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# Cognitive Predictors of Difficulties in Math and Reading in Pre-Kindergarten Children at High Risk for Learning Disabilities

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Cognitive skill differences that are apparent early in pre-kindergarten (pre-K) might provide predictive insights into risk for learning difficulties at school entry, particularly around early markers of risk for comorbid difficulties in early math and literacy. Domain-specific abilities (approximate number system or ANS acuity, phonological awareness) and domain-general abilities (working memory, vigilance, executive attention, and nonverbal IQ) were assessed in 493 children at the beginning of pre-K, to better understand how each uniquely contributes to risk for math difficulties (MD), and comorbid math and reading difficulties (MDRD). At the end of pre-K, standardized math and reading tests were used to form three risk groups (MD, MDRD, not-at-risk) with two severity cut points for math and reading (≤25th, ≤16th percentiles). Discriminant function analysis was used to determine whether and in what ways the groups differed on the cognitive variables. Both MD and MDRD-risk groups differentiated from the not-at-risk group on all variables except for ANS acuity, a finding that was convergent across severity cut points. The only significant contrast for ANS acuity emerged between the most severe MD only group and the not-at-risk group. Only vigilance or sustained attention supported the differentiation of MD risk from MDRD risk. Consistent with school-age studies of comorbidity, MDRD risk was also associated with the lowest levels of math and cognitive skills in this pre-kindergarten sample. Results reveal a potential specific role for sustained attention as an early risk factor for comorbid MDRD, a severe form of learning disability.

***Educational Impact and Implications Statement***

Children who have learning disabilities in both math and reading are at high risk for academic underachievement, school drop out, and difficulties in social functioning. The purpose of the current study was to determine whether it is possible to identify children at risk for both types of disabilities as early at the start of the pre-kindergarten year. Children who had difficulties in sustaining their attention at the start of pre-K were most likely to show the combination of significant difficulties in math and reading by the end of pre-K. The findings could be used to help identify young children at risk for significant learning difficulties and to inform early interventions.

*Keywords:* attention, cognition, comorbidity, math and reading, pre-kindergarten

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Comorbidity of learning disorders in mathematics and reading is associated with greater severity of academic difficulties, increased risk for affective and behavioral disorders (Willcutt et al., 2013), and lower response to some interventions (Fuchs, Fuchs, & Compton, 2013). Consequently, understanding comorbidity—it's developmental pathway(s), responsiveness to interventions, and its behavioral and neurobiological correlates—is highly important both for the science of learning disabilities as well as for education practice. Given the significant negative consequences of comorbid learning disorder, an important unaddressed question is whether there are early markers of risk for comorbidity. The current study was designed to answer this question in pre-kindergarten children.

## Comorbidity of Learning Disorders in School-Age Children

The prevalence of comorbid learning disorders is much higher than expected by chance (Badian, 1999; Gross-Tsur, Manor, & Shalev, 1996; Lewis, Hitch, & Walker, 1994). In a study of twins with disability defined at the 10th percentile on composite measures of academic functioning, more than 60% of children with disabilities in one of reading, math, or writing also had a disability in at least one of the other domains, and comorbidity for reading and math disabilities was 44% (Willcutt et al., 2019). In a European sample, the rate of comorbid reading, mathematics, and spelling difficulties was four to five times greater for children who were already experiencing significant difficulty in one academic domain than for children in the broader population (Landerl & Moll, 2010).

Although it is more typical for studies to focus on disabilities in reading *or* math, these statistics show that the comorbidity of learning disabilities is common. Of the small number of studies that have investigated comorbidity, a focus has been to determine how comorbid math disability and reading disability (MDRD) arises and what distinguishes individuals with comorbid learning disability from those with only reading disability (RD) or math disability (MD). In their review of cognitive and neurological bases of RD and MD, Ashkenazi, Black, Abrams, Hoeft, and Menon (2013) proposed three models of comorbidity: (a) an additive model such that MDRD includes the cognitive and neurological deficits from each specific learning disability; (b) a verbally mediated model in which impaired phonological or other linguistic processing that is often associated with RD also creates difficulties in math fact learning, sound-symbol mapping for Arabic numerals, and math word problems; and (c) a domain-general model in which impairments in processes that have a more global influence on learning, such as attention and working memory, lead to deficits in both reading and math, similar to the multiple deficit model for neurodevelopmental disorders proposed by Pennington, Willcutt and colleagues (Pennington, 2006; Peterson et al., 2017; Willcutt et al., 2013).

Willcutt et al. (2013) found that among school-age children with learning disabilities, deficits in phonological awareness and set shifting were uniquely associated with RD and MD, respectively, and comorbidity (MDRD) was associated with shared weaknesses in working memory, processing speed, and verbal comprehension, fitting both domain general and verbal-mediation models. In a study of second graders with RD or MD or MDRD or no learning disability, Cirino, Fuchs, Elias, Powell, and Schumacher (2015) found that students with no learning disability outperformed students with RD or MD, who in turn outperformed those with MDRD on most cognitive and academic tasks. Compared with their own overall cognitive level, the RD group had weaknesses on language measures including phonological awareness, rapid automatized naming (RAN), and verbal working memory, but demonstrated relative strengths in processing speed and nonverbal reasoning. The profile for students with MDRD was consistent with an additive model in that they displayed the same cognitive weaknesses as the RD group with the addition of weaknesses in processing speed and nonverbal reasoning (also see Moll, Göbel, & Snowling, 2015). Findings were similar whether the disability cut point was at the 25th percentile or at the 10th percentile. Although working memory was affected in all learning disability groups, it did not differentiate RD, MD or MDRD. In contrast, a meta-analysis on working memory and learning disabilities found that children with MDRD were characterized by weak verbal short term and working memory while children with MD without reading difficulties were characterized by weak visual-spatial short term and working memory (Szűcs, 2016).

A few longitudinal studies have also addressed issues of comorbidity in school-age children. Andersson (2010) found small deficits in visual-spatial working memory for MD and MDRD groups from Grades 3 to 6, with additional difficulties in verbal short-term memory (STM), processing speed, and set switching for the MDRD group. Vukovic (2012) found that, among children in kindergarten through third grade, phonological processing and early numerical skills accounted for both initial status and growth in mathematics in children with MD and with MDRD; working memory, STM, and processing speed did not differentiate the MD from the MDRD groups initially or over time. Geary, Hoard, Nugent, and Bailey (2012) found that students with the lowest levels of math achievement by 5th grade were those who had started first grade with both poor math achievement and poor reading.

More recently, some researchers have applied a dimensional approach to look at the *overlap* between reading and math across the continuum of both academic skills (Child, Cirino, Fletcher, Willcutt, & Fuchs, 2019; Peterson et al., 2017). In a group of second graders oversampled for MD, Child and her colleagues (Child et al., 2019) found that phonological awareness was most strongly related to reading. Although numerosity, assessed by an approximate number system or ANS acuity task (rapid discrimination of two large nonsymbolic quantities), was related to math and not reading, it was no more strongly related to math than were phonological awareness and working memory. Consistent with verbal-mediation and domain general models, working memory and phonological awareness were most strongly related to the overlap between math and reading. Using a similar design, Peterson et al. (2017) reported that phonological awareness, verbal comprehension, and processing speed accounted for the overlap between math and reading in line with both verbal-mediation and domain general models. In contrast to Child et al. (2019), working memory was only related to math.

To summarize, the findings from school-age studies are mixed. Some studies suggest that differences in severity of learning difficulties distinguishes MDRD from MD or RD. In other studies, findings seem to fit a verbally mediated model and/or a domain-general model. Comorbidity studies and dimensional overlap studies are not always in agreement about which cognitive correlates characterize MDRD or account for the overlap between reading and math. Working memory has been the most frequently studied cognitive variable, but, with the exception of the Szűcs (2016) meta-analysis, does not differentiate MDRD from MD.

