Mathematical Problem-Solving in Action: Teachers' Strategies and Approaches in the Classroom

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Abstract

The development of students’ mathematical problem-solving skills is contingent upon the approaches and methods employed by primary school teachers. This research endeavors to scrutinize the effectiveness of primary school teachers in their roles within the problem-solving process, with particular attention directed toward their inquiry techniques, instructional approaches, and responses to student errors. Employing as a explanatory mixed-design research, firstly quantitative data were obtained from the primary school teachers (N=116) and analyzed by applying the questionnaire form developed by the researchers; additionally, qualitative data were obtained by conducting focus group interviews with a subset of participants (N=6) based on the information obtained from these data. Thereby, teachers get opportunity to share their ideas in the process. Second phase of the research aimed to develop the active roles of the participants in the problem-solving process. The research outcomes underscore that teachers frequently gravitate toward questioning techniques and teaching methods that promote a nuanced comprehension of the problem. There also appears to be an underutilization of inquiries pertaining to prediction,
generalization, and collaborative group work. Furthermore, the research unveils an array of strategies employed by teachers to rectify student errors. These encompass commonly employed techniques such as question-answer, exemplification, and lecture responses, irrespective of the specific type of the question posed.


**Keywords:** Problem solving management, Primary school teachers, Teaching strategies, Teacher education

**Introduction**

We like to imagine that in the process of problem-solving, the folds of the human brain begin to *dance* while tingling. Why especially problem-solving? We live in a world where the advancement of information and communication technologies has brought about significant changes in both individual and social demands and characteristics (Erumit et al., 2019). Consequently, we are now required to possess a set of skills including problem-solving. In today's rapidly evolving educational landscape, the cultivation of problem-solving skills has emerged as a critical objective for educators worldwide. Problem-solving encompasses a range of cognitive *dancing* processes that empower individuals to analyse complex situations, generate innovative solutions, and make informed decisions. As societal challenges have been increasingly intricated, the ability to solve problems effectively has become essential for personal and professional success (Star & Strickland, 2008) with its role in increasing self-confidence and fostering creative and independent thinking (Guneri Yoyen et al., 2017; Sahin, 2004). Thus, problem-solving does not only serve as an objective within mathematics education but also holds a significant place in our lives.

Mathematical problem solving skill, which is a critical element of mathematical literacy, is an important skill in the learning and development of mathematics. It can be acquired and enhanced from the early stages of development (Miller & Nunn, 2001). Considering problem-solving as a learnable skill, the first essential requirement in solving problems is to have knowledge of the problem-solving process (Sahin, 2004). Within the process of mathematical problem-solving, Polya's (1957) stages of problem-solving are commonly used as a general framework. These stages encompass four steps: (1) understanding the problem, (2) devising a plan, (3) carrying out the plan, and (4) looking back. The initial step involves comprehending the problem and determining the given information and desired outcome. Subsequently, a plan tailored to the problem's structure is devised, followed by its execution. Evaluating the solution
and giving feedback entails discussing aspects such as accuracy, appropriateness, and alternative solution paths (Altun et al., 2007; Yilmaz, 2019). It would be beneficial for students if teachers clearly and explicitly presented these steps (Baroody & Wilkins, 1999). A clear understanding of the steps would support students’ problem solving skills as a whole. To enable students to acquire mathematical problem-solving skills, it is crucial for teachers to integrate appropriate problem-solving steps into their mathematics teaching. While the acquisition of four operational skills was deemed sufficient in the past, today it has become both a necessity and a prerequisite for developing higher cognitive skills (Guven & Ozcelik, 2017). As a challenging issue of the education system, teachers often tend to focus more on showing students how to perform mathematical procedures rather than enabling them to understand new concepts on their own (Burns & Lash, 1988). This emphasis on rote memorization-based education may indicate a prevalence of individuals in society who have not developed problem-solving skills, rather than fostering the growth of individuals who excel in problem-solving. Given the importance of problem-solving within mathematics instruction, teachers must possess comprehensive professional knowledge encompassing this area (Baki, 2012). Teachers should not only have a sufficient understanding of the subject matter but also be able to demonstrate their teaching skills (Gozel, 2016). This is necessary for achieving the desired success in mathematics teaching practices.

