INTRODUCING THE CONCEPT OF ENERGY: EDUCATIONAL AND CONCEPTUAL CONSIDERATIONS BASED ON THE HISTORY OF PHYSICS

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Abstract

In this research, an educational approach to the concept of energy is proposed. It is based on the history of physics. In 1854 Hermann Helmholtz gave a popular lecture on the recent discovery that energy is conserved. Such lecture is used as a guide to introduce the pupils within several nuances of this concept. Not much mathematics is used, so Helmholtz’s work, with several additions proposed here, is an excellent guide to understanding, from a qualitative point of view, the reasons that led scientists to establish the principle of conservation of energy. At the same time, it allows us to grasp two other concepts which are fundamental in reference to energy: work and heat. This panorama will be drawn in the first section. In the second one, some more mathematical and physical details on the teaching of energy in mechanics and thermodynamics will be offered. Finally, in the Conclusion, the interdisciplinary value of a historical approach to physics education will be pointed out.

Keywords: energy conservation, Helmholtz, physics history, physics education, science education

Introduction

Energy is probably the most important concept in physics because it pervades all the branches of this discipline. One speaks of mechanical energy, gravitational energy, thermal energy, electric energy, chemical energy, atomic energy, and rest energy. The most common definition presents energy as the physical quantity which measures the capability of a body to perform work. However, this definition is not universally accepted because energy has physical manifestations which cannot be completely reduced to the capability of a body or of a system to perform work. Therefore, probably a better definition of energy is the one given by the English Wikipedia: “Energy is the quantitative property that is transferred to a body or to a physical system, recognizable in the performance of work and in the form of heat and light”. That the one of energy is a problematic concept is illustrated by the fact itself that not all physicists agree on the definition of this notion. This is perhaps a unique case with regard to fundamental physical quantities. Some illustrious physicists, for example, Richard Feynman (1918-1988), prefer to define energy only through its property of being conserved without adding further specifications:
There is a fact, or if you wish, a law, governing all natural phenomena that are known to date. There is no known exception to this law—it is exact so far as we know. The law is called the conservation of energy. It states that there is a certain quantity, which we call energy, that does not change in the manifold changes which nature undergoes. That is a most abstract idea because it is a mathematical principle; it says that there is a numerical quantity which does not change when something happens. It is not a description of a mechanism, or anything concrete; it is just a strange fact that we can calculate some number and when we finish watching nature go through her tricks and calculate the number again, it is the same. (Feynman, Leighton, Sands 1963, p. 4-1).

This minimalist and abstract approach to the notion of energy is probably suitable to introduce operatively this concept while dealing with a course in physics at the university. For in that context, it is appropriate to introduce the concepts and their physical relations without necessarily posing a priori the question what a concept is. The students will understand its nature through its use in the different branches of physics. Besides pointing out that the great majority of the other physical concepts have, instead, a precise definition through one formula, it should also be remarked that the way in which Feynman introduces energy is too abstract for the pupils attending the last three years of the high school (aged 17-19), to whom this paper is dedicated.

Therefore, an educational itinerary in two steps is here proposed.

First step: a general idea of the concept of energy will be given. The best way to perform this task consists in explaining how the principle of the conservation of energy was reached in the history of physics. Such a story will also provide the learners with an intuitive, but sufficiently precise, idea of what energy is, why it was introduced in physics and how it is used. I will not follow the whole history of the concept of energy because, obviously, this would require a whole book, which is far beyond the purpose of this article. Instead, the work of Hermann Helmholtz (1821-1894) *Ueber die Wechselwirkung der Naturkräfte und die darauf bezüglichen neuesten Ermittelungen der Physik* ("On the interaction of the natural forces and the most recent determinations of physics connected to it", Helmholtz 1854) will be used as a guide in my educational proposal. Helmholtz, jointly with Robert Mayer (1814-1878), James Prescott Joule (1818-1889) and Ludvig August Colding (1815-1888), was one of the discoverers of the principle of energy conservation and, basically, of the modern concept of energy. The work mentioned above is a popularization as well as a succinct history concerning the discovery of this principle. It was written in 1854, whereas the scientific contributions on this topic by Mayer, Joule, Colding and Helmholtz himself date to the decade 1840-1850 (Mayer 1842, 1845; Joule 1845, 1847, 1850; for the works of Colding, written in Danish, see Kuhn 1977, pp. 66-103, Caneva 1998; Helmholtz 1847). This text is an excellent guide to enter all the nuances of the notion of energy which could be problematic for the learners. It is clear and has the merit to explain the concepts without using any mathematical apparatus, as far as this is possible. Therefore, it is ideal for an initial approach. I suggest dedicating six hours to this introduction because it is crucial that the pupils reach a clear, though qualitative, idea, of what energy is.

Second step: it consists in giving a quantitative determination to energy, to realize how it is used in the different branches of physics and to understand that this notion is the one which allows connecting such branches in a unitary vision. The best approach is to start with mechanics where the picture is easier and clearer. The notions of work,
kinetic energy and potential energy will be introduced as well as the principle of energy conservation for the conservative forces. After that, energetic considerations on the various motions, also including the harmonic one, should be developed to conclude with the concept of energy within gravity theory. This research will focus only on the principles. Therefore, it will not deal with the application of energetic considerations to the various motions.

