or including popular songs from different groups in lessons, though those songs are unrelated to the content of the material being taught (Adams, 2008; Wiest, 2001).

In the Western world, the traditional classroom has a Eurocentric focus, with a teacher who embodies Western characteristics (Bonner, 2009; Ernst-Slavit & Slavit, 2007). For students who come from diverse backgrounds, this type of classroom can be jarring. An inclusive and adaptable environment in combination with carefully selected teaching practices allows these students to learn about and adjust to new classroom practices.

Breaking Down Stereotypes

By incorporating a variety of cultures into their mathematics teaching, educators can break down some of the stereotypes in the mathematics world. Traditionally, students have been taught that mathematics was created by White, male, European mathematicians (Piccolino, 1998). Through the inclusion in the curriculum of mathematicians of all ethnicities and genders, students from diverse backgrounds will then see a place for themselves in the mathematics community (Piccolino, 1998). A culturally-based mathematics educator can

> . . . help to foster appropriate and desirable attitudes about mathematics and its role in our culture. In particular, multicultural activities can be instrumental in removing the stigma of mathematics as being an elitist discipline pursued by a select few. (Piccolino, 1998, p. 84)

> Culturally-based pedagogy can give all students, regardless of their learning preferences, the opportunity to learn mathematics. (Malloy & Malloy, 1998, p. 251)

Educators should see the value in allowing students to bring their previous experiences to the forefront of the mathematics classroom. All students have a vast knowledge drawn from various experiences in their community. While these experiences may not be the same for all individuals in the classroom, as a collective the knowledge pool contains a wealth of resources to improve classroom learning. Using teaching strategies that allow students to share their culture, languages, and experiences will make mathematical discussion richer (Gustein & Peterson, 2005; Nieto, 2004).

Two possible strategies that incorporate a culturally-based approach to teaching are cooperative learning and multiple representations. As examination of each follows.

Cooperative Learning

> Cooperative learning is the instructional use of small heterogeneous groups of students who work together to maximize their own and each other's learning. (Vaughan, 2002, p. 359)

This learning can focus on academic and social development (Lopata, Miller, & Miller, 2003). The instructional processes used in cooperative learning can range from simple to complex.

Bennett and Rolheiser (2001) call the simple processes *tactics* and the more complex processes *strategies*. Many researchers have studied and continue to study the use of cooperative learning in the classroom and the variety of strategies and tactics available for teachers to apply in the classroom (e.g., Bennett & Rolheiser, 2001; Ke & Grabowski, 2007; Stevens & Slavin, 1995; Van de Walle & Folk, 2005; Vermette & Foote, 2001).

The NCTM standards (2000) suggest that teachers create instructional programs in which students can communicate their mathematical thinking coherently and clearly to others, thereby solidifying a role for cooperative learning in the mathematics classroom. In addition, cooperative learning can be valuable in meeting the needs of a culturally diverse classroom (Brenner, 1994; Callahan, 1994; Hatfield, Edwards, & Bitter, 1997; Malloy, 1997; Zaslavsky, 1996). Cooperative learning allows for discussion and reflection, thus alleviating the stress for students of needing to come up with a quick response as required by some other teaching strategies (Hatfield et al., 1997).

Many educators employ cooperative learning techniques (Bennett & Rolheiser, 2001; Ke & Grabowski, 2007; Stevens & Slavin, 1995; Van de Walle & Folk, 2005; Vaughan, 2002; Vermette & Foote, 2001; Webb, Farivar, & Mastergeorge, 2002). The collaborative nature of cooperative learning gives students a chance to complete tasks and attain concepts they may not have been able to accomplish themselves (Paradis & Peverly, 2003).

The benefits of cooperative learning parallel Vygotsky's (1978) *zone of proximal development* (ZPD), which is the difference between what a learner can do on their own versus what they can do with help from others. Vygotsky (1978) states that adults (teachers and parents) or a child's peers can help student development, and that teachers can use cooperative learning to increase the understanding of mathematics by students of all backgrounds.