## Relations of Domain General and Domain Specific Abilities to Early Math and Literacy

In contrast to a focus on comorbidity of MD and RD, studies of younger children have investigated concurrent and longitudinal predictors of early math and early literacy. Using the ECLS-K 2011 data, Morgan and colleagues (Morgan et al., 2017) found that at the beginning of kindergarten, verbal working memory and cognitive flexibility (the ability to respond to changing rules) uniquely predicted difficulties in *both* reading and math by first grade. In pre-kindergarten children, a variety of executive functions have been related to growth in both math and reading (Welsh, Nix, Blair, Bierman, & Nelson, 2010); however, some studies suggest that executive functions are more highly related to the growth in math compared with reading (e.g., Bull, Espy, & Wiebe, 2008; Fuhs, Nesbitt, Farran, & Dong, 2014; Schmitt, Geldhof, Purpura, Duncan, & McClelland, 2017; Willoughby, Blair, Wirth, & Greenberg, 2012; Willoughby, Magnus, Vernon-Feagans, & Blair, 2017). Working memory is a stable, direct predictor of math across long developmental windows (e.g., Barnes et al., 2014; Geary, Nicholas, Li, & Sun, 2017).

In the early childhood literature, many measures of executive function have been found to predict early math, consistent with findings that executive function is a singular construct in young children (Blair, Ursache, Greenberg, & Vernon-Feagans, 2015). Even in preschoolers, however, executive function measures are often complex and draw on several more foundational abilities, the primary one being attention (Garon, Bryson, & Smith, 2008). Attention is the ability to focus on a task (referred to as vigilance or sustained attention) and ignore irrelevant information (referred to as executive attention). Attention is required in both the early perceptual stages of information processing as well as in higher-level integrative processing (Stevens & Bavelier, 2012).

Given its developmental precedence in higher level executive processes (Wass, Scerif, & Johnson, 2012) and its ubiquitous role at all levels of cognition, what is known about the relation of attention to math and reading? The ability to sustain attention at age 4 is a direct and significant predictor of math and reading 16 years later even controlling for many academic, language, and demographic variables (McClelland, Acock, Piccinin, Rhea, & Stallings, 2013); fluctuations in attention in the preschool years predict concurrent and first grade math outcomes (Isbell, Calkins, Swingler, & Leerkes, 2018); attention predicts growth in math from the preschool to early school years (e.g., Steele, Karmiloff-Smith, Cornish, & Scerif, 2012); teacher ratings of inattention at the beginning of kindergarten is a significant predictor of later achievement in reading and math (Duncan et al., 2007); and early and persistent inattention is associated with lower math and reading achievement, drop out (Rabiner, Godwin, & Dodge, 2016) and less response to reading interventions (Miller et al., 2014; Rabiner & Malone, 2004). These findings suggest that attention might be important for academic learning in general.

Although these studies suggest that early executive functions and attention predict growth in math, and sometimes reading, they do not address risk for disabilities in reading, math or both in pre-kindergarten children. More generally, there is a paucity of information in both school-age and early childhood studies regarding whether basic attention abilities might serve as a marker of risk for more pervasive learning difficulties in both math and reading.

## Domain-Specific Correlates of Math and Reading

Some researchers have hypothesized that number sense as measured by ANS acuity tasks might be to math as phonological awareness is to reading (Butterworth, Varma, & Laurillard, 2011; Libertus, Feigenson, & Halberda, 2011; Mazzocco, Feigenson, & Halberda, 2011). Although several meta-analyses have now shown significant, but small relations of ANS acuity and math in school-age children and adults (e.g., Chen & Li, 2014; Schneider et al., 2017), the relation of ANS acuity to math may be larger in younger children than it is in older children and adults (Fazio, Bailey, Thompson, & Siegler, 2014) and may particularly characterize pre-kindergarten children with very low math knowledge (Purpura & Logan, 2015). However, questions have been raised about what ANS acuity actually measures given that individual differences in task performance can be explained by visual-spatial working memory (Bugden & Ansari, 2016) and by the ability to ignore distracting information (Fuhs & McNeil, 2013). In the current study, ANS acuity was tested as the domain-specific math predictor in the context of both visual-spatial working memory and the ability to ignore distracting information. This method provides a rigorous test for the role of ANS acuity in the prediction of risk for MD and MDRD in pre-kindergarten children. Phonological awareness was tested as the domain-specific predictor related to reading.

There are several important issues pertaining to MDRD comorbidity that have yet to be addressed and which this study was designed to investigate. The main question is whether there are early predictors that can reliably distinguish very young children at risk for only one learning disorder versus those at-risk for difficulties in both math and reading. The ability to identify children prior to the start of formal schooling (i.e., pre-kindergarten) at greatest risk for later severe learning difficulties affords opportunities for early intensive intervention. Second, because attention is important for all information processing and is foundational to other math- and reading-related cognitive correlates such as working memory, we asked whether risk for MDRD might be conferred by poor attention, a hypothesis not previously tested in the comorbidity literature. Third, we asked whether ANS acuity predicts risk for MD or MDRD in pre-kindergarten children in the presence of two executive functions which are hypothesized to account for individual differences in ANS acuity. Fourth, we were interested in determining to what extent very early risk for MDRD shares characteristics with school-age risk for MDRD; that is, whether MDRD in very young children is also associated with the most severe deficits in math and reading.

This study follows the method used in Cirino et al. (2015) with school-age children in which cognitive profile analysis was conducted at two achievement cut points, the rationale for which is discussed below. Our hypotheses and research questions were as follows: (a) Pre-kindergarten children with comorbid MDRD risk will have lower reading achievement and lower math achievement than children with MD risk (the severity hypothesis); (b) Both learning risk groups (MD and MDRD) will have lower literacy and math achievement than children not at risk as well as lower levels of domain-general abilities; (c) The MDRD and MD groups will have lower ANS acuity than the group without risk for learning difficulties and the MDRD-risk group will have lower phonological awareness than the MD-risk and no-risk groups; and (d) Given the important role of attention at all levels of information processing, will attention be a specific risk factor for comorbid MDRD risk?

# Method

This study involved secondary data analyses from a randomized controlled trial of a mathematics tutorial intervention for children with very low mathematics knowledge at the beginning of the pre-kindergarten (pre-K). Details of the parent study, including the sample and procedures, are described more fully in a main impacts paper (Barnes et al., 2016). The study was conducted in pre-K programs in Houston, Texas (16 public schools within an urban school district) and in the bay area of California (29 schools within three school districts or local education agencies). Children came from 95 pre-K classrooms across two states and two cohorts. Children were accepted for participation based on parental consent, if they were 4 years of age before the first intervention session, spoke English or Spanish as their first language, and scored at a very low level on a math screening measure at the beginning of pre-K that has been shown to strongly predict kindergarten math achievement (Barnes et al., 2016). Of 1700 children who were screened, 518 were consented and were randomly assigned to one of three conditions: a math intervention condition, a math intervention plus attention training condition, or a business-as-usual (BAU) instruction condition.

## Participants

Data from 493 children were used in the profile analysis when the cut point was set at the 25th percentile and 490 children with the cut point at the 16th percentile (our rationale for using these cut points is described below). The numbers of children in the profile analyses is smaller than the number consented into the parent study (518) because (a) we only used cases that had data on every variable and (b) we excluded children with only RD risk given that their numbers were too small to analyze (see below). The sample was 46.7% female; 71.7% Hispanic; 17.9% African American; 1.7% Asian American; 2.2% Caucasian; 3.7% mixed race; and ethnicity was not reported for 2.8%. The mean age of children at pretest was 4.5 years. All children were from low-income families and were eligible to attend pre-K programs in their respective states (see Barnes et al., 2016 for more details on sample).

## Measures Used for Identifying At-Risk Subgroups

Children were assessed in their first or preferred language on all measures. Measures that were not standardized in Spanish were translated into Spanish and back-translated into English. The Test of Early Mathematics Ability, 3rd edition (TEMA-3) had previously undergone this translation process with permission from and consultation with the test authors. For tasks where stimuli and responses were primarily nonverbal, translation was restricted to instructions (e.g., working memory, ANS acuity). Standardized English and Spanish versions were used for language-sensitive tests (e.g., phonological awareness, Letter–Word Identification). There were no level differences or interactions due to language of assessment.