The next step will be the introduction of energy in thermodynamics. Here, there is a conceptually difficult step which is represented by the notion of heat. It is crucial to offer a clear explanation of this concept because it is a bridge between mechanics and thermodynamics and allows to fully understand the value of the principle of energy conservation. If energy is introduced in an appropriate manner, the pupils should be ready to understand the seminal role played by another notion connected to energy, that of entropy. Thermodynamics is definitely the key to fully understanding the concept of energy and a particular care should be devoted to this section of physics.

Finally, electricity and electromagnetism should be introduced. Here energy should be connected with another crucial concept of physics, in fact, the most important one, at least in contemporary physics, that of field. It is clear that the notion of field should be introduced while dealing with gravity, but, as Einstein and Infeld suggest (Einstein-Infeld 1938, pp. 125-152), electricity and, afterwards, electromagnetism represent areas of physics in which the importance of the field concept shines through more clearly than in Newtonian gravitational theory. In spite of the fact that electricity and electromagnetism are fundamental sections of physics, I will not deal with them because mechanics and thermodynamics are sufficient to explain the itinerary here developed.

Two remarks are necessary: 1) I restrict my considerations to the teaching of classical physics, thus excluding relativity and quantum mechanics; 2) on the teaching of the energy concept a huge and specialized literature exists (see, only to give examples of significant papers, Arons, 1999; Bächtold, 2017; Bächtold & Munier, 2019; Bécu-Robinault & Tiberghien, 1998; De Berg, 1997; Demkanin, 2020; Duit, 1981, 1987; Goldring & Osborne, 1994; Kaper & Goedhart, 2002; Koliopoulos & Ravanis, 2001; Kubsch et al., 2021; Lehrman, 1973; Mai et al., 2021; Sexl, 1981; Solomon, 1985; Van Heuvelen & Zou, 2001; Van Roon et al., 1994; Warren 1982).

I am a historian of science and mathematics, not an expert in science education. Therefore, I have no claim to replace the profound debate on this topic with my considerations. I only hope that some of the ideas here expounded can be useful in an educational context.

Energy and Energy Conservation in the Story Told by Helmholtz

Helmholtz tells that during the 17th and the 18th century, there were many attempts to create machines and automatons which produced a perpetual motion. This means that the machine is self-powered and, in addition, performs any activity that man desires. There was no known physical principle which, a priori, prevented from constructing such a machine. However, all the attempts carried out by the most skilled inventors failed, so that in 1775 the Paris Academy resolved to no longer consider any proposal or project aimed at realising perpetual motion. However, these failures as well as the desire to determine a physical quantity which expressed what exactly man requires from
a machine led the physicists to introduce one of the fundamental notions of their entire science: that of work. Consider a water wheel as that proposed in Fig. 1B, which is activated by water falling from above.

**Figure 1A**
*An Undershot Water Wheel. The Water Under the Wheel is Made to Move, so That It, in Turn, Sets the Wheel in Motion*

**Figure 1B**
*An Overshot Water Wheel. Water Falls from above Onto the Wheel Blades and Sets Them in Motion*

The wheel axle can be fitted with small protrusions that catch the handles of heavy hammers as they rotate to lift them up and drop them down. When the hammers fall, they strike a metal mass beneath them and transform such a mass. Ergo, the work of the machine consists in lifting a weight. Therefore, first of all, the machine has to win the weight of hammer mass $m$, that is $mg$. This means that, if the weight is doubled, the work also is. On the other hand, the effectiveness of the hammer blow on the metal mass depends not only on its weight, but also on the height $h$ from which it falls and is proportional to such height. It is easy to understand that the expounded reasoning is also valid if the displacement is not perpendicular and if the force is not that of gravity. It holds for every displacement and for every force. Thus, the physicists had the idea to offer a quantitative determination to the term work and to define it as the product of the force by the displacement of the body. The first one to clearly define the concept of work was the French physicist Gaspar-Gustave de Coriolis (1792-1843, Coriolis 1829). It should be pointed out that a force can produce work only if it has a component tangential to the displacement, if its direction is perpendicular to the displacement the force cannot produce any work. Therefore, if $\theta$ is the angle between the direction of the force and that of the displacement the infinitesimal work $dW$ is defined as the product of the force $F$ by the displacement $ds$ by the cosine of the angle $\theta$ through the formula $dW=Fds\cos\theta$. Using the concept of scalar product, which was not yet completely defined when Helmholtz wrote, it is $dW=F\cdot ds$. It is now necessary to remark that the three Newtonian principles teach us that in order to lift a hammer of mass $m$ at the height $h$, it is at least necessary to use an equivalent mass of water which falls from the height $h$. Experience shows us that, in almost every concrete case, the mass of the water has to be bigger than $m$ or the
height bigger than \( h \).

