A Comparison of Mathematics Textbooks from Turkey, Singapore, and the United States of America*

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Abstract
The purpose of this study was to compare 6th grade Turkish, Singapore, and American mathematics textbooks in terms of certain features of textbook design. Textbooks were compared based on their visual design, text density, internal organization, weights of curriculum strands, topics covered, and content presentation. The results revealed varied assumptions for student learning and choices of design. Singapore books reflected simple features of text density and enriched use of visual elements, fewer number of topics, and an easier inner organization to follow. American books were mainly designed as reference books. Turkish books reflected a measured middle way between the two and reflected a design that valued active student learning. However, Turkish books could use ideas to improve visual design and presentation of certain topics.

Key Words
Mathematics Textbooks, International Comparative Studies, Content Analysis, Mathematics Education.

The mathematical performance of Turkish students in the international comparative studies such as the Trends in International Mathematics and Science Study (TIMSS-R) and Programme for International Student Assessment (PISA) have alarmed stakeholders that there are serious problems in mathematics education of students in Turkey (Alacaci & Erbas, 2010). Mainly as a part of an initiative for joining the European Union (EU), there have been several reform efforts in Turkey in the pursuit of adaptation of the EU standards and norms in social and political fields, including education. One of those reform initiatives is the update of the primary education curriculum and textbooks in elementary (1st grade to 8th grade) and in secondary (9th grade to 12th grade) level. As of 2008-2009 academic year, the gradual implementation process was completed and all students in all grades started to use the new curricula and textbooks. It is widely accepted that textbooks assume three important roles in education: (i) to serve as a guide to decide which topics to be taught, (ii) help teachers to organize topics and materials in an order, (iii) provide teachers with ideas and activities for teaching students (Altun, Arslan, & Yazgan, 2004; Delice, 2005; MEB EARGED, 2003; Robitaille & Travers, 1992; Woodward & Elliott, 1990). Even though mathematics textbooks play a big role in teaching and learning mathematics throughout the world; there is a clear dearth of research on the actual use of textbooks in mathematics education (Li, Zhang, & Ma, 2009). Two reasons that are linked to each other are the difficulty in collecting data on textbook use and the lack of a theoretical framework in doing so (Rezat, 2006). Furthermore, for students, one of

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the primary factors that play into classroom learning is the opportunity to learn. Textbooks are important indicators of students’ opportunity to learn as they reflect the intended curriculum for schools. The purpose of this study is to compare Turkish, Singapore, and American 6th grade mathematics textbooks based on certain features of textbook design. In particular, textbooks were compared with regards to elements of visual design, text density, internal organization, relative weights of curriculum strands, number of topics covered, and styles of content presentation.

Theoretical Framework
The conceptual and theoretical framework in this study comprised of activity theory suggested by Vygotsky (Rezat, 2006) and reader oriented textbook theory suggested by Weinberg and Wiesner (2011).

Activity Theory and Textbook Use: One theoretical perspective that is helpful in understanding the role of textbooks in mathematics classroom comes from an interpretation of activity theory (Rezat, 2006). According to this theory, humans conduct activities in a culturally mediated context by using an embedded artifact to reach an object. Activity theory attempts to explain the role of textbooks in teaching and learning of mathematics from a sociocultural perspective by using “subject-mediating artifact-object” triad. 'Textbook use' is one such activity. Humans use artifacts called mathematics textbooks to reach an object in a culturally formed system. In other words, textbook use is influenced by the educational system in which it is used (Li, 2007). Textbooks are both a pedagogical tool as well as a marketed product, textbooks are an instrument for learning as well as an object of learning, and textbooks address both students and teachers. These dimensions capturing the dual roles of mathematics textbooks make it impossible however to understand the role of human activity called “textbook use” by a single triad of “subject-mediating artifact-object” supplied by Vgotsky-inspired theoretical tool, as shown in Figure 1. Rezat (2006, pp. 411–413) proposed four traits forming a tetrahedron model together as shown in Figure 2. In particular, especially in developing countries access to textbooks and mathematical achievement are strongly related (Fuller & Clarke, 1994; Heyneman, Farrell, & Sepulveda-Stuardo, 1978; Schiefelein & Simmons, 1981). It is also known that the weight of a topic in a textbook in terms of problem types and numbers has a positive correlation with the levels of learning of that topic (Ball & Cohen, 1996; Garner, 1992; McKnight et al., 1987; Olson, 1997; Schmidt, McKnight, & Raizen, 1997; Schmidt et al., 2001). For example, Törnroos (2005) tried to find a correlation between the number of pages devoted, type and number of exercises and problems about a topic and students’ levels of learning of that topic. He found a significant correlation between type and number of questions and student learning of the topic. Furthermore, there was found a high correlation between the emphasis on a topic in mathematics textbook in terms of number and type of questions and students’ rate of success in similar items in tests like PISA and TIMSS (Törnroos, 2005).