Teachers’ role within the multifaceted process of developing problem-solving skills cannot be ignored. Especially, primary school teachers hold a significant role in shaping the learning environment and engaging students in activities that foster problem-solving (Sahin, 2004; Soylu & Soylu, 2006; Orgovanyi-Gajdos, 2016). They serve not only as conveyors of subject-specific knowledge but also as facilitators, guides, and mentors, nurturing the cognitive abilities of primary school students (Silver & Stein, 1996; OECD, 2019). While primary school teachers play a pivotal role in fostering problem-solving skills through various teaching strategies and approaches; they create an inclusive and supportive classroom climate that encourages students to take risks, explore diverse solutions, and learn from mistakes as well. By fostering a safe and non-judgmental environment, teachers empower students to develop the confidence and resilience required for effective problem-solving (Vosniadou, 2019). The comprehensive utilisation of teaching techniques that actively engage students in problem-solving tasks fosters critical thinking, collaboration, and creativity (Hmelo-Silver, 2004; Jonassen, 2000). In
conclusion, the application of traditional and contemporary approaches significantly enhances students' problem-solving abilities.

Upon reviewing the relevant literature, it becomes evident that a conspicuous lacuna exists in the realm of scholarly investigations pertaining to problem-solving within the domain of primary school teachers. Existing research primarily explores the problem-solving process of mathematics teachers and student teachers (Avcu, 2012), their performance (Gumus & Umay, 2017), employed strategies (Gokkurt-Ozdemir et al., 2018; Gurbuz & Guder, 2016), solution methods (Pehlivan, 2011), and their skills and beliefs on problem solving (Pekgoz, 2020). Furthermore, research examines the mathematical problem-solving process of primary school teachers. While Altun et al. (2007) investigated the non-routine problem-solving skills and thoughts of prospective primary school teachers; Evrekli, Inel and Turkmen (2011) explored the problem-solving skills of teacher candidates; Aylar (2017) investigated the pedagogical content knowledge of teacher candidates regarding problem-solving; Yilmaz (2019) examined the problem-solving process of teacher candidates, and Dindyal et al. (2021) explored the effect of primary school teachers' mathematical problem-solving knowledge on students problem-solving performance. Ozrecberoglu and Caganaga (2017) present a research which is closest to the framework of this study; however they focus on mathematics teachers and classroom management. For this reason, in this study, it is aimed to examine the active roles of primary school teachers and to reveal how they manage in the mathematical problem-solving process.

To this end, our study is guided by the following research questions:

1. How do primary school teachers perceive the mathematical problem solving process?
2. How do primary school teachers manage the mathematical problem solving process?
3. How do primary school teachers find a solution in the mathematical problem solving process?
4. Which solution method do teachers prefer to correct student errors?

**Method**

**Research Design and Sample**

This study analysed the role and management of primary school teachers in the problem-solving process. For this purpose, explanatory mixed-method research (Creswell, 2003) was preferred. The participants in the study were selected by random sampling method (Yildirim & Simsek,
and consisted of 116 primary school teachers. Quantitative data were obtained by applying the questionnaire form developed by the researchers, while qualitative data were obtained by conducting focus group interviews with a subset of participants ($N=6$).

**Research Instruments and Procedure**

The questionnaire form consisting of both open-ended and rating questions (see Appendix) was used, and sent to the participants electronically. The form was created by making use of the problem-solving studies in the literature, and to ensure content validity, a pilot study was conducted with five teachers who were not included in the original research data. As a rating question, the teachers were asked which guiding questions they directed to the students during the problem-solving process, how often, and with which method they carried out the problem-solving process. The open-ended questions were based on the teachers' views on the concept of maths problems. Finally, the questions asked in TIMSS (2011) and solved incorrectly by the students were shown to the teachers. They were asked how they would correct such incorrect solutions.

The second phase of the research was conducted qualitatively with the participation of six teachers interviews. Teachers were first given a 4-hour training on managing and supporting the problem-solving process, and they were received mathematics problems developed by the researchers to be applied in the classroom by supporting students with the questions and methods in the form. Teachers observed the students while applying the problems and shared these recordings with the researchers during the evaluation process. Finally, a focus group interview was conducted with these teachers, and the implementation process was discussed. Conducting concurrent interviews with a group of participants fosters an environment conducive to uninhibited expression of opinions. Group dynamics further facilitate in-depth exploration of participants' perspectives and viewpoints, as established in scholarly literature (Merriam, 2013; Yildirim & Simsek, 2013). Focus group interviews, which typically consist of 4 to 12 participants (Edmunds, 2000; Gibbs, 1997; Kitzinger, 1995; MacIntosh, 1993), utilize open-ended questions in a conversational style (Bas et al., 2008; Krueger, 2014). In this case, 6 primary school teachers were invited to share their perceptions which led the collected data undergo meticulous word-by-word analysis. The content analysis method is then employed to categorize the data.
Data Analysis