So far, we have analysed the work necessary to lift the hammer to a height \( h \). But now, let us wonder another question: why does the hammer modify the metallic mass if the hammer itself moves and not if it is at rest? The answer is rather obvious: work has also to be a function of velocity. This is conspicuous, Helmholtz claims, in the case of the projectiles. They are inoffensive if they are at rest, but lethal when moving quickly. The movement of a mass considered as a quantity able to produce work was called *living force* (*vis viva*). The notion of *vis viva* had already been used by Huygens, Leibniz and the Bernoullis so that, unlike the concept of work, it had already an important role in physics. Nowadays (apart from a factor \( \frac{1}{2} \)) we call this quantity *kinetic energy*. The novelty of the years 1830-1850 is the strong connection between living force and work.

If our hammer would fall on a very elastic lamina, in the best circumstances, it would bounce to the same height (not higher) from which it is fallen. This means that the living force can produce the same quantity of work as that from which it was generated. Numerous examples of communication of *vis viva* to produce work can be given: a man winding a watch communicates to its mechanism a living force that the watch returns over the next twenty-four hours to overcome the friction of its wheels and air. Work is, thence, a way to communicate a living force between two physical systems. Such a living force can be communicated to produce another work. But it never happens that in these processes the living force is bigger than the work through which it has been communicated.

The mathematical theory confirms what our examples and our reasoning have shown: machines do not produce any impulsive force, but simply communicate the kinetic energy given to them through work, which can, thus, be seen as the energy exchanged between two systems when a displacement takes place. Machines are, ergo, mechanisms which transform energy. When this law was established and proved, it was evident a *perpetuum mobile* to be impossible: if the received energy is used to produce work, the machine loses a part of its energy and progressively will stop.

Now I add a consideration which is not present in Helmholtz’s story, but which can be useful for the students. We have seen that work is expressed as the scalar product \( F \cdot ds \), which, in the case in which \( F \) is gravity force, can be written as \( mgh \). This quantity can be transformed into kinetic energy. When a body of mass \( m \) is at the height \( h \), but is at rest, it produces no work. However, as soon as the gravitational force acts on the body, work is produced. There is the potentiality to produce work. When the movement begins and the body reaches the soil, work \( mgh \) is carried out. Therefore, it is only natural to define a function which indicates the work performed on the body when it passes from the height \( h \) to the soil. This function of the coordinates is called *potential energy* and the difference between its initial and final values indicates the work performed on the body. On the other hand, if the entire kinetic energy of the source is transformed into the kinetic energy of a machine, the work performed by the machine is equal to the difference between its final and initial kinetic energy. This means that the sum of the initial potential and kinetic energy is equal to the sum of the final potential and kinetic energy. Furthermore, work can be interpreted as the way in which energy is transported from a system in the state \( A \) to the system itself in the state \( B \) or between two different systems. This means that mechanical energy (the sum of kinetics and potential energy) is

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conserved. Are things so plain? Let us come back to Helmholtz.

Until now only motive forces have been considered, but in nature there are many phenomena which are not directly connected to motive forces: let us think of heat, electricity, magnetism, light, and chemical forces. They have different connections with the motive forces. However, in any natural process, there are also mechanical effects. This means that mechanical work can be also produced through not exclusively mechanical processes. Let us think of an easy example: If a container with gas is closed by a moving piston carrying weights when the gas is heated it expands because of the increased kinetic energy of its particles and the piston with the weights rises (Fig. 2).

**Figure 2**  
*Visualization of the Mechanism Presented in the Running Text*

![Visualization of the Mechanism](image-url)

Here, heat generates work. Therefore, could perhaps a *perpetuum mobile* be created using non-mechanical forces? Is this possible?

On the other hand, it is well known that any motion on a rough surface produces heat. Therefore, what is exactly the relation between heat and movement, heat and work?

All the attempts to construct a *perpetuum mobile* based on heat failed. Therefore, the physicists changed their perspective and began to wonder why neither this kind of *perpetuum mobile* can exist. The first one who offered a satisfying answer to this question was Robert Mayer in 1842. He was a physician, not a physicist, and, while working in Giava he noticed that the windswept waves were hotter than the water of the calm sea. His attention was also captured by another apparently strange and interesting phenomenon: Lavoisier understood that the animal heat is the result of a combustion process. On this basis, he realized that the change in blood colour as it passes from the arteries to the veins is the sign of the oxidations of tissues. In order to maintain the body’s temperature, the production of heat must be associated with a loss of heat. This loss depends on the environment temperature. Therefore, the production of heat also depends on temperature and, hence, the oxidative processes depend on temperature. This means that such processes diminish in hot climates. Ergo, in these climates venous blood and arterial blood should have more similar colours than in cold climates. This
was what Mayer saw: in the tropics, venous blood is less blue, i.e. less oxidised, than in Europe (see Cappelletti in Helmholtz 1967, note 4, pp. 223-225). Mayer wondered then how our organism produces heat and what the relation between our mechanical activity and the heat of our body is. At the same time, Colding and Joule arrived at conclusions analogous to Mayer’s as to the relation between heat and movement. Joule, in particular, was able to reach a precise determination through the following brilliant experiment, which can be summarized as follows: he considered a watertight container filled with water. Inside it were paddle-shaped wheels rotating on an axle (Fig. 3). On the outside, tied to two pulleys, were two weights that could descend in free fall. The apparatus was equipped with a thermometer. The weights had a well determined height and, therefore, a precise capacity to perform work, a potential energy. Their final kinetic energy was less than their potential energy. At the same time, the temperature of the water during the descent of the weights had increased. Joule then interpreted heat as a mechanical equivalent of work, i.e. a way of transferring energy. In this case, the potential energy of the weights had been transformed partly into kinetic energy and partly into heat energy. The experiment was repeated several times in different circumstances always giving the same results (Joule 1845, 1850).