**Test of Early Mathematics Ability, 3rd edition (TEMA-3).** The TEMA-3 (Ginsburg & Baroody, 2003) is an individually administered test of informal and formal mathematical knowledge. Items assess numerical abilities, including number sense, number fact knowledge, arithmetic calculation and problem solving. The TEMA-3 is for children aged 3 to 8 years. Test–retest reliability ranges from .82 to .93, and alternate-form reliability is .97. Criterion predictive validity ranges from .54 to .91 with other standardized math assessments. Within-sample test–retest reliability was .85. Performance on this test at the end of the pre-K year was used to determine risk for math difficulties.

**Letter–word identification.** The Letter–Word Identification subtest from the Woodcock-Johnson Tests of Academic Achievement-3rd Edition NU (WJ-3 NU; Woodcock, McGrew, & Mather, 2007) and Identificación de letras y palabras from the Batería III Woodcock-Muñoz (Muñoz-Sandoval, Woodcock, McGrew, & Mather, 2005) assess print knowledge and word reading. Early items require the child to identify individual letters (upper case letters followed by some lower case letters) and later items include reading words. Reliability is high for this age with internal consistency at .98. Within-sample internal consistency was .87. Performance on this test at the end of the pre-kindergarten year was used to determine risk for reading difficulties because letter naming abilities at the end of pre-K are strong predictors of later reading achievement (Catts, Fey, Zhang, & Tomblin, 2001; Piasta, Petscher, & Justice, 2012). Throughout the remainder of the paper we refer to both the English and Spanish versions as Letter–Word Identification.

**Criteria for identifying subgroups.** We used percentile ranks from testing at the end of pre-kindergarten to classify students into three groups. Children who scored at or below the cut score (whether the 25th or 16th percentile, depending on the set of analyses) on the TEMA-3 but above the cut score on Letter–Word Identification were classified as at risk for difficulty with math (MD risk). Those who scored at or below the cut points on both the TEMA-3 and on Letter–Word Identification were identified as at risk for difficulties in both math and reading (MDRD risk). Children who scored above the cut point on both measures were labeled as being not at risk for math or reading difficulty (not at risk).

Defining subgroups according to levels of achievement is challenging, especially given an absence of agreed-upon absolute performance standards or achievement levels that invariably constitute risk. Learning difficulties are often characterized by chronically low achievement relative to peers, therefore the use of relative performance to peers (through percentile cut scores), although not ideal, can provide at least some basis for categorizing students according to low achievement on measures in which low performance is predictive of subsequent learning and achievement difficulties. Any cut score can be considered arbitrary, to an extent. However, consideration of aspects of a percentile cut score such as the magnitude of deviation from the mean, or the use of particular cut scores in prior research, can support the rationale for using certain cut scores for defining risk status. In this study, we examined cognitive skill profiles among subgroups of students defined by achievement levels consistent with how the research community has commonly defined risk and/or low achievement status.

In this study we used two cut points in separate analyses to define risk: The 25th percentile and the 16th percentile. We selected the 25th percentile to be consistent with how *risk* for subsequent and more intensive difficulties has been commonly defined in research and the field. The 25th percentile stands at the lower bound of the interquartile range (i.e., the 25th to 75th percentiles, the middle 50% of a population), and thus the 25th percentile has often been viewed as the lower bound of the average range. The 25th percentile has been used as a cut point to define difficulties, risk status, or low achievement in reading and mathematics either alone or on a comorbid basis (e.g., Cirino et al., 2015; Dirks, Spyer, van Lieshout, & de Sonneville, 2008; Francis, Shaywitz, Stuebing, Shaywitz, & Fletcher, 1996; Geary, Hoard, Nugent, & Byrd-Craven, 2008; Vukovic & Siegel, 2010). Additionally, the 25th percentile is proximal to the 30th percentile, which Torgesen (2000) recommended as a criterion level for determining student success on various measures of achievement and has been used in several studies to denote risk status (e.g., Simmons et al., 2008; Vellutino, Scanlon, Zhang, & Schatschneider, 2008).

In our second set of analyses we used the 16th percentile as a cut point that is more consistent with significant difficulties, and a percentile level that has been used often in research to define learning disability status in math and reading. The 16th percentile falls 1 *SD* below the mean, falls outside of the interquartile range, and represents a significant departure from the average range of achievement. The 16th percentile has been used as a cut point to designate severe difficulties or disability status in mathematics or reading (e.g., Bryant et al., 2016; Compton, Fuchs, Fuchs, Lambert, & Hamlett, 2012; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Mammarella, Hill, Devine, Caviola, & Szűcs, 2015; Seethaler & Fuchs, 2010; VanDerHeyden, Broussard, & Cooley, 2006). Scores between 1 to 1.5 standard deviations below the mean are also recognized by the *Diagnostic and Statistical Manual of Mental Disorders*, fifth edition (*DSM–5*) as representing significant risk for specific learning disability (Tannock, 2013). Therefore, although the use of percentile cut scores is not without limitations, the practice has an extensive history in research and captures deviation from the mean that is characteristic of learners with or at-risk for learning disabilities.

Although some studies have also used the 10th percentile as a cut-score (Cirino et al., 2015; Morgan, Farkas, & Wu, 2009), the resulting group size for MDRD risk would have been too small for analysis, which is consistent with what is known about limitations on the floors of academic tests for children who are at the entry level of the test. We did not have enough children to create an RD-risk-only group, which is likely attributable to the original study inclusion criteria based on low mathematical knowledge (i.e., rather than early literacy skill).

Using the 25th percentile to define risk, there were 185 (38%) classified as MD risk, 87 (18%) classified as MDRD risk, and the remainder ($n$ = 218, 44%) considered not at risk. Using the 16th percentile as the risk cut off, 164 (33%) students were classified as MD risk, 43 (8%) as MDRD risk, and 286 (58%) as not at risk.

## Measures Used to Investigate Differences Among At-Risk Subgroups

The analyses included data from pretest measures of domain-specific and domain-general cognitive skills known to be related to reading and mathematics allowing for the use of time precedence in the design to test early predictors of later academic achievement. A broad measure of informal mathematical knowledge designed for pre-kindergarten children (Child Math Assessment) was also used as a predictor of group membership (see Cirino et al., 2015 for a similar approach). All measures used to investigate profiles and differences among subgroups were administered at the beginning of the pre-K year. The Child Math Assessment was also used at the end of the pre-K year to determine whether performance differed according to risk group status.

**Nonverbal IQ.** The Matrices subtest from the Kaufman Brief Intelligence Test, Second Edition (Kaufman & Kaufman, 2004) assesses nonverbal cognitive ability in individuals from 4 to 90 years of age. Examinees are shown a page with one picture at the top and a choice of pictures below. The child must choose one of the pictures on the bottom that is related to the picture at the top. The items involve concrete stimuli (people and objects) as well as more abstract stimuli (designs and symbols). The subtest demonstrates internal consistency reliability of .86 for individuals 4–18 years of age.

**Working memory.** A preschool-friendly visual-spatial working memory task was used (Bisanz, Sherman, Rasmussen, & Ho, 2005). In this task, modeled after the standard Corsi-Blocks task, children must replicate the series of jumps between lily pads that a frog makes starting with a span of 1 and going to a span of 7. There are two trials at each span level and the test is discontinued when both trials at a particular span level are missed. Children's total accuracy score was used. Internal consistency for this task in 4- and 5-year old children is .70 and relations of concurrent performance on measures of phonological awareness and vocabulary range from .22-.26 and from .26-.31 with nonsymbolic arithmetic and number naming (LeFevre et al., 2010). Test–retest reliability in the current sample was .70.

**Phonological awareness.** The Phonological Awareness subtest of the Test of Preschool Early Literacy (Lonigan, Wagner, Torgesen, & Rashotte, 2007) or the Spanish Preschool Early Literacy Assessment (Lonigan, 2012) were used. These subtests contain elision and blending items. Internal consistency for both tests exceeds $\alpha$ = .89 (Goodrich & Lonigan, 2017; Lonigan et al., 2007). Raw scores were used and converted to sample-based Z scores.