Through interactions with English speakers of various abilities, ELLs can develop their language skills by working with

their peers (Barwell, 2007; Moschkovich, 2002). Teachers should encourage students to communicate their understanding and explain their thinking processes to their peers to deepen their mathematical understanding and develop their communication skills (Barwell, 2007; Winsor, 2007). Working in groups also allows ELLs to utilize their peers to support their development of understanding classroom practices and these peers can also act as interpreters (Garrison & Mora, 1999).

The discourse that arises from cooperative learning structures can improve an ELL's language ability as well as familiarize them with the mathematics classroom community. Moschkovich (2002) describes discourse as being important in helping individuals identify themselves in their native community as well as a new member of the North American culture. ELLs and other culturally diverse students will benefit from this discourse.

While working in groups, students are able to imitate culturally-established mathematical practices with more capable peers or through interactions with the teacher (van Oers, 1996). Over time, students who are beginning to grasp the dominant norms will be able to more successfully function in the classroom community on their own. Through their interactions with others, these students will begin to internalize the new and different norms (Ohtani, 2007; van Oers, 1996). While cooperative learning can be an effective teaching tool, incorrect implementation of the strategy in a diverse mathematics classroom could negate possible benefits.

Recommended Implementation

Although cooperative learning can allow a group of students to work together and draw from their diversity, initially there are risks that students may feel isolated and excluded from the interactions of the group. This can stem from students' not having the skills to work in cooperative groups or not understanding the goal of the task, as well as the danger that certain students will dominate the group.

Tschannen-Moran and Hoy (2000) caution teachers to consider the group characteristics of their class before implementing cooperative learning. Students may not be socially, cognitively, or emotionally prepared to work with their peers and teachers may need to address these issues before using a teaching strategy that could cause tension between students rather than act as a vehicle for community building and academic growth (Barnes & Todd, 1977). As such, the role of the teacher is important when implementing cooperative learning.

Social and cognitive skills are necessary components of an effective cooperative group. Barnes and Todd (1977) explain that, without these skills, groups were unable to work as a unit and destructive interactions will occurr. Examples of necessary social skills include: "the ability to control progress through the tasks, the skills to manage competition and conflict, and the ability to modify and use different viewpoints as well as the willingness to give mutual support" (1994, p. 5).

Critical cognitive skills include: "constructing meaning for a given question, inventing a problem, setting up hypotheses, using evidence, and recreating experience" (Cohen, 1994, p.5). If a teacher realizes that the students do not have these necessary skills, the teacher may need to vary the type of activity used in order to focus the task on developing the skills of the group before focusing on the academic content (Tschannen-Moran & Hoy, 2000).

By explicitly stating the goal of the task, teachers can ensure that all students understand what is expected of them (Tschannen-Moran & Hoy, 2000). In this way, a dominant student will not be able to impose assumed goals on the rest of the class. All students in a multicultural classroom can also benefit from a clearly-stated goal since there will then be less of a risk of misinterpretation. If a teacher notices that a group is going off-task, it is the teacher's responsibility to remind the group of the goal and ensure that all members subscribe to that goal. To ensure this dedication, positive interdependence should be created (Bennett & Rolheiser, 2001).

Positive Interdependence

Johnson and Johnson (1991) define *positive interdependence* to be the dual responsibility of individual group members achieving the goal and the full class ensuring that the rest of the group members—all members of the class—also achieve the goal.

> Positive interdependence exists when students perceive that they are linked with groupmates in such a way that they cannot succeed unless their groupmates do (and vice versa), or that they must coordinate their efforts with the efforts of their groupmates to complete a task. (Johnson & Johnson, 1991, p. 55)

The development and assignment of student roles is one way to ensure positive interdependence. This strategy involves the teacher assigning roles to each member of the group, with all roles deemed necessary for the completion of the task. The assigning of roles is also a strategy to ensure that all members of the group participate equally (Cohen, Lotan, Scarloss, & Arellano, 1999; Johnson & Johnson, 1991).

Students will feel competent and willing to engage in the group if they feel comfortable in their roles. Student will also feel empowered if they can bring their previous experiences to their assigned roles. There are a wide variety of roles that can be used, such as materials manager, recorder, encourager, or summarizer (Cohen et al., 1999; Parr, 2001).