Figure 3
The Device Used by Joule Here is Presented in Two Slightly Different Forms. The Explanation in the Running Text Refers to the Figure on the Left

Joule was, thus, able to determine the nature of heat: it is similar to that of work. Both of these magnitudes are a way of transferring energy and transforming it into different forms. Through this experiment and through other ones presented in further papers Joule was also able to determine the mechanical equivalent of heat. It was 4.155 J/cal (today we know it is 4.186 J/cal). Thanks to these experiments, Joule demonstrated that heat and mechanical work could be converted directly into each other, while keeping their overall value constant: in hydraulic and mechanical machines, friction transforms the lost mechanical power (work) into heat and, vice versa, in thermal machines, the mechanical effect produced (work) is derived from an equivalent amount of heat.

Joule’s discovery was crucial because most physicists believed that heat was a substance which passes from a hotter body to a colder one, something similar to humidity
which is water passing from a body whose water’s density is greater to a body whose water’s density is smaller. As a matter of fact, Joule’s experiments proved that heat is not a substance but a way of transferring energy. Joule began working on the concept of heat when he realised that a wire through which an electric current was passing became hot. If heat had been a substance, this should not have happened as the passage of heat should only have occurred in the presence of two bodies having different temperatures: i.e., no change in temperature should have been noticed. As a matter of fact, the idea of heat as a substance had already been challenged by the experiments of Benjamin Thompson (1753-1814), Count Rumford, conducted in the late 18th and early 19th centuries. Thompson had noticed that with friction an indefinite amount of heat could be generated without any apparent passage of heat flow. But if heat was not a substance, what was it? Joule, with his experiments, gave the answer: like work, it is a way of transforming and transporting energy.

The picture begins now to be clearer. There is a quantity which is conserved: energy. It has various forms. We have seen potential, kinetic and thermal. Mechanical energy is not conserved in every process because, if a process produces heat, a part of mechanical energy is lost through heat and becomes thermal energy. In most cases, it is impossible to re-transform completely such energy into kinetic energy and a part of it is lost in the environment, but it does not disappear. Simply it is not anymore usable to produce movement.

Let us now come back to Helmholtz: since heat is a form of energy transformation, this implies that no new energy can be created through heat and that, hence, neither a Perpetuum mobile of the second kind can be constructed.

It is paramount to point out that heat is produced in any phenomenon, not only in the mechanical ones: chemical bonds produce heat, the passage of current in a wire produces heat, and so on. This means that there is a chemical energy, an electric energy which will have specific peculiarities, but which are subject to the general law of conservation of energy.

Now there is a further important step addressed by Helmholtz: when is it possible to convert heat in mechanical work? The research of Sadi Carnot (1796-1832) published in 1824 and of Rudolf Clausius (1822-1888) in the period 1857-1877 established that this is possible only when heat passes from a hotter body to a colder one and, also in this case, the transformation of heat in mechanical work is only partial. The passage of heat from a hotter body to a colder one is a natural process. The opposite process cannot take place naturally. If a body cannot be further cooled, its heat is, so to speak, trapped. The thermal energy of the body can in no way be converted into mechanical, chemical or electrical energy. Therefore, as Helmholtz claims, if all bodies in nature had equal temperatures, it would be impossible to transform any part of their heat into work. That is, any transformation would be impossible. Hence, in the universe, there is a part of heat which is transformable and a part which is not. However, heat from warmer bodies tends to pass continuously into less warm bodies through conduction and radiation. That is, there is a tendency towards thermal equilibrium. In every movement, some mechanical energy is converted into heat through friction and collisions. The same happens in chemical and electrical processes. This means that the portion of heat that cannot be converted into work increases over time. When thermal equilibrium is reached, which
necessarily will happen, no more transformation will be possible in the universe.

