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The Interaction of Worked-Examples/ Self-Explanation Prompts and Time on Algebra Conceptual Knowledge

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Abstract

Success in Algebra I often predicts whether or not a student will pursue higher levels of mathematics and science. However, many students enter algebra holding persistent misconceptions that are difficult to eliminate, thus, hindering their ability to succeed in algebra. One way to address these misconceptions is to implement worked-examples and self-explanation prompts, which have been shown to improve students' conceptual knowledge. However this effect seems to be greater after a delay. The current study sought to explore such time-related effects on algebra conceptual knowledge. In a year-long random-assignment study, students either studied worked-examples and answered self-explanation prompts (n = 132) or solved typical isomorphic problems (n = 140). A three-way mixed ANCOVA (pre-algebra knowledge x condition x time) found a significant condition by time effect. The growth of algebra conceptual knowledge was greater for students studying worked-examples than for those solving typical problems.

Keywords: worked-examples; self-explanation prompts; algebra; conceptual knowledge

Introduction

Algebra I is often considered to be a gate-keeper course, meaning that a student's success in the course often determines whether he or she will continue on to a higher level mathematics or science course (U.S. Department of Education, 1997). Furthermore, students in the United States tend to struggle mastering algebra concepts, potentially contributing to the lower enrollment of U.S. college students in mathematics and science related majors compared to competing countries.

The newly implemented Common Core State Standards (CCSSI, 2010) stresses the importance of both procedural and conceptual knowledge of mathematics content. However, especially when it comes to algebra, students hold persistent misconceptions, which hinder their ability to master the content. In fact, students often enter Algebra I holding strong misconceptions that may impact their success mastering algebra content (Brown, 1992; Chiu & Liu, 2004; Kendeou & van den Broek, 2005). For instance, misconceptions such as believing that the equals sign is an indicator of operations to be performed (Baroody & Ginsburg, 1983; Kieran, 1981; Knuth Stephens, McNeils, & Alibali, 2006), that the negative sign represents only the subtraction operation and does not modify terms (Vlassis, 2004), that subtraction is commutative (Warren, 2003), and that variables cannot take on multiple values (Booth, 1984; Knuth et al., Kuchemann, 1978) are all thought to be critical. Holding such misconceptions have been shown to hinder students' success in problem solving (Booth & Koedinger, 2008).

A large body of research supports the notion that eliminating mathematics misconceptions is not an easy task. In fact, many students continue to hold these misconceptions after traditional classroom instruction (Booth, Koedinger & Siegler, 2007; Vlassis, 2004). Often in order to challenge a student's misconception, one must directly draw out and confront the faulty thinking (Donovan & Bransford, 2005). A combination of worked-examples and self-explanation prompts has been used to do just that.

Worked-examples, which are mathematics problems with worked-out solutions, provide the opportunity to point out common misconceptions to students. Some textbooks offer a small number of worked-examples, often at the beginning of a chapter or section. However, research indicates that interleaved worked-examples, alternating between worked-examples and problems for students to solve, are more beneficial to learning (Clark & Mayer, 2003; Sweller & Cooper, 1985).

Furthermore, the benefit of worked-examples can be improved with the inclusion of self-explanation prompts, which are questions that prompt students to explain their reasoning. When students self-explain, they are able to integrate various pieces of knowledge, fill gaps in their own knowledge, and make new knowledge explicit (Chi, 2000; Roy and Chi, 2005). Students at all ability levels who are prompted to self-explain learn more than those who do not self-explain (Chi, de Leeuw, Chiu & Lavancher, 1994).

Often if a textbook uses a worked-example, it displays a correct problem solution. However, incorrect worked-examples have also shown benefits to learning. In empirical laboratory students, students who are asked to explain the errors in incorrect solutions, as well as explain effective strategies in correct examples, learn more than students who are asked to only explain correct examples (Durkin & Rittle-Johnson, 2009; Siegler & Chen, 2008).