**Attention.** The Child-Attention Networks Test (Rueda, Rothbart, McCandliss, Saccomanno, & Posner, 2005) is a computer-based, preschool-friendly standard flanker task that measures vigilance and executive attention. The task is the measure of attention in the National Institutes of Health Toolbox for the Assessment of Neurobiological and Behavioral Function – Cognition Battery (Bauer & Zelazo, 2014). Test–retest reliability is .92 and convergent validity with the WPPSI-III Block Design is .60 (Zelazo & Bauer, 2013). The child is asked to determine which way the middle fish in a set of three is swimming and catch it with a net using the "Z" or "?" key on a laptop. To familiarize the child with the task there was extensive off-computer practice with cards followed by practice on the computer, and scaffolding was provided as needed. Test trials (cued and uncued) consisted of four blocks of 17 trials each. On cued trials, bubbles appeared on the screen before the fish appeared; on uncued trials the fish appeared without being cued by bubbles. Trials were also either congruent or incongruent. On congruent trials the middle and flanking fish all swim in the same direction; on the incongruent trials the middle fish swims in the opposite direction to the two flanking fish. Accuracy on congruent trials (a composite measure of cued and uncued congruent trial accuracy) was used as the measure of vigilance because the congruent trials require sustained attention over time (press the button on the keyboard that matches the direction in which the middle fish is swimming), but do not require the inhibition of competing or irrelevant information. Accuracy on incongruent trials (a composite measure of cued and uncued incongruent trial accuracy) was used as the measure of executive attention or ability to inhibit responding to an irrelevant stimulus, as incongruent trials require children to focus on the direction of the middle fish and ignore the opposite direction in which the flanking fish are swimming. Composite scores were used because performance on cued and uncued congruent trials and on cued and uncued incongruent trials were highly correlated, respectively. In this sample, test–retest reliabilities were .80 for the composite score for congruent trials (measure of sustained attention) and for the composite score for incongruent trials (the measure of executive attention). Response time data were not used due to low rates of accuracy common in preschool children (see Davidson, Amso, Anderson, & Diamond, 2006).

**Approximate number system (ANS) acuity.** ANS acuity (i.e., perception and discrimination of quantities without counting) was measured with Panamath (Panamath.org, 2010–2011). Forty trials were administered following four practice trials. For each practice trial, a picture of Elmo and Cookie Monster were displayed on the computer screen, each standing inside a box of blue dots and a set of yellow dots, respectively. The children were familiarized with the characters and then were told, "Elmo and Cookie Monster will have dots in their boxes. If Elmo has more dots, touch Elmo's box. If Cookie Monster has more dots, touch Cookie Monster's box." They were then queried on what they would do if Elmo has more dots or Cookie Monster has more dots to ensure understanding. The examiner recorded the response and pressed a key to advance to the next computer screen, which had a fixation cross to which the child was directed. Once the child was attending, the examiner pressed a key to advance to the test screen. For each test trial the child was shown a new set of stimuli and asked "Who has more dots?" Feedback was provided on each practice trial but not on test trials. Children were discouraged from counting. Total accuracy was used in analyses (Mazzocco et al., 2011). Within sample test–retest reliability was .92.

**Child Math Assessment.** The Child Math Assessment (Klein & Starkey, 2012) is an individually administered assessment of preschool children's informal mathematical knowledge across a broad range of concepts and skills, including number, arithmetic operations, space and geometry, measurement, and patterns. This assessment comprises nine tasks, with multiple items per task, with task difficulty appropriate for children from three to five years of age. Test–retest reliability over a 2-week interval is .91, and internal consistency is .92. Test–retest reliability in the study sample was. 92. With respect to concurrent validity, scores were found to be positively related to TEMA-3 scores, $r = .74$, $p < .01$ among 4- and 5-year-old children. A total proportion correct score was used in analyses.

## Analysis Plan

Discriminant function analysis (DFA) was used to identify whether and in what ways MD-risk, MDRD-risk, and not-at-risk children differed on selected cognitive variables, including vigilance, executive attention, ANS acuity, working memory, phonological awareness, and nonverbal IQ, and on the Child Math Assessment. Age, treatment condition, and state were modeled as covariates because these variables were found to affect end of pre-K outcomes on the test of math achievement (TEMA-3) that was used to form the risk groups; however, there was no variation in slopes for treatment effects across schools and classrooms so these variables were not treated as covariates in the model (see Barnes et al., 2016). There was also no effect of treatment condition on the categorization of groups in the current study (*p* values ranged from .26 to .54).

Per convention, we began by testing for differences across groups on a set of linearly combined dependent variables. A significant omnibus MANCOVA test was followed by a discriminant function analysis (DFA). DFA predicts membership in one of two or more groups based on weighted combinations of that same set of dependent variables (though conceptualized as predictors in DFA), assuming the omnibus test of group differences (per the MANCOVA) differs statistically from 0. Canonical correlations and univariate tests of group differences were used to estimate the contribution of individual variables to the function. Canonical coefficients represent the bivariate correlation between each variable and the discriminant function that separates groups. Coefficients greater than .33 were interpreted for distinguishing between MD-risk, MDRD-risk, and not-at-risk groups (Tabachnick & Fidell, 2007). Because there were more than two groups, cognitive profile differences were evaluated with three planned multivariate contrasts: MDRD risk versus not at risk, MDRD risk versus MD risk, and MD risk versus not at risk. In each set of comparisons, effect sizes were calculated as the standardized mean difference (in raw scores or in scale scores depending on the measure) between pairs of groups using the pooled standard deviation to estimate variance (Hedges' *g*). These effect size estimates are in Table 3, with those .25 standard deviation or higher considered to be "substantively important" ("What Works Clearinghouse", 2017, p. 14). In this study, positive effect sizes favor the first group in each pair of contrasts (e.g., for MD vs. MDRD, MD is the first group and MDRD is the second group). Analyses using the 25th percentile to define the risk cut point was performed followed by the same set of analyses using the 16th percentile as the risk cut point.

# Results

The standard scores for reading and math achievement for the three groups at both the 25th and 16th percentile cut points are reported in Table 1. Table 1 also includes the proportion correct scores on the Child Math Assessment at the end of prekindergarten, the measure of math knowledge that was not used to form the risk groups. When risk was defined at the 25th percentile (on Letter–Word Identification and/or TEMA-3),

Table 1
*ANOVA Comparison of Math and Reading Scores for Groups at the 25th and 16th Percentile Cut Points*

| Measure | *n* | *M* | *SD* | Bonferroni comparison | |
|---|---|---|---|---|---|
| | | | | MD risk | MDRD risk |
| 25th percentile | | | | | |
| TEMA | | | | | |
| MD risk | 185 | 81.16 | 7.11 | | |
| MDRD risk | 87 | 76.52 | 6.67 | .00 | |
| Not at risk | 218 | 99.9 | 7.17 | .00 | .00 |
| CMA | | | | | |
| MD risk | 185 | .55 | .15 | | |
| MDRD risk | 87 | .50 | .15 | .13 | |
| Not at risk | 218 | .65 | .13 | .00 | .00 |
| WJ Letter-word ID | | | | | |
| MD risk | 185 | 101.09 | 6.55 | | |
| MDRD risk | 87 | 82.53 | 6.15 | .00 | |
| Not at risk | 218 | 106.43 | 7.65 | .00 | .00 |
| 16th percentile | | | | | |
| TEMA | | | | | |
| MD risk | 164 | 77.52 | 6.20 | | |
| MDRD risk | 43 | 74.91 | 6.00 | .20 | |
| Not at risk | 286 | 97.38 | 7.86 | .00 | .00 |
| CMA | | | | | |
| MD risk | 164 | .52 | .15 | | |
| MDRD risk | 43 | .46 | .15 | .04 | |
| Not at risk | 286 | .65 | .13 | .00 | .00 |
| WJ Letter-word ID | | | | | |
| MD risk | 164 | 97.8 | 8.09 | | |
| MDRD risk | 43 | 78.56 | 5.31 | .00 | |
| Not at risk | 286 | 104.89 | 8.05 | .00 | .00 |

*Note*. TEMA = Test of Early Mathematics Ability – 3; CMA = Child Math Assessment; MD = math disability; MDRD = comorbid math disability and reading disability.

similar to studies with school-age children, a severity effect was evident such that children with MDRD risk scored lower on the TEMA-3 than the MD-risk group. The groups did not differ on the Child Math Assessment. When the 16th percentile (on Letter–Word Identification and/or TEMA-3) was used to define risk the MDRD-risk and MD-risk groups did not differ on the TEMA-3, but they did differ on the Child Math Assessment such that the MDRD-risk group had lower scores. The MD-risk group was significantly lower than the not-at-risk group on Letter–Word Identification at both cut points; however, average reading scores for the MD-risk group were much higher than those for the MDRD-risk group.