Through a series of concatenated reasoning, we have led the students to understand, albeit almost only qualitatively, the concepts of energy, work, heat and the principle of conservation of energy. With the final considerations on thermal equilibrium, we came to the threshold of one of the most important and complex concepts in physics: that of entropy. It is true that energy is not created and not destroyed, it is only transformed, but it is transformed in a way that progressively the capability to do work is lost by a system. To introduce the concept of entropy, one might say, intuitively, that entropy measures the capacity of a system to perform work and the way in which it loses this capacity. Entropy tells us how far a system is from the equilibrium state. Objects in contact with different temperatures have low entropy. As the heat passes from the hotter body to the colder one, entropy increases until it reaches the maximum when the two bodies have the same temperature. At this point, there is no more heat transfer. In this situation, it is no longer possible to create work from heat. Energy is not disappeared, but it is lost in the environment and cannot be utilized. This means that the entropy of a system increases over time and only for completely isolated systems it is constant over time. However, there is a way to present entropy, which is connected to the one described, but is even more profound. In order to perform this task, we must abandon Helmholtz and turn to the work of the great Ludwig Boltzmann (1844-1906). He realized that entropy has to do with the number of ways in which the microscopic states of atoms and molecules in a system can be changed without changing the macroscopic properties of the system itself. Example: let us consider a box in which there is a certain number of gas atoms. They cannot be distinguished from each other. To simplify the situation as much as possible, suppose there are only six atoms at the beginning. Suppose that all the atoms are in the left side of the box. In how many ways can this configuration be realized? Obviously only in one way. Instead, how many configurations are possible in which five atoms are on the left part of the box and one atom is on the right part? An elementary reasoning proves that there are six configurations. With regard to the disposition 4-2, there are 15 configurations. An easy calculation shows that the biggest number of configurations is realized when the disposition of the atoms is three in the left side of the box and three in the right side. There are 20 of these configurations. Therefore, if one looks at the box at an arbitrary time, he has a high probability to see the disposition 3-3. Boltzmann found that entropy $S$ is given by the following formula $S = k \log W$, where $k$ is a constant and $W$ represents the number of possible microscopic configurations of a system which produce the same macroscopic state of the system. In our example the disposition 6-0 has entropy $S = k \log 1 = 0$, the disposition 5-1 has entropy $S = k \log 6$, the disposition 4-2 has entropy $S = k \log 15$ and the configuration 3-3 has entropy $S = k \log 20$. Obviously, the proposed example is unrealistic because the number of particles in any container is enormously bigger than six (for example in a room there is an average of $10^{26}$ molecules of air). When the number of particles increases (suppose it to be $2n$) the possibility to have the disposition $n-n$ (namely a uniform disposition) is incomparably bigger than any other disposition. This is the reason why the systems tend to have the most uniform possible disposition. Suppose now that in the left part of a box divided by a septum there is a hotter gas and in the left side a colder gas. What happens when one removes the septum? The particles, on the basis of the above reasoning, tend to reach a uniform distribution. This means that the left side will tend to become colder and the right side hotter, so that
a uniform distribution of temperature is reached. This is the reason why heat passes from hot bodies to cold bodies and not vice versa (for a good and elementary discussion of entropy from which the approach here proposed is drawn see Amedeo Balbi’s lesson on this subject. It is available on Youtube, see References). The opposite transition is not impossible, but is statistically so unlikely that it does not, in fact, occur in nature. Therefore, the systems tend progressively to lose their potentiality to perform work and tend to the thermal equilibrium. The universe, as a whole, seems, thence, destined to the so-called thermal dead.

**Quantitative Determination of Energy**

In the previous section, the general concept of energy has been explained in connection with the related notions of work and heat. The pupils should have understood that energy is a concept which pervades all the branches of physics and links them in a sole theoretical picture. This is the main idea behind this paper. However, when a quantitative determination of energy must be given, it is appropriate to consider energy in the single sections of physics. Such approach is more comfortable for the students and, basically, it is the traditional one. I will briefly analyse the situation in mechanics and thermodynamics, focusing, particularly, on the latter given its seminal importance for the topic here presented.

**Mechanics.** Let us recall that, given a force \( F \) and an infinitesimal displacement \( ds \), the infinitesimal work is defined as

\[
dW = F_T \, ds = m \frac{dv}{dt} \, ds = m dv \frac{ds}{dt} = mvdv
\]

where \( F_T \) indicates the component of \( F \) tangential to the displacement.

By integrating, it is possible to determine the total work necessary to move a particle from point \( A \) to point \( B \), so that

\[
W = \int_A^B F_T \, ds = \int_A^B m \frac{dv}{dt} \, dv = \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2
\]

where \( v_B \) indicates the speed in \( B \) and \( v_A \) that in \( A \). This formula is important because it indicates that the work developed by the force \( F \) between \( A \) and \( B \) does not depend either on the functional form of \( F \) or on the trajectory of the particle between \( A \) and \( B \), but only on its mass and on the half square of the initial and final velocity. By defining the quantity \( E_k = \frac{1}{2} m v^2 \) as **kinetic energy**, the explained reasoning shows that

\[
W = E_{K,B} - E_{K,A}
\]

This means that the **work performed on a particle is equal to the variation of its kinetic energy**. This result is also known as the theorem of living forces because, as previously clarified, in the past kinetic energy was called living force.
It is appropriate to stress that this proposition can also be obtained, though in a less precise manner, through reasoning which is independent from the use of integrals: since \( L = F \cdot s \) and \( F = ma \), it is \( L = m \cdot a \cdot s \). As the body is subject to a constant force, its motion is uniformly accelerated, so that from kinematics it is known that

\[
v_f^2 - v_i^2 = 2a \cdot s
\]

where \( v_f \) indicates the final speed of the body and \( v_i \) the initial one. Therefore, it is

\[
s = \frac{v_f^2 - v_i^2}{2a}
\]

so that

\[
L = m \cdot a \cdot s = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2
\]