While the use of worked-examples and self-explanation prompts have been shown to improve learning, often there is a delayed-effect, meaning that the effect is larger on a delayed post-test rather than immediately after the intervention. For instance, Adams and colleagues (2014) found that while students solving isomorphic problems with feedback and students studying incorrect examples did not differ significantly at immediate posttest, students in the incorrect example group scored significantly better on a delayed posttest compared to the problem-solving group. This suggests that the worked-example/self-explanation effect may improve over time.

Current Study

This study applies previous laboratory research supporting the use of both correct and incorrect worked-examples paired with self-explanation prompts to the classroom. While highly controlled laboratory studies are necessary when developing theories, applied studies are needed in order to investigate the limits of generalization.

We explore the effects of studying worked-examples and answering self-explanation prompts compared to solving typical isomorphic problems on students' algebra conceptual knowledge. We hypothesize that students who study worked examples and answer self-explanation prompts will have less algebra misconceptions and, therefore, will have higher conceptual knowledge compared to those who solve traditional isomeric problems.

Finally, the current study will explore students' conceptual knowledge growth over the course of a full school year, extending the evidence to support a delayed effect by providing longitudinal evidence from repeated interventions.

Methods

Participants

Participants included 562 Algebra I students from 28 classrooms (12 teachers) from five school districts across the United States. The sample was 49% female. Students were classified as underrepresented minority (URM; Black, Hispanic, biracial) or non-URM (White, Asian); 65% of the students were classified as URM. Participants were also socioeconomically diverse, with 52% coming from families who qualified for the Free or Reduced Lunch program (FRL).

Due to the restrictions of repeated-measures ANOVA, only students who completed all four quarterly exams were included in the analysis. Due to natural attrition (i.e. students leaving the school or absence on the day of the quarterly exam) the sample was reduced to 272; 51% female, 61% URM, 50% FRL.

Classrooms were randomly selected to either complete problem- or example-based worksheets yielding 14 problem-based (n=140) and 14 example-based (n=132) classrooms. Of the 12 teachers, eight taught one class of each condition; however, two teachers instructed two classes of the problem-based condition and one class of the example-based condition, while two others instructed two example-based classes and one of the problem-based class.

Procedure

Intervention During the school year, teachers taught the algebra content using their own typically teaching methods; however, they were asked to sporadically assign the 42 study-worksheets at times they deemed appropriate during the year. Teachers did not have to assign the worksheets if they did not cover that material in their curriculum. On average, teachers assigned 27 worksheets (ranging from 15 to 40) throughout the year. There was no significant difference in the number of worksheets assigned between groups, with the problem-based group completing an average of 28 worksheets and the example-based group completing an average of 26 worksheets, p >.05. Teachers were given the freedom to assign the worksheets in any order and were told to treat the assignments as they would any other assignment in their class; however they were instructed to have students complete the assignments during the class period, not for homework. Students were allowed to work together if the teacher typically permitted that behavior. Each assignment took about 20 minutes to complete.

The worksheets of both conditions contained four problem-sets (with two math problems similar to each other per set). The problem-based worksheets contained four regular problem-sets where students were asked to simply solve each problem, similar to a typical math worksheet. The example-based worksheets replaced one math problem within each set with a worked-example and self-explanation prompt(s). Students in this group were instructed to study the worked-example, answer the self-explanation prompt, and complete the second math problem on their own. Each example-based worksheet contained two correct worked-examples and two incorrect worked-examples. See Figure 1 for sample problem- and example-based problem sets.