For the analysis of the data, the responses in the form consisting of rating questions were descriptively expressed as frequency and percentage. The qualitative data obtained in the study were analysed by content analysis. The main feature of qualitative data analysis is the process of coding by dividing the data into small parts (Yıldırım & Şimşek, 2016). To reach consistent codes and themes, the researchers completed the process by convincing each other and getting a common truth. Collecting data in the long term, obtaining supervision permission from the participants, voluntary participation option, obtaining expert opinions on data collection tools and process, and obtaining ethical committee approval are accepted as factors that increase the validity and reliability of the research (Creswell, 2003). It was aimed to obtain strong and convincing results by collecting both qualitative and quantitative data as well as by providing the options consolidating the validity and reliability.

Results

How Do Primary School Teachers Perceive The Mathematical Problem Solving Process?

Teachers were asked to describe their perception of problem-solving as a term. Their answers were summed in Table 1. According to the table, primary school teachers perceive problem-solving as thinking skills, reflecting real life, arithmetical knowledge, understanding the question, modeling and representation, and applying problem-solving steps and reasoning. The responses collected under the theme of thinking skills consist of abstract thinking and analytical thinking. The other response includes the reactions of finding solutions to uncertainty and complexity.

Table 1

Teachers’ Problem-Solving Descriptions

<table>
<thead>
<tr>
<th>Themes</th>
<th>Frequencies (f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thinking skills</td>
<td>60</td>
</tr>
<tr>
<td>Reflecting real life</td>
<td>32</td>
</tr>
<tr>
<td>Arithmetic</td>
<td>9</td>
</tr>
</tbody>
</table>
How Do Primary School Teachers Manage The Mathematical Problem Solving Process?

It was asked two different scaled questions to reveal how teachers manage the problem-solving process. The first one elicits questions teachers ask (Table 2), while the latter seek the methods they use (Table 3). According to Table 2, while most teachers frequently asked questions 2, 3, and 4 to the students during problem-solving, they asked questions 8, 9, 10, and 11 less often. It is also revealed that teachers frequently asked predictive questions as their years of seniority increased. As a result of the interviews conducted with teachers, the following statements of the teachers support these findings: "Students do not care about prediction. Prediction is equivalent to making a guess." "Students have difficulty in transferring their solutions to one another." "It is useful to move from the child's immediate environment and make it more understandable."

Table 2

Questions Teachers Ask Students During Problem Solving Process

<table>
<thead>
<tr>
<th>Questions</th>
<th>Never</th>
<th>Sometimes</th>
<th>Usually</th>
<th>Always</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Do you know the meaning of the terms in problem?</td>
<td>3</td>
<td>3</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>2. What did you understand about the problem?</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3. What is given in the problem?</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>
4. What are the requirements in the problem? 1 1 2 2 20 17 93 80
5. Can you explain the problem with your words? 2 2 18 16 48 41 48 41
6. Can you draw figure/table explaining the problem? 7 6 14 12 43 37 52 45
7. How do you plan to solve the problem? 2 2 8 7 46 40 60 52
8. Can you predict the answer to the problem? 8 7 27 23 50 43 31 27
9. Can you compare your result with your prediction? 7 6 35 30 38 33 36 31
10. Have you solved any similar problem before? 9 8 33 28 44 38 30 26
11. Can you apply the solution of the similar problem for this problem? 8 7 22 19 53 46 33 28
12. What do you think about the solution? 4 3 17 15 40 34 55 48
13. Have you verified the result? 3 3 16 14 52 45 45 39
14. Why did you solve the problem this way? 3 3 22 19 51 44 40 35
15. Can you solve other problems in this way? 6 5 30 26 51 44 29 25
16. Can you construct a similar problem yourself? 7 6 23 20 53 46 33 38

To explain the research question more clearly, teachers were asked to write down which methods they used in the problem-solving process and how often. Table 3 presents that teachers frequently use methods 2, 6, 7, 8, and 9 in the problem-solving process. However, they use methods 1 and 4 less often.
### Table 3