The next concept which is necessary to introduce is that of a conservative force. A force is defined conservative if its dependence from the position \( r \) of the particle is such that work \( W \) can be expressed as the difference between the values considered in the initial and final points of a quantity \( E_p(r) \) which is called potential energy. It is a function of the particles’ coordinates. Therefore, if \( F \) is conservative, it is

\[
W = \int_A^B F \cdot dr = E_{p,B} - E_{p,A}
\]

Namely: work is equal to the difference between the potential energy in the initial point and in the final point. Thence, potential energy is a function of the coordinates such that the difference between its values in the initial and final positions is equal to the work performed on a particle to move it from the initial to the final point. This implies that the work performed by a conservative force is independent of the trajectory. Taking into account Equation 1) we have that

\[
E_{K,B} - E_{K,A} = E_{P,A} - E_{P,B}
\]

Namely

\[
E_{K,B} + E_{P,B} = E_{K,A} + E_{P,A}
\]

This means that mechanical energy is conserved in the case that all forces are conservative.

However, in nature there are many non-conservative forces: friction is an example. Sliding friction opposes displacement. Therefore, it is obvious that the work performed by friction does not depend only on the initial and final points of the trajectory traversed by a body, but also on the length of such a trajectory. The longer the trajectory, the greater the work done by the friction forces. In such conditions, mechanical energy is not conserved. This depends on the fact that when a body moves on a rough surface, an
old acquaintance of ours comes into play: heat. Hence, as we have seen in the previous section when heat is produced the quantity of mechanical energy does not remain constant but decreases. Thus, heat can also be interpreted as the intermediary quantity between mechanics and thermodynamics, the sector of physics to which now we turn.

**Thermodynamics.** The first quantity which is necessary to consider is temperature. Be given a system $C$ of particles $m_1, m_2, \ldots, m_n$ whose speeds are $v_i, v_2, \ldots, v_n$ in the reference frame of $C$. The average kinetic energy of every particle is

$$E_{K,m} = \frac{1}{n} \left( \sum_{i=1}^{n} m_i v_i^2 \right).$$

If all the particles have the same mass, this formula is transformed into

$$E_{K,m} = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{2} m v_i^2 = \frac{1}{2} m \left( \frac{1}{n} \sum_{i=1}^{n} v_i^2 \right) = \frac{1}{2} m v_{qm}^2,$$

where $v_{qm}$ is defined as mean-square velocity. Its formula is

$$v_{qm}^2 = \frac{1}{n} (v_1^2 + v_2^2 + \cdots v_n^2).$$

Temperature $T$ of a system of particles is an intensive quantity correlated to the kinetic energy of the system calculated in the reference frame of the system itself. Basically, the higher the average kinetic energy of particles composing the system, the higher its temperature. Temperature is not a measure of the amount of heat in a system simply because there is no point in asking how much heat a body possesses. Heat, as we have seen, indicates the passage of energy between two systems, it is not a property of a single system, it is a quantity which correlates two systems. However, temperature has a relation to heat. For, with notable exceptions, if heat is supplied to a system, its temperature increases, whereas if heat is removed from it, its temperature decreases; in other words, an increase in the temperature of the system corresponds to an absorption of heat by the system, whereas a decrease in the temperature of the system corresponds to a release of heat by the system.

After this premise on the notion of temperature, it is appropriate to define the notion of a thermodynamic system: A system is a portion of space delimited by a surface which separates the interior of the system from the exterior.

The complementary of a thermodynamic system is the environment, defined as the set of things that do not belong to the system. A thermodynamic system is isolated if it can exchange neither energy nor matter with the environment; is closed if it can exchange energy but not matter with the outside world; is open if it can exchange both energy and matter. The first principle of thermodynamics is the general law of conservation of energy. It states that
The Internal Energy of an Isolated System is Constant

Let us now connect heat, internal energy, and work of a system. When one supplies a body or system of bodies with an amount of heat $dQ$, it will partly increase its internal energy by an amount $dU$, while it will partly produce work $dW$, so that the relation

$$dQ = dU + dL \quad 2)$$

holds. If the body performs a transformation or a cycle of transformations at the end of which the state of the system is the same as the initial one, one speaks of a closed cycle. At the end of a closed cycle, the internal energy is the same as the initial one. This means that $dU=0$, so that, indicating by $Q$ the sum of all the $dQ$ and by $L$ the sum of all the $dL$, it will be

$$Q = L.$$

This equation indicates a very important fact: whenever a system completes a closed cycle, the work obtained and the heat expended are equal. This is the precise statement of the first principle of thermodynamics which, in addition to enshrining the conservation of energy, shows the equivalence between heat and work. If, instead, the cycle is not closed, equation 2) must be used. For the gases, equation 2) can assume a more expressive form: suppose that a gas with pressure $P$ is inside a container whose wall can expand very slowly until reaching a form whose difference from the initial one is infinitesimal (Fig. 4).