The means, standard deviations, and correlations among variables are reported in Table 2. The bivariate correlations among the six outcome variables were small ($r$s = .14–.34). Distributions for the outcome variables were normal. Skewness and kurtosis values within each group were no larger than 2, and the $p$ values for the Shapiro–Wilk statistics did not differ statistically from 0, suggesting no violations of the normality assumption. Further, Box's test provided support for the assumption that group-specific covariance matrices were nondifferent ($p < .07$). We transformed scores on the six cognitive measures to z scores so that all could be plotted using a common metric. The profile plots using the 25th and 16th percentile cut points are provided in Figure 1. Table 3 reports the effect sizes among the three groups.

## Results Using the 25th Percentile as the Cut Point for Risk

Group means adjusted for age, treatment condition, and state differed on the set of cognitive variables (Wilks' $\Lambda = 0.865$, $F[12, 942] = 5.93$, $p < .01$). With three groups, two discriminant functions were estimated. The first of these two functions explained 86% of the variance. The second function explained 14%. The first discriminant function significantly differentiated between groups ($p < .001$). The second did not ($p = .08$). Accordingly, we excluded the second function from further consideration.

To identify the subsets of variables that define each discriminant function, we examined correlations between the discriminant func-

Table 2
*Performance by Group on the Cognitive Variables*

| Variable | MD | | MDRD | | Not-at risk | | Correlations | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | *M* | *SD* | *M* | *SD* | *M* | *SD* | 2 | 3 | 4 | 5 | 6 |
| 25th percentile | | | | | | | | | | | |
| 1. Phonological awareness | −0.14 | 0.91 | −0.22 | 0.89 | 0.22 | 1.05 | .215$^{**}$ | .179$^{**}$ | .238$^{**}$ | .213$^{**}$ | .214$^{**}$ |
| 2. Working memory | 2.47 | 2.08 | 2.33 | 2.08 | 3.03 | 2.09 | | .257$^{**}$ | .329$^{**}$ | .144$^{**}$ | .192$^{**}$ |
| 3. ANS acuity | 27.70 | 7.73 | 29.25 | 6.77 | 28.70 | 8.19 | | | .340$^{**}$ | .162$^{**}$ | .136$^{**}$ |
| 4. Attention – Vigilance | .81 | .18 | .75 | .21 | .85 | .16 | | | | .158$^{**}$ | .280$^{**}$ |
| 5. Attention – Executive | .46 | .24 | .46 | .22 | .57 | .27 | | | | | .155$^{**}$ |
| 6. Nonverbal IQ | 90.46 | 14.66 | 88.57 | 13.66 | 93.96 | 12.99 | | | | | |
| 16th percentile | | | | | | | | | | | |
| 1. Phonological awareness | −0.16 | 0.87 | −.35 | 0.94 | .16 | 1.03 | | | | | |
| 2. Working memory | 2.36 | 2.12 | 2.07 | 2.15 | 2.99 | 2.05 | | | | | |
| 3. ANS acuity | 27.66 | 7.99 | 27.53 | 6.78 | 29.01 | 7.73 | | | | | |
| 4. Attention – Vigilance | .78 | .20 | .72 | .22 | .86 | .16 | | | | | |
| 5. Attention –Executive | .46 | .23 | .48 | .48 | .55 | .27 | | | | | |
| 6. Nonverbal IQ | 89.93 | 14.28 | 86.77 | 14.56 | 93.50 | 13.15 | | | | | |

*Note*. ANS = approximate number system. Values represent Z scores for Phonological awareness, raw scores for Working memory and ANS acuity, proportion correct for Attention, standard scores for Nonverbal IQ.
$^{**}$ Correlation is significant at .01 level. Correlations are independent of the cut-score.

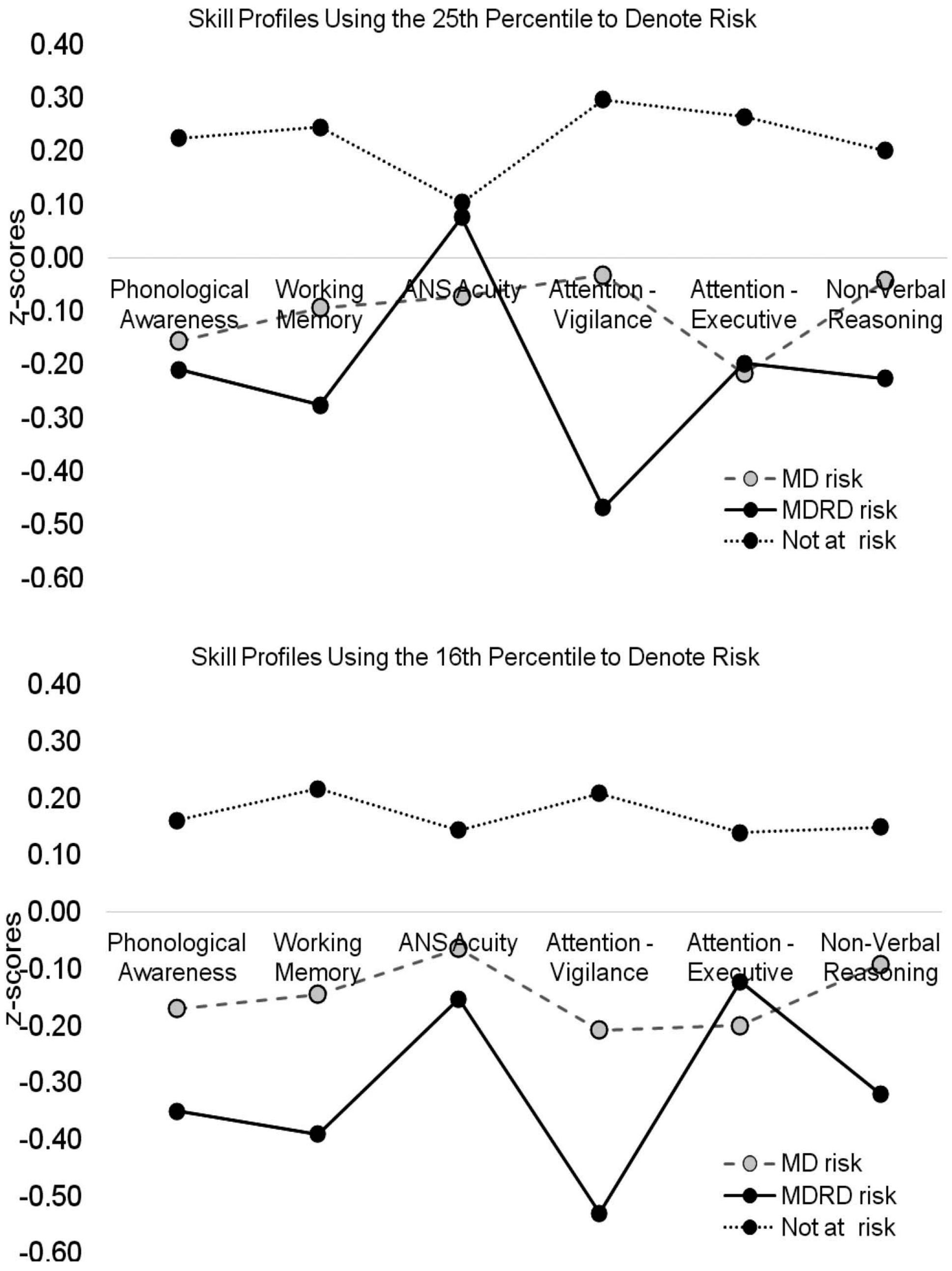

*Figure 1.* Skill profile plots of risk groups when the 25th and 16th percentiles were used to denote risk status. ANS = approximate number system; MD = math disability; MDRD = comorbid math disability and reading disability.