**Methods Teachers Use During Problem Solving Process**

<table>
<thead>
<tr>
<th>How often teachers use the methods</th>
<th>Never</th>
<th>Sometimes</th>
<th>Usually</th>
<th>Always</th>
</tr>
</thead>
<tbody>
<tr>
<td>Methods</td>
<td></td>
<td>Frequency</td>
<td>Percentage</td>
<td></td>
</tr>
<tr>
<td>1. I make students work in small groups/ with a peer.</td>
<td>5</td>
<td>41</td>
<td>35</td>
<td>43</td>
</tr>
<tr>
<td>2. I support students to solve the problem using objects.</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>42</td>
</tr>
<tr>
<td>3. I ask students to solve problems individually.</td>
<td>0</td>
<td>11</td>
<td>10</td>
<td>53</td>
</tr>
<tr>
<td>4. I encourage students to use Web 2.0 technology tools.</td>
<td>18</td>
<td>36</td>
<td>31</td>
<td>40</td>
</tr>
<tr>
<td>5. I ask students to solve problem with different method.</td>
<td>0</td>
<td>19</td>
<td>16</td>
<td>58</td>
</tr>
<tr>
<td>6. I help students to understand mathematical language.</td>
<td>0</td>
<td>4</td>
<td>3</td>
<td>31</td>
</tr>
<tr>
<td>7. I ask questions to help students realise their mistakes.</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>27</td>
</tr>
<tr>
<td>8. I ask students to explain the solution to the class.</td>
<td>1</td>
<td>8</td>
<td>7</td>
<td>39</td>
</tr>
<tr>
<td>9. I encourage students to solve problem with models.</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>41</td>
</tr>
</tbody>
</table>
How Do Primary School Teachers Find A Solution In The Problem Solving Process?

According to Table 4, primary school teachers use various methods when their students struggle with problem-solving. According to the table, the most frequently given answers are comprehension, giving hints about the problem, and concretisation and modeling. During the interviews, one teacher said: "Group work was very effective, but it was difficult to form heterogeneous groups." As a result of these interviews, teachers stated that they understood the importance of guiding students by asking them questions during problem-solving. In conclusion, it is seen that the information obtained from the interviews with the teachers and the information obtained from the questionnaire form support each other.

Table 4

Teachers' Methods of Guiding Students

<table>
<thead>
<tr>
<th>Themes</th>
<th>Frequencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comprehension studies</td>
<td>41</td>
</tr>
<tr>
<td>Giving a hint</td>
<td>31</td>
</tr>
<tr>
<td>Concretisation and modeling</td>
<td>25</td>
</tr>
<tr>
<td>Supporting discussion</td>
<td>21</td>
</tr>
<tr>
<td>Example&amp; method diversification</td>
<td>17</td>
</tr>
<tr>
<td>Problem simplification</td>
<td>14</td>
</tr>
<tr>
<td>Part to whole</td>
<td>11</td>
</tr>
<tr>
<td>Motivation</td>
<td>8</td>
</tr>
<tr>
<td>Relate to real life</td>
<td>7</td>
</tr>
<tr>
<td>Dramatization</td>
<td>5</td>
</tr>
</tbody>
</table>

Which solution method do teachers prefer to correct student errors?

Teachers were asked how they can guide students to correct their errors and shown in Table 5. According to the table, teachers use various methods to correct student errors. Among these
responses, question-answer, exemplification, and lecture responses are frequently preferred, regardless of the question type.

Table 5

*Teachers’ Methods of Correcting Students’ Mistakes*

<table>
<thead>
<tr>
<th>Themes</th>
<th>Q1(f)</th>
<th>Q2(f)</th>
<th>Q3(f)</th>
<th>Q4(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Repeating</td>
<td>7</td>
<td>20</td>
<td>6</td>
<td>17</td>
</tr>
<tr>
<td>Explaining</td>
<td>20</td>
<td>13</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>Illustration/simulation</td>
<td>21</td>
<td>26</td>
<td>14</td>
<td>16</td>
</tr>
<tr>
<td>Question and answer</td>
<td>31</td>
<td>21</td>
<td>17</td>
<td>15</td>
</tr>
<tr>
<td>Guiding</td>
<td>16</td>
<td>20</td>
<td>7</td>
<td>12</td>
</tr>
<tr>
<td>Giving a hint</td>
<td>9</td>
<td>15</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>Solving by themselves</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>Concretisation and modeling</td>
<td>12</td>
<td>6</td>
<td>39</td>
<td>8</td>
</tr>
<tr>
<td>Reminding</td>
<td>13</td>
<td>7</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