There is also empirical evidence supportive of the model in Figure 2b, that students reach mathematical knowledge through their teachers. Teachers rely heavily on textbooks in planning lessons and developing explanations of mathematical concepts in their instruction. For example, 75 to 90 percent of mathematics instructional time in US classrooms are run based on the framework provided by textbooks (Tyson & Woodward, 1989; Woodward & Elliott, 1990); in Japan most teachers teach no more or less than what is in the textbooks (Fujii, 2001), and in Norway and Spain most lessons are taught in close fidelity with textbooks (Schmidt et al., 1997).

They access the information in the textbook when the teacher borrows concept explanations, metaphors, definitions, theorems or other pedagogical tools from textbook, or when she assigns problems to students from the textbook. Figure 2(c) refers to teacher’s use of textbook for his own professional development in a context more general than the need to prepare a particular lesson, and more as a source of reference to supplement his mathematical knowledge. Other than these three ways, students can learn mathematical knowledge directly from their teachers; however, we do not focus on this face of the tetrahedron as it does not involve textbook use directly.
textsbooks tended to increase by grade levels from 1-4, 5 to 8 and from 9 to 12, for example, the USA (Schmidt et al., 2001).

Corresponding to Reza's component of the model of textbook use shown in Figure 2c, there is evidence that textbooks serve as an important resource for teachers' understanding of the intended curriculum. National curriculum goals are best-operationalized and understood through mathematics textbooks for teachers in many countries (Bierhoff, 1996; Haggarty & Pepin, 2002; Harries & Sutherland, 1998; Howson, 1995; Schmidt, McKnight, Valverde, Houang, & Wiley, 1997). In summary, teachers benefit highly from textbooks about how to explain a topic, which activities to use, and textbooks emerge as an important input for teachers' instructional practices and plans (Fan & Kaeley, 2000; Robitaille & Travers, 1992).

Reader Oriented Textbook Theory: Suggested by Weinberg and Wiesner (2011), reader oriented textbook theory attempts to understand the design features of mathematics textbooks. According to this perspective, the real function of a textbook comes from the process through which textbooks can relate to its readers and the process by which readers construct meaning from the textbooks. In fact, textbooks can be described to fall between two poles: text oriented and reader-oriented textbooks. Textbooks that have a text-oriented approach presume readers will take the objective knowledge in the order in which it is given in a text. The language typically used by author of textbook written in this orientation for example says that non-living objects such as graphs can show a mathematical entity to the reader. Mathematical knowledge can be correctly attained by following the order and organization of the content in the textbook. Alternative to this is for the reader to see the mathematical relationships himself or herself. Reader oriented textbooks on the other hand do not limit the textbook-reader relationship into one correct path, rather take into account subjective interpretation of mathematical explanations and take into account it is possible to develop different understandings of mathematical concepts. Text oriented textbooks assume readers analyze the content of the textbook, reader oriented textbooks assumes readers will construct their personal meanings from the textbook. Further, reader oriented textbooks point out why learning this content is important, ask questions to the reader to help them look back and consolidate the learning from the textbook, and suggest questions and topics for discussion among peer learners (Weinberg & Wiesner, 2011). In short, reader oriented textbooks are designed to include processes to help derive meaning from a textbook.