tion and variables in the model (see Table 4). These "factor structure coefficients" (Huberty & Olejnik, 2006) denote the simple (bivariate or canonical) correlations between each variable and the discriminant function and provide a meaningful framework for assigning substantive labels to a function (not unlike interpreting factors in an EFA). The largest factor structure coefficients were associated with the attention measures, namely, vigilance (congruent trial accuracy; $r = .63$) and executive attention (incongruent trial accuracy; $r = .63$). Other variables that met the $r > .33$ threshold included phonological awareness ($r = .59$), working memory ($r = .45$), and nonverbal IQ ($r = .41$). ANS acuity did not meet the cut-off ($r = .08$). The mean discriminant score for each group (i.e., group centroids) indicated that students at risk for MDRD had a much lower score on this function (−.479) than students not at risk (.348) or students at risk for MD (−.185). Given our interest in contrasting across three groups (MDRD risk vs. not at risk, MDRD risk vs. MD risk, and MD risk vs. not at risk), we fit two models. DFA allows $n - 1$

Table 3
*Effect Size Estimates (Hedges g)*

| Variable | MD vs. MDRD | MD vs. Not at risk | MDRD vs. Not at risk |
|---|---|---|---|
| 25th percentile | | | |
| Phonological awareness | .09 | −.36 | −.43 |
| Working memory | .07 | −.27 | −.33 |
| ANS acuity | −.21 | −.13 | .07 |
| Attention – Vigilance | .30 | −.24 | −.56 |
| Attention – Executive | .01 | −.42 | −.42 |
| Nonverbal IQ | .13 | −.25 | −.41 |
| 16th percentile | | | |
| Phonological awareness | .22 | −.33 | −.51 |
| Working memory | .14 | −.30 | −.45 |
| ANS acuity | .02 | −.17 | −.19 |
| Attention – Vigilance | .29 | −.45 | −.81 |
| Attention – Executive | −.08 | −.33 | −.25 |
| Nonverbal IQ | .22 | −.26 | −.50 |

*Note*. MD = math disability; MDRD = comorbid math disability and reading disability.

contrasts, where *n* is the number of groups. With three groups, two models were necessary to accommodate the relevant contrasts.

**Comparisons of MDRD-risk and not-at-risk subgroups.** Students at risk for MDRD scored significantly lower (Wilks' $\Lambda = 0.89$, $F[6, 470] = 10.02$, $p < .01$) on all cognitive attributes except ANS acuity. Vigilance ($r = -.78$), executive attention ($r = -.44$), phonological awareness ($r = -.42$), working memory ($r = -.50$), and nonverbal IQ ($r = -.41$) defined the discrimination between the MDRD-risk and not-at-risk groups. Effect sizes were appreciable for all variables with the exception of ANS acuity (see Table 3), with the largest effect size observed for vigilance ($g = .56$) followed by phonological awareness, executive attention, and nonverbal IQ, respectively ($gs = .43, .42, .41$).

**Comparison of MD-risk and MDRD-risk subgroups.** The contrast of MD risk and MDRD risk was also statistically significant (Wilks' $\Lambda = 0.96$, $F[6, 470] = 3.29$, $p = .004$), with vigilance ($r = .80$) contributing to the discriminant function. The effect size for vigilance as assessed by congruent trial accuracy was .30 favoring the MD-risk subgroup (see Table 3).

**Comparison of MD-risk and not-at-risk subgroups.** Finally, the MD-risk and not-at-risk subgroups also differed in their performance across cognitive measures (Wilks' $\Lambda = 0.92$, $F[6, 470] = 7.09$, $p < .01$). Compared with the not-at-risk group,

Table 4
*Canonical Structure Coefficients and Univariate Test Results*

| | 25th percentile | | | 16th percentile | | |
|---|---|---|---|---|---|---|
| Variable | Correlation coefficient | *F* value | *p* values | Correlation coefficient | *F* value | *p* values |
| MDRD vs. Not at risk | | | | | | |
| Phonological awareness | −.42 | 10.77 | .00 | −.46 | 9.74 | .00 |
| Working memory | −.50 | 15.45 | .00 | −.54 | 13.76 | .00 |
| ANS acuity | −.03 | .04 | .84 | −.27 | 3.41 | .07 |
| Attention – Vigilance | −.78 | 37.43 | .00 | −.89 | 36.83 | .00 |
| Attention – Executive | −.44 | 11.68 | .00 | −.22 | 2.37 | .13 |
| Nonverbal IQ | −.41 | 10.08 | .00 | −.46 | 9.91 | .00 |
| MD vs. MDRD | | | | | | |
| Phonological awareness | .09 | .17 | .68 | .33 | 1.20 | .27 |
| Working memory | .31 | 1.95 | .16 | .44 | 2.16 | .14 |
| ANS acuity | −.26 | 1.34 | .25 | .16 | .29 | .59 |
| Attention – Vigilance | .80 | 12.69 | .00 | .85 | 7.92 | .01 |
| Attention – Executive | −.04 | .02 | .88 | −.17 | .33 | .57 |
| Nonverbal IQ | .31 | 1.93 | .17 | .49 | 2.59 | .11 |
| MD vs. Not at risk | | | | | | |
| Phonological awareness | −.59 | 14.85 | .00 | −.50 | 11.59 | .00 |
| Working memory | −.52 | 11.73 | .00 | −.55 | 14.12 | .00 |
| ANS acuity | −.28 | 3.25 | .07 | −.32 | 4.85 | .03 |
| Attention – Vigilance | −.53 | 12.24 | .00 | −.79 | 29.46 | .00 |
| Attention – Executive | −.73 | 22.92 | .00 | −.53 | 13.09 | .00 |
| Nonverbal IQ | −.37 | 5.86 | .02 | −.37 | 6.53 | .01 |

*Note*. MD = math disability; MDRD = comorbid math disability and reading disability; ANS = approximate number system.

students at risk for MD scored significantly lower on all cognitive attributes except ANS acuity. The largest effect sizes were observed for executive attention ($g = .42$) and phonological awareness ($g = .36$; see Table 3).

In a separate analysis using the Child Math Assessment score at the beginning of pre-kindergarten to predict risk status, children not at risk scored significantly higher than children with MDRD risk ($p < .01$) and MD risk ($p < .01$). However, children with MDRD risk were not significantly lower than children with MD risk on the Child Math Assessment at the beginning of pre-kindergarten ($p = .52$).

## Results Using the 16th Percentile as the Cut Point for Risk

The same analyses were repeated using the 16th percentile to denote risk status. There were significant overall differences in group means based on the MANCOVA results, $F(12, 946) = 6.08$, $p < .01$. The findings comparing risk groups for the 16th percentile were the same as those at the 25th percentile (see Table 3 and Figure 1) and are discussed together below. The analysis using the Child Math Assessment score at the beginning of pre-kindergarten to predict risk status also revealed the same findings as the 25th percentile; children with MDRD risk and MD risk at the end of pre-K had lower scores on the CMA than those without risk at the beginning of pre-K.

Results for the univariate *F* tests and estimates of the canonical correlations at the 16th percentile were also similar to those at the 25th percentile with two exceptions (see Table 4). First, the MDRD-risk versus not-at-risk group did not differ on executive attention ($p = .13$). Second, the MD-risk group was significantly lower on ANS acuity than the not-at-risk group ($p = .03$).