Involving All Students

Teachers can also use an open-ended task that cannot be completed by one student alone to eliminate the problem of dominant personalities in the group (Cohen, 1994; Cohen et al., 1999). Cohen's (1994) idea of "complex instruction" draws from the notion that group tasks require the experiences, repertoires, and strategies of all group members in order to achieve the higher-order thinking necessary to complete the task. The diversity of knowledge that can be found in a multicultural classroom needs to be shared to accomplish the cooperative task. All students need to be active participants in order for the group to be successful. ELLs and students from non-dominant groups will feel more valued when their experiences are necessary to complete the task. This type of task also strengthens the positive interdependence of the group.

A teacher's actions can also support the idea that "each individual brings valuable and different abilities to the [cooperative] task" (Cohen et al., 1999, p. 85). Cohen and colleagues state that assigning competence to low-status students allows all students to feel valued in the group. In a multicultural classroom, ELLs and students from non-dominant groups are not automatically low-status, however, language challenges and unfamiliarity with North American classroom practices may prove to be a deficit (Gutierrez, 2002). A public statement that elevates a student's expectations for competence and the group's expectations for the student should specifically address the strength of the potentially low-status student and how that student can contribute that strength to the group task. Cohen and Lotan (1997) found that such status interventions increased the participation of low-status students without inhibiting the participation of high-status students.

In a diverse classroom, students may have preconceived notions about who in the classroom may have a more valuable contribution to make in cooperative group tasks. Although there have been many studies which show the benefits of hetero-

geneous groupings, there are risks that low-status students may not feel valued in such groups. In the multicultural classroom, ELLs and non-dominant groups are at risk for feeling excluded or devalued. Careful implementation and consideration by the teacher can diminish these risks.

Multiple Representations

Although social interactions in the form of cooperative learning are beneficial, teachers can also look for more ways to give students in their multicultural classroom an opportunity to understand mathematics. The use of various representation forms, which help students to make connections and communicate their mathematical understanding in multiple representation forms, is another highly effective strategy.

There are many types of representation forms and they can be grouped into two categories: internal and external (Pape & Tchoshanov, 2001). *External representations* come in the form of numerical tables, physical materials, pictures, symbols, graphs, and algebraic formulas. Another lens through which educators can view these external representations is as cultural tools (Cobb & Yackel, 1996; van den Heuvel-Panhuizen, 2001).

When students use various forms, they have more opportunities to communicate their thinking. A representation form can stimulate dialogue with peers and teachers, and enable students to discuss the merits of their chosen representation form and be able to compare it to other forms (Pape & Tchoshanov, 2001). In the beginning phases of using representations, students may not be as eloquent with their representation form and this discussion can help them refine their knowledge and their ability to represent their mathematical understanding (DiSessa, Hammer, Sherin, & Kolpakowski, 1991).

Once a student is able to represent a concept in different ways and can explain how each of the representation forms relates to the others, it can be said that they truly understand the concept (Lesgold, 1998). Similar to cooperative learning, using multiple representations gives all students opportunities for discussion to deepen their mathematical understanding. Students can also practice and develop their communication skills.

For ELLs, the challenge of communicating their understanding in a written and oral form can be overcome by teachers exposing these students to as many forms of representation as possible. This exposure allows students to improve their English communication skills and, in the context of learning a new concept, use language-neutral representations to further scaffold their learning (Moschkovich, 1999; 2002).

Since multiple representations allow students to scaffold their learning, teachers may start with material that is familiar to students' because of their cultural background and experiences (Gutierrez, 2002). By getting to know the backgrounds of the students and encouraging students to bring their experiences into the classroom and share the strategies that they would naturally use, teachers can nurture their students' construction of knowledge.

Teachers will then scaffold the students' understanding and use representations that build towards a desired norm or level of understanding (Wiest, 2001). Visual and kinesthetic representations often span across groups and may be a good starting point. Teachers need to have the ability to take students from their cultural base and stretch them to consider other possible viewpoints and representation forms (Malloy & Malloy, 1998). Through the use of multiple representations, students will have a broader repertoire of strategies with which to tackle problems and thus develop their skills so as to be adaptable to various classroom methods that would otherwise be unfamiliar.