Reader oriented textbook theory can provide useful insights for textbook design. For example, does a given textbook present itself solely as a source of mathematical definitions, theorems, operations, symbols and problems, or does it suggest ways for the students to internalize this information? These ‘ways’ can be asking questions to help see relationships between new and earlier content, suggesting questions and topics of discussion among peers for reflections, presenting multiple ways to solve problems in addition to the most “direct” way and help compare these ways, bringing alternative ways of understanding concepts and operations to the consideration of the reader by using for example, talking balloons and pictures. These are desirable features of textbook design according to reader-oriented theory, even though it does not suggest that without these ways it is impossible to learn or understand the mathematical information. But rather predicts that lack of these features deducts from the utility of mathematics textbooks. Reader oriented theory provides a general perspective for thinking about textbook design, but does not suggest prescriptive ways for textbook design.

Method

Textbooks Compared in the Study

To identify parameters of textbook design, we compared student editions of 6th grade mathematics textbooks from Singapore (Kheong, Ramakrishnan, & Soon, 2008), the United States of America (Bell et al., 2007) and Turkey (Aktas et al., 2007). Neither teacher editions nor guidebooks nor study workbooks accompanied to the textbooks were included in the analysis. The Turkish textbook was the official textbook for 6th grade mathematics published by the Turkish Ministry of Education. Most students in Turkey were using this textbook as it was approved and distributed free of charge to students by the Ministry. Singapore textbook was also approved by the Singapore Ministry of Education and had a big market share. The US textbooks, on the other hand, had roughly a good 20% market share and reflected the reform movement in mathematics education (Malzahn, 2002). The best-selling textbooks for Turkey, Singapore and the US were selected considering that they would give a good picture of problems available to the students in these countries.
Analysis of the Textbooks

The analysis of the textbooks was carried out in terms of the following issues: (i) visual design and readability (i.e., text density in 5% of the pages chosen randomly, and use of visual elements such as photos, drawings, tables, figures, graphs, diagrams, models, icons, and speaking balloons in 10% of the pages chosen randomly in all three textbooks); (ii) content structure (i.e., how chapters/units are organized in terms of number of units and their contents throughout the textbooks); (iii) weights of content areas (i.e., weights of each of the following content strands: numbers and number concept, statistics and probability, geometry, algebra, and measurement (National Council of Teachers of Mathematics [NCTM], 2000) throughout the textbooks); (iv) subject headings (i.e., content focused on subject teaching in all three textbooks were coded and classified primarily based on the content titles defined in Milli Eğitim Bakanlığı [MEB] (2004)); (v) prevailing approaches to the content presentation throughout the books. The followings were identified and analyzed in terms of their sequencing and role in presenting the content: student-centered activities, topic explanations, real-life and/or realistic connections, technology and manipulative use, problems and exercises. The whole coding process was carried out by two researchers and at least 85% consistency was sought between them. Disagreement was resolved by discussion until full agreement was reached.

Results

The results showed varied assumptions of student learning and varied choices of design options in different countries. Singapore book in particular reflected distinctive design features with low text density and higher use of visual elements, fewer number of topics covered, a clear and simple inner organization, and a style of presentation with explicit directions of use for students. Singapore book contained mixed exercises for review for distributed practice of learning. American book on the other hand gave the impression of a reference book rather than a book that can be directly used in instruction with a relatively high text density and higher number of topics containing additional non-instructional information. This feature of American textbooks had been also observed elsewhere (Porter, 1989). There were fewer structural elements that facilitated how students should interact with the textbook. Turkish books reflected a measured middle way between the two countries in many design features such as text density, use of visual elements and the number of topics covered. Turkish books presented topics using student activities and examples with explicit connections to real world contexts.