**Figure 4**
The Figure Referred to the Situation Described in the Running Text

![Figure 4](image_url)


The gas exerts the pressure $P$ on the walls and therefore performs the work $W$. If $a$ indicates the element of surface, for the exerted force $F$ the equation $F = P \cdot a$ holds. Being $dl$ the length element, the element of volume will be $adl$, so that $W = F \cdot ds = P \cdot a \cdot dl = P \cdot dV$. Hence equation 2) gets the form
\[ dQ = dU + P\, dV. \]

In the previous experience, we have supposed that the walls move very slowly. Suppose the opposite situation: be given a gas in part \( A \) of the box \( AB \), while being part \( B \) empty (Fig. 5).

**Figure 5**
*Image Representing the Situation Described in the Running Text*

Remove suddenly the septum. The gas will expand, but this expansion implies no work. Therefore \( dW = 0 \). It is evident that \( dQ = 0 \) too, so that \( dU = 0 \). In this experience, there is no change in internal energy. Suppose now to make this experience with a perfect gas. It is possible to note that the gas’ temperature does not change. Therefore, when internal energy does not vary, the temperature of a perfect gas is not modified while varying its pressure and volume. Ergo, to each value of \( U \) a single value of \( T \) corresponds and conversely. Thus, one reaches this important conclusion: in a perfect gas internal energy is a *function only of the gas’ temperature* (many of the ideas here presented are drawn from Toraldo di Franca 1976, chapter III).

Internal energy is connected to numerous important properties and quantities of a system. The first of them is the *free energy of a system*. It represents the quantity of macroscopic work (change in the kinetic energy) that a system can perform on the environment. It depends on the temperature, pressure, and concentration of the considered chemical species. There are various kinds of free energy. For example, Helmholtz free energy is the internal energy when a transformation with constant volume and temperature is considered. Gibbs free energy represents free energy in transformations performed with constant pressure and temperature. Another important quantity connected with internal and free energies is enthalpy. Given a thermodynamic system, its enthalpy \( H \) is defined as the sum of internal energy plus the product of pressure by volume

\[ H = U + pV. \]

Enthalpy indicates several significant properties of a thermodynamic system. In particular:
1) In an isobaric transformation (constant pressure) in which only mechanical work is performed, the variation of enthalpy indicates the heat that the system exchanges with the environment.

2) In an isochorobaric transformation (constant volume and pressure) the variation of enthalpy coincides with heat exchange and with the variation of internal energy during the process.

3) In an isobaroentropic transformation (constant pressure and entropy) the variation of enthalpy expresses the variation of free energy.

Enthalpy is subject to a rather complex mathematical treatment which, obviously, cannot be proposed in all its aspects to the pupils of the last three years in high school. However, it is important that these concepts are introduced and explained because the learners should understand that almost all of them have been introduced to clarify the complex relations between energy, work and heat. This is the original problem from which thermodynamics was born in the first half of the 19th century and it is a difficult task. In order to clarify this complex situation, the concepts presented here (and also others) have been created.

Let us move now to the last topic of our itinerary: entropy and the second principle of thermodynamics.

The purely mechanical phenomena are reversible. In principle, nothing within mechanics prevents to reverse the time-harrow and to reverse the phenomenon. On the other hand, according to what we have seen in the previous section on entropy, the thermodynamical phenomena, generally speaking, are not reversible: if we have a box divided by a septum and a gas is contained in a part of the box, when we remove the septum, gas will be distributed in the entire box. For the statistical reasons described above, the opposite process, in which the whole gas comes back in a part of the box, will not take place. The harrow time is irreversible.

An investigation that analyses a physical phenomenon in its entirety will, however, shows that there are no purely mechanical phenomena. Example: the Moon and the Earth rotate around the barycentre of their system. The principles of conservation of mechanical energy and of angular momentum should guarantee that the situation does not change over time. In fact, things are not so simple: the Moon rotating around the Earth causes tides, which cause the parts subject to them to heat up and thus dissipate mechanical energy. The Earth-Moon system thus loses mechanical energy. The Moon continuously moves away from the Earth, which slows down its rotation period. The opposite process does not take place because the whole phenomenon is not purely mechanical, but is thermodynamical and heat is involved. The only reversible phenomena in thermodynamics are those which occur near equilibrium: if two bodies A and B are in contact, heat passes from the hotter A to the colder B. However, if their difference of temperature is negligible, an infinitesimal variation of the initial conditions is sufficient in order to make B hotter and A colder, so that heat can pass in the opposite direction. However, in the physical reality, no properly reversible phenomenon exists. This situation is stated by the second principle of thermodynamics which can be expressed by two formulations:
A) It is impossible for the only result of a transformation to be the passage of heat from a body at a given temperature to one at a higher temperature. This formulation is due to Clausius.

B) It is impossible for the only result of a transformation to be the production of work at the expense of heat supplied by a single source at a fixed temperature. This formulation is due to William Thomson, Lord Kelvin (1824-1907).

The two postulates are equivalent. For example, let us suppose B) does not hold. Then, it is possible to obtain work by cooling seawater. Through friction, we could transform this work into heat and supplying heat to a higher temperature source, so violating A).