# Discussion

The goals of this study were to investigate the domain-specific and domain-general cognitive profiles associated with outcomes in early mathematics and reading at the end of pre-K and to determine whether these cognitive profiles differentiated children with risk for comorbid difficulties in math and reading (MDRD risk), children at risk for difficulties in math (MD risk), and children deemed not at risk for learning difficulties. Novel aspects of the study included (a) investigation of domain-general and domain-specific risk factors for difficulties in math and reading at the pre-kindergarten level, (b) an investigation of ANS acuity as a predictor of risk for MD in the context of cognitive correlates (i.e., visual-spatial working memory and inhibition of irrelevant information) found in other studies to account for performance on ANS acuity tasks, and (c) an exploration of whether foundational attention abilities (sustained attention and executive attention) might be associated with comorbid risk for difficulties in both early math and reading.

Four main findings emerged from this study. First, similar to studies with school-age children, we found a severity effect for MDRD risk in this pre-K sample. When the cut point for risk status was set at the 25th percentile, math achievement on the measure used to form the groups was lower in the MDRD-risk group than in the MD-risk group. When the cut point was set at the 16th percentile, performance on a broad assessment of informal math skills that was not used to form the groups (i.e., the Child Math Assessment) was lower for the MDRD-risk group compared with the MD-risk group. Second, and also similar to studies of comorbidity with school-age children, the MDRD-risk and MD-risk groups had lower levels of cognitive and reading skills compared with the not-at-risk group. Third, findings for domain-specific skills did not fit predictions. There was a nuanced pattern of effects for ANS acuity such that ANS acuity only discriminated children not at risk from those with severe math difficulties not accompanied by difficulties in reading. Phonological awareness was lower in both MDRD and MD groups than in the not-at-risk group, a finding consistent with that in school-age studies of comorbidity; however, phonological awareness was not significantly lower in the MDRD-risk group versus the MD-risk group. Fourth, a direct child measure of attention, specifically, vigilance (the ability to maintain attention over time), was the only cognitive predictor that discriminated children with comorbid MDRD risk from those with MD risk alone, and there was consistency in this finding across cut points.

## Severity of Academic Difficulties in Children With MDRD Risk

Consistent with studies of comorbidity in school-age children that have revealed that academic deficits in MDRD are more severe than those for children with MD (Cirino et al., 2015; Willcutt et al., 2013), we found that by the end of the pre-K year, children with MDRD risk (at or below the 25th percentile in math and reading) had more severe deficits on the TEMA-3, a standardized measure of number and operations, than children with only MD risk. With the cut point at the 16th percentile, the MDRD-risk and MD-risk groups did not differ on the TEMA-3, but did differ on the Child Math Assessment. Because the TEMA-3 does not measure numerical knowledge below 36 months of age, it may be that the test is not sensitive enough to detect differences in mathematical knowledge between the groups at the lowest levels of achievement; in other words, this lack of difference between MD and MDRD-risk groups at the 16th percentile cut point might reflect a floor effect. Note that average math achievement for the MD-risk group was at the 5th percentile for the 16th percentile cut point, thus their very low performance at the lower cut point may have made it less likely that statistically significant differences with another group could be found. Some support for this view is provided by the findings for the Child Math Assessment. The Child Math Assessment may have been better able to discern differences in severity at very low levels of math because it is a test specifically designed to assess a broad range of informal mathematical skills at pre-K.

Similar to comorbidity studies of school-age children we also observed that preschool children at risk for MD demonstrated early literacy skills that were significantly lower than children not at risk, but significantly higher than in the MDRD-risk group. Such findings suggest that the acquisition of math and reading skills depends, in part, on common or overlapping cognitive pathways. It is worth noting, however, that the average early reading scores for the MD-risk groups were well within the average range—at the 45th and 53rd percentiles depending on the cut point used to define the groups.

Preschool children at risk for MDRD or MD by the end of pre-K had lower levels of math knowledge than those not at risk (based on the Child Math Assessment) at the *beginning* of pre-K; however, level of informal math knowledge at the beginning of pre-K did not discriminate the MD-risk group from the MDRD-risk group (see Cirino et al., 2015 for a similar finding in school-age children vs. Jordan, Hanich, & Kaplan, 2003). This suggests that tests of domain specific skills at the beginning of pre-K, prior to formal schooling, might not be sensitive enough to discriminate those children at risk for the most severe learning difficulties by the end of pre-K.

Findings of greater severity of math and reading difficulties in the comorbid MDRD-risk group at this very young age point to the need for studies of whether early signs of comorbidity are stable predictors of significant later pervasive learning difficulties. If comorbid MDRD risk status is relatively stable across time, targeting children with MDRD risk in pre-kindergarten for intensive reading and math interventions would be critical given the negative long term academic and social consequences associated with comorbidity (Willcutt et al., 2007, 2013).

## Domain General and Domain Specific Predictors of MD and MDRD Risk

In keeping with several school-age cognitive profile studies, preschool children with MDRD risk or MD risk had lower levels of most cognitive abilities (i.e., phonological awareness, working memory, nonverbal IQ, vigilance, executive attention) than children deemed not at risk; however, the severity of these cognitive deficits varied by group. The MDRD group scored lower than the other two groups on the variables that differentiated the groups—about a half standard deviation below the grand mean, whereas the not-at-risk group scored a third of a standard deviation above the grand mean, and the MD-risk group scored in between. These findings suggest that comorbid MDRD in pre-K children is also associated with more severe deficits in the cognitive abilities that are associated with acquiring reading and mathematics.

The effects for phonological awareness were large and consistent with Vukovic's (2012) longitudinal findings that phonological awareness was a strong predictor of growth in early mathematics regardless of MDRD or MD status. Explanations for the relation of phonological awareness to reading are well established (Bryant & Goswami, 2016). Longitudinal studies also show an association of linguistic and phonological skills for many, but not all, mathematics outcomes (Barnes et al., 2014; Hecht, Torgesen, Wagner, & Rashotte, 2001; Krajewski & Schneider, 2009; LeFevre et al., 2010; but see Passolunghi, Vercelloni, & Schadee, 2007). Explanations for this relation include the involvement of phonological skills (the precision of phonological representations) in direct retrieval of math facts from memory (De Smedt, Taylor, Archibald, & Ansari, 2010) and in other aspects of arithmetic learning and/or performance that rely primarily on verbal codes (Krajewski & Schneider, 2009).

In keeping with most school-age studies of children with learning disabilities, working memory (visual-spatial, in this case) was lower in children with MDRD and MD risk compared with children not at risk. Working memory did not discriminate the MDRD and MD groups, which is consistent with some studies (e.g., Cirino et al., 2015; Vukovic, 2012; Willcutt et al., 2013), but not others (Peterson et al., 2017; Szűcs, 2016). Because we did not have a measure of verbal working memory we could not test whether different types of working memory might have differentiated the groups as in the Szűcs, 2016 meta-analysis. In a recent meta-analysis, however, type of working memory (verbal, numerical, visual-spatial) did not moderate the relation of working memory to any aspect of math (Peng, Namkung, Barnes, & Sun, 2016), suggesting that it is the executive component of working memory (i.e., concurrent holding and processing of information regardless of type of material) that is most related to math. Our findings are consistent with this view that low working memory, particularly the central executive component of working memory, characterizes children at the beginning of pre-kindergarten at risk for MD or comorbid MDRD. Working memory is a stable predictor of math across the early elementary and middle school years with domain-specific skills becoming more predictive over time (Geary et al., 2017).

In terms of hypotheses regarding domain-specificity, we predicted that ANS acuity would discriminate MDRD and MD groups from the no risk group. Although ANS acuity was once proposed as a core deficit in MD akin to phonological awareness for RD (Butterworth et al., 2011; Mazzocco et al., 2011), more recent findings question this view. Meta-analyses have shown relations of ANS acuity with a range of math outcomes to be significant but small (Chen & Li, 2014; Schneider et al., 2017). Further, several studies suggest that visual spatial working memory and inhibition (Bugden & Ansari, 2016; Fuhs & McNeil, 2013; Purpura & Simms, 2018) and cardinality (Chu, van Marle, & Geary, 2015; Purpura & Simms, 2018) may explain the relations of ANS acuity with math. The ability of ANS acuity to discriminate the groups in the current study was rigorously tested because both visual spatial working memory and inhibition (ability to ignore irrelevant information in the executive attention task) were included in the analyses. We found that ANS acuity was the *only* variable that did *not* differentiate the groups with one exception: the only significant contrast was at the 16th percentile cut point such that the MD group had lower ANS acuity than the not-at-risk group; however the effect size was small. The findings are most consistent with Purpura and Logan (2015), who found an effect of ANS acuity on early math knowledge in the presence of working memory and response inhibition, but only at the lowest quantile of math performance in pre-kindergarten children. Whether there is a subset of children with severe math difficulties but intact early reading whose difficulties in learning math arises, in part, from a core deficit in numerosity awaits further investigation.