The finding of this study reveal the different ways mathematics textbooks could be made reader-friendly beyond merely presenting mathematical knowledge. For example, unlike other textbooks, Singapore book had some unique aspects within its unit organization that we believed made it more reader-friendly for students and teachers compared to the other two textbooks. Mathematical explanations in a unit started with a “warm up.” If there is more than one way to perform an operation (for example while calculating direct proportions), or solution of a given problem, these are explicitly identified and explained as method 1 and method 2. Following mathematical explanations was a section called “let’s work together.” In this section, students were invited to work in pairs on solving problems similar to the worked-out problems. This is followed by a section called “let’s try now” in which students were asked to do individual practice. In a section called “let’s sum up”, students were given an overall view of the topic under consideration and were asked to reflect on what they just learned. Finally, in the last section of a unit entitled “it is time to think now,” students were given a challenging problem more difficult than the ones that have been dealt so far. It is probably aimed to give a sense of accomplishment and successful finishing of the unit. These organizational components were repeated in different units in the same pattern. We believe these sections are but one way to help the reader relate with the textbook and the contents inside, going beyond “here is the mathematical knowledge, take it.” Turkish mathematics textbook presented content in the context of multiple student activities and application and practice of what is learned in new problems. An aspect unique to Turkish book was asking students to pose a problem in addition to solving them according to the steps outlined by Polya (1973). American textbook on the other hand, simply started the unit with a real life example of the use of new content, followed by simple and direct explanation of the content, and concluded with individual practice and application. Another important distinction of Singapore text was in its design of exercises given at the end of a unit. These exercises were expected to be worked out in classroom and were not limited to the unit but contained problems from all the units placed before that unit. Mixed exercises are known to be
effective to reinforce student learning than exercises limited to a particular unit. Because it helps to revisit earlier topics at multiple points in time by giving an opportunity for distributed practice and further facilitate making connections across topics (Anderson, Reder, & Simon, 2000; National Research Council [NRC], 1999; Slavin, 2006). Conclusion, Discussion and Recommendations

In this study, Rezat’s (2006) models of textbook use and Weinberg and Wiesner’s (2011) reader-oriented textbook theory were used to interpret findings. A mathematics textbook serves student learning in more than one way; as student reads explanations from the textbook, as student individually works on exercises to reinforce learning, as student solves problems assigned for homework from the textbook. Also, textbooks serve as a resource for teachers to revisit and refresh the mathematical knowledge he/she will teach, and textbooks suggest ways to explain and plan for classroom activities for teachers (Rezat, 2006). It is postulated that well-designed textbooks reflect explicit design features to facilitate these multiple functions for students and teachers. We conjecture after Weinberg and Wiesner (2011) that the value of a textbook lies not only in how elegantly it presents or explains mathematical knowledge, but also how it reflects a healthy model of learning (for students) and teaching (for teachers) to create meaning by its users. To go with an analogy, good product designers worry not only about designing a well functioning product inside a box, but also plan ways for the users to interact with the product by preparing carrying handles on the box and by designing user guides. Similarly, good textbooks not only present mathematical knowledge, but also suggest ways for students and teachers how to interact with it and construct meaning from it. From this perspective, Singapore book reflects a simple yet powerful model for its users by suggesting an implicit model of student learning; that is “understand, share, practice, solve problems, reflect and summarize, and solve more challenging problems.” This implicit model is engineered by explicit structural elements within the textbook. It is accomplished by simplicity and explicit organization. Turkish books reflect a different route centered around multiple activities, solved examples, applications and problem solving. Although Turkish book emphasizes student engagement through activities, the whole process of learning seemed to be designed with a higher level of precision in the Singapore book. American book on the other hand reflects an assumption of “more is better” with a crowded design both in text density, number of pages and the number of topics covered. However, the engineering of the interaction with the textbook by its users seemed to be somewhat crudely developed compared to the other two textbooks.

The different design features in the three textbooks reflect a complex list of decisions and choices made while writing a textbook. These choices and the interactions among these choices can be interpreted and perhaps hypotheses can be formed as to their effectiveness. Perspectives such as Rezat’s (2006) model of textbook use and Weinberg and Wiesner’s (2011) reader oriented textbook theory can be useful in this regard. However, the outcome of these choices, hypotheses formed about them and the resultant design of mathematics textbooks for their ability to facilitate student learning and motivation has to be evaluated by empirical research (e.g., Morrison, Ross, & O’Dell, 1988).

References/Kaynakça


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