Student voice and discussion allows students to talk about how they come up with mathematics solutions. The students may have learned similar concepts in their native context and they may have different strategies for solving problems (Zaslavsky, 1996). Teachers must be ready to accept these different solutions and allow students to bring the strategies that they learned from their native culture into the classroom. By restricting the types of representations that students can use, teachers are limiting their students' abilities to draw from their background.

The ability to choose appropriate representation forms is also beneficial for ELLs. By allowing ELLs to practice communicating their ideas, teachers can help students develop their language skills as well as contribute to the mathematics learning of the classroom. In doing this, teachers do not need to be the expert with all representation forms and cultural norms. Student sharing can allow for students to be each other's teachers and move the classroom community from a teacher-led to a student-led environment.

Combining the Approaches

If used in conjunction with cooperative learning, students would gain exposure by seeing their peers use representation forms that they themselves may not have naturally chosen to use. The students thus have a chance to see mathematics problems solved in different ways, build on other students' knowledge, and even add some of these previously foreign representation forms to their own knowledge base (Ahmed, Clark-Jeavons, & Oldknow, 2004; Mevarech & Kramarski, 1997).

In this instance, the zone of proximal development is revisited and students will improve their mathematical understanding through exposure to other, more complex representational forms as presented by their peers. Teachers can facilitate the scaffolding of more complex representation forms and allow students to discuss the representational form and their corresponding mathematical understandings that they bring with them to the group.

Teachers need to use all forms of representation equally and not show bias towards a certain form, since otherwise students will pick up on this preference and tend to favor the form chosen by the teacher (Herman, 2007). The more that a student is exposed to different representation forms and the more time that they have to learn to use those forms, the more confident and comfortable they will feel with constantly being able to change their chosen method of representation.

The students will then grow up with the idea that there is not one correct method and that they should be able to use any appropriate method at any time. This representational fluency and ability to adapt may help the students feel comfortable in a different cultural setting with a different set of representational norms than what they were used to previously.

Cultural tools are

> . . . gradually appropriated by the pupils as cognitive means for regulating their personal mathematical activity. Thus, process of symbolizing of cultural tools is characterized by changes of their function from collective use to private one. (Ohtani, 2007, p. 4-40)

The combination of cooperative learning and multiple representations also allows students of different backgrounds to bring to the classroom discourse the cultural tools with which they are already familiar.

Through cooperative learning, students can heighten their mathematical understanding by sharing their understandings as represented through their own tools and learn about other students' cultural tools. As the community of practice begins to include all of these different tools, students will reach a certain comfort level with different apporoaches and different tools. When that occurs, students will be able to use a tool that was once unfamil-

iar and use it in their own construction of mathematical understanding (Lave & Wenger, 1991).

The positive benefits of using both cooperative learning and multiple representation forms will produce a more successful student in the multicultural classroom.

Conclusion

As teachers continue to improve their teaching practices to meet the standards set by NCTM (2000), they must be aware of the needs of the learners in today's classrooms. With the increasingly diverse classroom, North American teachers cannot rely on teaching practices that may previously have been appropriate for a mainly Eurocentric classroom. Students today come to the classroom with many different experiences, backgrounds, and languages. Their needs and strengths must be attended to in order to ensure that they receive an equitable opportunity to learn mathematics.

Teachers must allow students of all backgrounds to help develop mathematics practices specific to the new classroom community. Culture is ever changing and adapting, so all members of the school and classroom community—students and teachers—can and will contribute to defining a new set of classroom practices that meets the needs of all members (Nieto, 1992). Teachers' decisions can allow students to participate in creating classroom practices to which all members of the classroom can subscribe (Nieto, 1992).

While teachers may not be able to know everything about all of the cultures that students bring to the classroom, the use of teaching strategies such as cooperative learning and multiple representations will offer students of all backgrounds a chance to feel included in the classroom community. Students from diverse backgrounds will have the opportunity to bring their culture to the classroom and teachers, as well as students, can and should see these diverse contributions as strengths and opportunities for further growth rather than as obstacles to overcome.

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