Similar to findings in school-age studies of comorbidity, phonological awareness was lower in both MD and MDRD-risk groups than in children without risk for disability. However, the prediction that phonological awareness would be lower in the MDRD group than in the MD group was not supported. Despite average Letter–Word Identification scores in the MD-risk group and below average Letter–Word Identification scores in the MDRD group, the two groups did not differ on the univariate tests of significance for phonological awareness. In preschool children, phonological awareness tasks have stronger relations with executive function measures than do letter naming tasks (Blair et al., 2015; Willoughby et al., 2012). Because pre-kindergarten letter identification abilities are strong predictors of first grade word reading (Piasta et al., 2012), we do not think that the lack of a significant difference in phonological awareness on the univariate comparison of the MDRD versus the MD groups invalidates the study findings. Rather, we suggest that when phonological awareness is measured at the beginning of pre-kindergarten using elision and blending, it constitutes a novel task for low SES children that draws heavily on linguistic, memory, and attentional resources to a greater extent than similar tasks in older children.

Despite the importance of attention for all levels of information processing, its status as a foundational skill from which more complex executive and other cognitive processes develop (Garon, Bryson, & Smith, 2008; Posner & Rothbart, 2007), and the strong links of inattention with learning disabilities (reviewed in Tannock, 2013), child measures of attention have not been previously investigated in the school-age studies of comorbidity. In this study, vigilance (i.e., the ability to maintain focus across time) produced large effects when comparing children not at risk to those with MD risk and MDRD risk. Executive attention (i.e., the ability to focus on relevant information and inhibit or ignore irrelevant information) also showed large effects in three of four comparisons involving MDRD risk or MD risk versus children not at risk. Furthermore, and critical to the main goal of the study, was the finding that deficits in vigilance discriminated the comorbid MDRD-risk group from the MD group at both cut points.

These findings suggest that at the start of pre-kindergarten, children with poor ability to sustain attention across time are those who may be most at risk for the most significant learning difficulties in both reading and math. Replication of this finding is important to determine whether the assessment of sustained attention abilities at the beginning of pre-kindergarten might add to the prediction of pervasive learning difficulties prior to formal instruction in school. We say this keeping in mind that domain-specific assessments of math knowledge and phonological awareness at the beginning of pre-kindergarten were not sensitive predictors of which children went on to have the most severe math and reading difficulties at the end of pre-kindergarten.

Behavioral measures of inattention (i.e., ratings of attention-related behaviors) show strong relations with mathematics (e.g., Cirino, Fletcher, Ewing-Cobbs, Barnes, & Fuchs, 2007; Greven, Kovas, Willcutt, Petrill, & Plomin, 2014; Raghubar et al., 2009). In school-age populations ratings of inattention are a particularly strong predictor of long-term academic deficits (Rabiner et al., 2000; Sayal, Washbrook, & Propper, 2015), are characteristic of many students who do not adequately respond to intervention (Al Otaiba & Fuchs, 2002), and have been implicated as characteristic of individuals with persistent reading problems (Shaywitz & Shaywitz, 2008). However, assessing attention using teacher-endorsed behavioral attention questionnaires poses difficulties for determining exactly what is being measured. For example, teacher ratings of behavioral aspects of attention may be affected by teacher perceptions of student academic functioning rather than being more independent assessments of attention behaviors (Fuchs et al., 2006). Given that many studies of attention and math have used teacher- or parent-rated measures of child attention, we also examined data from the Children's Behavior Questionnaire (Rothbart, Ahadi, Hershey, & Fisher, 2001), which includes teacher-rated assessments of attentional focus, inhibitory control, and hyperactivity in preschool children. In our sample, we did not find that teacher-endorsed measures of attention behaviors discriminated the groups in the same way as performance on the direct child measure of attention. Questions on the Child Behavior Questionnaire regarding overt attention-related or disruptive behaviors in pre-kindergarten children may not adequately capture difficulties in those internal aspects of attention (i.e., vigilance) that are important for learning (Chun, Golomb, & Turk-Browne, 2011).

## Limitations

Despite the novel findings from this pre-K study of risk for comorbid learning difficulties, there are several limitations. First, were unable to include an RD-risk group in the analyses because the sample of RD risk only children was small in this study. This was likely because participants for the parent study were recruited based on very low math knowledge at the beginning of pre-kindergarten. Specific math screening at the start of pre-K was intended to select a sample of children who were at risk for math difficulties more so than for reading difficulties, and the very small RD-risk subgroup does suggest some early specificity in academic difficulties. Studies of very young children that do not overselect for one type of academic difficulty would be helpful as would dimensional studies of young children (e.g., Blair, Ursache, Greenberg, & Vernon-Feagans, 2015) that look at the domain-specific and domain-general cognitive abilities that differentiate reading from math and that contribute to their overlap (e.g., Child et al., 2019; Peterson et al., 2017). With respect to this latter point, it should also be noted that the current study did not include measures of cognitive abilities (e.g., information processing speed) that have been associated in some school-age studies with the overlap of difficulties in reading and math. Nor did the study include measures of math language skills at the beginning of pre-kindergarten, which have strong predictive power for classification of math difficulties at the end of the pre-K year (Purpura, Day, Napoli, & Hart, 2017). Although the inclusion of large subgroups of English- and Spanish-speaking children in the study might affect generalization of the findings to other pre-K populations, it is also an advantage that the findings are consistent across these two pre-K language groups, which make up the largest majorities of the pre-K student population in the United States. Finally, a weakness of our study, as well as several studies of comorbidity, concerns the use of single indicators to measure constructs of interest, both for cognitive correlates as well as with respect to classification of disability groups. Latent factors would also allow one to use a latent class analysis approach as an alternative to cut points.

Math knowledge at school entry is a strong predictor of later school achievement (Duncan et al., 2007), and low math knowledge at the beginning of kindergarten is a strong and relatively stable predictor of later math disability (Morgan et al., 2009). Given the very young age of our participants, however, we are unable to say that these children have difficulties in math or reading beyond "at-risk" status. Although we followed children to kindergarten in this study, the main focus of the study was mathematics and we did not assess reading achievement in kindergarten. Future studies should follow pre-kindergarten children deemed to be at risk for learning disabilities in math and/or reading into the early primary grades to determine the sensitivity and specificity of early direct measures of child attention for predicting MD, RD, and particularly MDRD—learning disability in its most severe form.

## Conclusions

This is the first study to examine cognitive profiles that distinguish pre-K children at risk for MD and MDRD from each other and from children not considered to be at risk. Similar to studies of older school-age children, pre-K children with MDRD risk had more severe difficulties in both academic domains as well as in several reading- and math-related cognitive abilities than children with MD risk and those not at risk. These findings are consistent with a view that comorbidity results in the most severe form of learning disorder and suggest this pattern may be discernable as early as the end of the pre-kindergarten year. The study also produced three new findings. First, ANS acuity distinguished children with very low math achievement but intact early literacy from those not at risk for learning difficulties. Whether one source (of many) of early severe difficulties in math arises from a deficit in basic numerosity discrimination requires further investigation. Second, direct child measures of attention, particularly vigilance—the ability to focus or sustain attention over time—was the strongest discriminator of children at risk from those not at risk. Third, vigilance was the only cognitive ability that discriminated the MD risk from the MDRD-risk group, consistent with a domain-general model of comorbidity in which impairments in attention are purported to have a global influence on learning, leading to problems across academic domains (Ashkenazi et al., 2013). Individual differences in the ability to sustain attention across time should be further studied for their ability to predict, at an early age, increased risk for and severity of learning difficulties. Such information might inform early identification procedures as well as early interventions.

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