The second principle of thermodynamics offers this picture of the physical world: a source of heat is more valuable the higher its temperature because the greater the amount of heat that can be converted into work. Suppose some of the heat falls from a higher to a lower temperature. No real transformation is reversible. Therefore, a part of the heat will remain trapped at the lower energy and will be irrecoverable for the purpose of producing work. The energy that descends to a lower temperature degrades and becomes less and less usable. Mechanical energy can be fully converted into work, but not the reverse. When the universe had reached the same temperature in all its parts there would be thermal death. No discernible phenomenon could occur. Clausius clarified this situation through the concept of entropy: suppose that a system performs a reversible transformation, during which a machine supplies the heat $Q$ at the temperature $T$ to the system. We will say that its entropy $S$ in increased of the quantity $Q/T$. Thus, when a system is at the temperature $T$ and receives the quantity of heat $dQ$, its entropy increases of the quantity

$$dS = \frac{dQ}{T}.$$ 

Thence, passing from state $A$ to state $B$ entropy increases of the quantity

$$\int_A^B \frac{dQ}{T}.$$ 

Consider a Carnot machine, namely a thermodynamical cycle on a gas given by four transformations: an isothermal expansion, an adiabatic expansion (that is a transformation in which no exchange of heat between the system and the external environment takes place), an isothermal compression and an adiabatic compression, which return the gas to its initial condition. If a Carnot machine subtracts the heat $Q_1$ from a source whose temperature is $T_1$ and pours the quantity of heat $Q_2$ to a source whose temperature is $T_2$, the relation $T_1/T_2 = Q_1/Q_2$ holds. In this case, the increment of entropy is null because the system acquires the entropy $Q_1/T_1$ and loses the entropy $Q_2/T_2$, which are equal. However, we know this is only an ideal situation. In the universe, the phenomena are irreversible and, in this case, the relation $T_1/T_2 > Q_1/Q_2$ holds, that is $Q_1/T_1 < Q_2/T_2$. Thence, in an irreversible transformation entropy always increases. Ergo, the second principle of thermodynamics can also be formulated as follows:
In an isolated system, entropy is an increasing function of time, namely

since no real transformation is perfectly reversible, it follows that in an isolated system, entropy will always increase. Therefore, energy degrades and, if the universe is an isolated system, it will be destined to thermal death.

I will not deal here with Boltzmann’s definition of entropy because what is expounded is sufficient for my aims.

Conclusions

The main purpose of this work has been to give learners a general conceptual overview of the notion of energy. The basic idea here expressed is that, before considering the mathematical details concerning the various forms of energy, it is appropriate to introduce the concept of energy following a historical approach as it is particularly suitable for the pupils to gain the essence of this notion, which is so important in physics. A further idea is that, while speaking of energy, it is difficult to prescind from thermodynamics because this branch of physics is that through which it is possible to clarify all the nuances of energy as well as its connection with another fundamental notion, that of entropy. Therefore, the suggestion here developed is to propose an itinerary in which six hours (or how many the teacher will consider appropriate) are dedicated to introducing conceptually and historically the notion of energy. At this stage, it is advisable to make limited use of mathematics, though it is impossible to completely avoid it. Afterwards, namely after that the learners have acquired a series of general ideas on energy, this concept can be introduced in mechanics developing the mathematical details appropriate for young people aged 17-19. Later on, energy has to be introduced in thermodynamics. Given the importance of this section of physics in relation to the notion of energy, particular care has been dedicated to this topic, which allows us to understand the deeper implications of the physics of the reversible and irreversible. As it is natural, entropy and its relations with energy play here a pivotal role.

It is paramount to stress two aspects of this paper:

1) The idea behind it has been to discuss the basic principles and not the applications of such principles to the single aspects of physics, for example, as to mechanics, the application of the concept of energy to the different kinds of motions, or to collisions, or to the study of gravitation and, as to thermodynamics, the application of energy concept to the different kinds of transformations, to the notion of specific heat, to the kinetic theory of gases and so on.

2) Other branches of physics, such as electricity and electromagnetism might have been included in this discussion. However, the arguments put forward seem to me to be sufficient to clarify the point of view presented here, and adding new material would have overburdened the work.

In this period the terms multidisciplinarity and interdisciplinarity are widely used, but the concrete examples of an interdisciplinary education are not very numerous. Behind this work, there is the idea to offer an interdisciplinary approach to the concept of energy, in which history of physics becomes an important support in an educational
context. A consideration which the teachers might propose concerns, e.g., the fact that the problem of work, heat and energy was posed and solved when machines became essential for the economy of the Western countries and while the industrial revolution was developing. It is not a coincidence that words such as work and energy were used to denote physical quantities. In the common language, they are clearly referred to the activity of man. In physics they lose this anthropocentric meaning, but maintain the idea of an activity exerted on a system, though not necessarily by man. This is an example which shows that theoretical physics is not extraneous to the economic structure of society, although it would be a big mistake to think of an automatic link between the two. However, there is undoubtedly a link. It would be interesting for the teacher of physics to discuss these topics jointly with the teacher of history, thus proposing an attempt of an interdisciplinary education.

It is not important to offer a complete or a completely precise history of the way in which the concept of energy has been developed. This is the task of a historian of science not of a teacher or an expert in science education. What is important, is to appropriately select sections of the history of science, or part of the works of an author, which can be used in science education. Such an operation has been developed in this work as to the notion of energy.

**References**


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