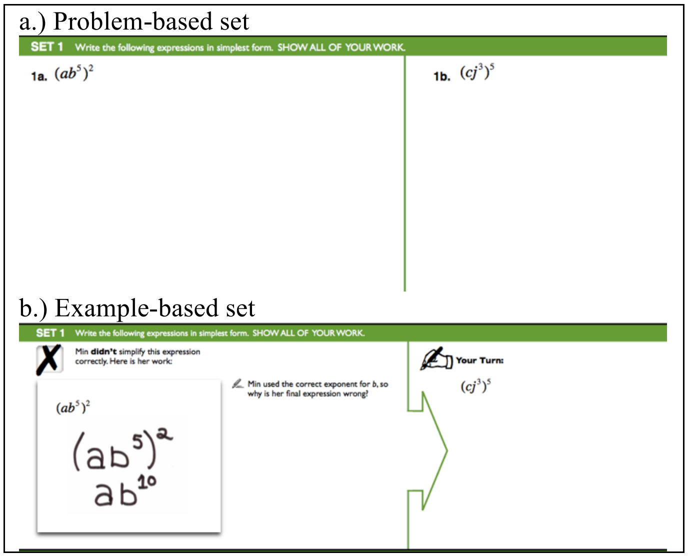

Figure 1. Sample problem- and example-based problem sets.

Assessment At the beginning of the school year, all students were given a pre-test assessing their pre-algebra knowledge. Throughout the school year, students were given four quarterly exams. The four exams contained the same 18 items, however teachers were asked to only assign the test items taught to date; therefore, students were not answering items containing content they were not already taught. This exam assessed both procedural and conceptual algebra knowledge. At the conclusion of the year, students were given a post-test, consisting of 10 Algebra I standardized-test release items.

At the end of the school year, each school provided the researchers with student demographic information, such as gender, ethnicity, and free or reduced lunch qualification. Finally, teachers completed a survey answering questions about their use of the worksheets. The survey contained questions such as "How often did you review the worksheets with the students after completion?"

Measures

Algebra Conceptual Knowledge The quarterly benchmark exams consisted of 18 items, each of which had multiple parts, yielding a total of 71 sub-items. Of these 71 sub-items, 46 measured students' conceptual knowledge of algebra content. We operationally define conceptual knowledge as an understanding of the core features in problems for a given topic (e.g. Booth, 2011). Algebra conceptual knowledge scores were calculated for each quarter by dividing the number of correctly answered items by 46. This score does not take into account the number of items attempted since each teacher assigned a different number of items each quarter.

Pre-algebra Knowledge The pre-algebra exam was given at the start of the school year before students completed any study-worksheets. This exam covered content necessary for the success in an algebra course, such as the understanding of equality and difference between coefficient and constant. This exam consisted of 11 items with 71 sub-items. Pre-algebra knowledge scores were calculated by dividing the total number of correctly answered items by 71.

Teacher Reports At the end of the year, teachers were administered a survey about their experience in the study. In one item, they were asked about the frequency with which they reviewed study assignments in class. Teachers responded by selecting one of the following options: 0-20% of the time, 20-40% of the time, 40-60% of the time, 60-80% of the time or 80-100% of the time. Teachers' responses were recoded into a 1 (0-20%) to 5 (80-100%) scale.

All measures were scored and coded by two researchers, checking for internal and external consistency.

Results

The following analysis explores the effects of time, pre-algebra knowledge and condition on students' algebra conceptual knowledge. Pre-algebra knowledge was included in the model because student' prior-knowledge is known to greatly influence their future learning. While other outcome measures (i.e. procedural knowledge and standardized test release items) were collected, they are beyond the scope of this study focused on conceptual knowledge growth. The other measures will or are presented in other reports. Finally, URM status and rate of teacher review were included as covariates because differences were found between conditions.

A three-way mixed ANCOVA was run to understand the effects of pre-algebra knowledge, condition, and time on algebra conceptual knowledge. The rate of review and minority status were included as covariates. Using Greenhouse-Geisser estimates, the interaction between condition, pre-algebra knowledge and time was not statistically significant; however, there was a statistically significant two-way interaction between time and all between-subject variables. See Table 1 for results.