**Discussion**

In this study, participating primary school teachers stated that when students had difficulty while solving problems, they used various strategies and tactics that would help students understand the subject. In addition, attempts to understand the problem, the participant teachers generally preferred techniques such as presenting insights about the problem, encouraging conversation and peer teaching, concretizing, question-answer and using different representations. Other strategies included discussion, giving examples, breaking down and simplifying problems, moving from concrete to abstract solutions, arousing and developing
positive beliefs in students, connecting the problem to real-life scenarios, and involving them in dramatic activities. Participants mostly stated that students lacked in using mathematical expressions correctly, they were too impatient in the problem-solving process, and focused on solving as many questions as possible. This response differed in the two separate phases of this study. In the first stage, teachers often stated that they helped students understand the language of mathematics, whereas in the second stage, they emphasized that students had difficulties in the process of applying mathematical expressions. At the end of the questions in the first stage of the research, the participants were asked for their opinions about they would correct students’ mistakes or wrong answers for the problems in TIMSS (2011). The participants stated that they often prefer using concrete objects during the problem solving process, help students to understand the language of mathematics, ask guiding questions, explain the solution to the whole class, and facilitate problem solving through representation. On the other hand, symbolization and retelling were the least-preferred strategies. In addition, the fact that digitalization is not preferred at all can be interpreted as a result of the material or the fact that the participants do not know how to use the technology while teaching mathematics.

Only 30% of the participants preferred the concretization strategy in the questions with pictures which can be easily brought into the classroom environment. There will be no effort made to develop a learning process that uses mathematical connection abilities to solve mathematical issues if teachers do not integrate the learning materials into the students' lives (Kenedi et al, 2019). Therefore, the percentage of the teachers who used materials is poor in this respect. The teachers helped the students to shape their conceptual understanding according to their answers for the given questions about concept knowledge and the application of it.

Repetition, reminders and giving examples are among the most effective ways to build a better conceptual understanding for the students who forget concepts or make mistakes. For instance, most participants generally asked, "What did you understand from the problem? What are the given information and the desired results in the problem?" and benefited from such questions. However, questions like “Can you guess the answer? Can you compare your guess with the result you got? Have you solved a similar problem before? Can you apply the solution you got from a similar problem to this problem?” were not used as much as the previous ones. These results are consistent with the results of a study on the effect of the problem-solving strategies in the classrooms are more effective than scientific approaches to students’ abilities in
communication, creativity, problem-solving, and mathematical reasoning (Tambunan, 2019). In light of their results, it may be appropriate to discuss teachers' teaching models because it was seen both Tambunan’s (2019) and the current study that teachers generally preferred the same strategies for all questions when appropriate answers were obtained from four different learning outcomes. For example, a teacher tried to guide students through narration followed the same lesson when asked about guessing or geometric shapes. The results obtained from the interviews, which constitute the second stage of the research, also confirm the teachers’ answers in the first stage. Primary school teachers had the opportunity to evaluate both their students and themselves within the scope of these practices. According to the explanations based on teachers' self-evaluations during the implementation process, modeling and group work in the problem solving process are very effective in terms of students' understanding of the problem. Also, in the context of interviews carried out within the scope of the research, it is interpreted as promising results that teachers emphasize the concept of estimation and express its positive effect.

Conclusion

Today's changing educational landscape requires the cultivation of problem-solving skills as a critical objective for educators. In the development of problem solving skill, teachers’ perception, the way teachers express the concept of mathematical problem solving, handle the process in the classroom and guide their students are important. The methods and approaches that primary school teachers follow are worth to investigate since students encounter this sophisticated skill at an early age through their primary school teachers. The strategies teachers use during this process and how they manage it were examined in this study. The way teachers characterize the problem-solving process in the classroom is crucial for gaining the basic problem solving skills. As the findings of this study showed once again, since problem solving process is multidimensional, following a suitable strategy for the structure of each problem, and selecting appropriate methods and techniques according to the academic status of students are the key factors of developing students’ mathematical problem solving skills.

Recommendations

Studies on reading comprehension skills, which almost all of teachers participating in the research complain about, should be given priority. In addition, permanent learning should be
provided by creating a connection between new situations and previous knowledge and should be reinforced with applications in similar problem situations. Considering that each student's learning process is different from each other, different solution methods should be presented to students during the learning process to improve their problem solving skills, and hence, students can choose the most suitable method for a problem and apply it to similar problem types.

References


**Appendix**

The survey link can be accessed from QR code below.