Table 1. Greenhouse-Geisser estimates for 3-way ANCOVA for algebra conceptual knowledge.

	df	*F*	*p*	partial η^2
Quarter	2.513	10.826	<.001	.055
Quarter x URM	2.513	4.333	.008	.023
Quarter x Rate of Review	2.513	17.900	<.001	.088
Quarter x Pre-algebra	201.058	1.482	<.001	.389
Quarter x Condition	2.513	3.991	.012	.021
Quarter x Condition x Pre-algebra	118.121	.957	.608	.195
Residual	467.459			

See Table 2 and Figure 2 for condition by time estimated marginal means. At quarter 1, the example-based group scored slightly lower than the problem-based group; however, by quarter 4, the example-based group outscored the problem-based group.

Table 2. Condition by time estimated marginal means with 95% confidence intervals.

				95% CI	
Condition	Quarter	Mean	SE	Lower	Upper
Problem-based	1	.179	.008	.163	.195
	2	.321	.011	.299	.343
	3	.435	.012	.413	.458
	4	.516	.016	.484	.548
Example-bases	1	.169	.008	.153	.186
	2	.332	.011	.310	.354
	3	.447	.012	.423	.470
	4	.568	.017	.535	.601

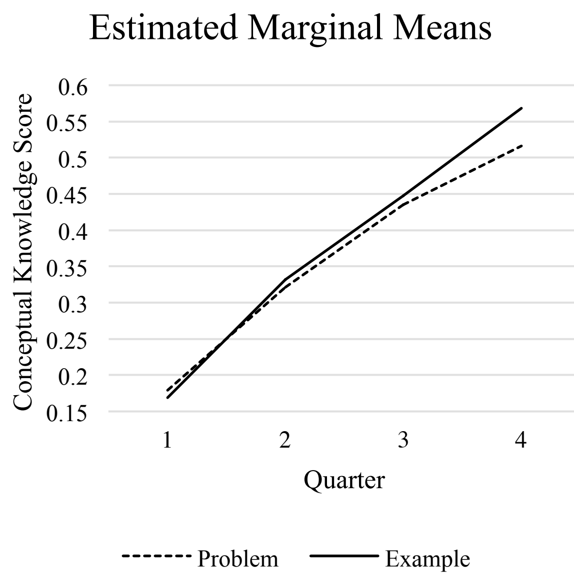

Figure 2. Condition by time estimated marginal means.

Discussion

Due to the nature of the quarterly exams, it was expected that students would score better over time. As mentioned in the procedure section, the conceptual knowledge portion of the quarterly exam consisted of 46 sub-items. However, students only attempted to answer the items in which they were familiar with. Therefore, students attempted to answer more items as they covered additional content over the course of the school year, leading to potential increased scores over time. However, we were more interested in the interaction between treatment and time. It was hypothesized that there would be differences in the rate of algebra conceptual knowledge growth between the example-based and problem-based groups.

As predicted, this analysis revealed a significant condition by time interaction. At the end of quarter 1, students solving typical algebra problems, in the problem-based group, scored slightly better than students in the example-based condition. However by the end of quarter 2, the opposite occurred. Students studying worked-examples and answering self-explanation prompts scored slightly higher than those in the problem-based group. This gap continued to widen throughout the remainder of the school year. By quarter 4, example-based students scored an average of 5 percentage points higher on the algebra conceptual knowledge test than the problem-based students, which is supported by previous studies finding a delayed effect (i.e. Adams et al., 2014).

The limitations of this study include a sample restricted to those present for all four quarterly exams. In addition, although a within-teacher design controlled for teacher-related variables, it is possible that there was some contamination across classrooms. For instance, some teachers reported using a few of their own worked-examples with their problem-based classroom. The current analysis was based on linear growth; further studies should consider using a more robust analysis in order to account for possible quadratic or cubic growth curves.

This analysis adds to the current body of research by providing evidence from the classroom to support laboratory findings. It also extends our understanding of the short-term benefits of worked-examples and self-explanation prompts by offering longitudinal data. Our findings emphasize the need to measure learning over longer time intervals.

Based on these findings, it is suggested that teachers interleave worked-examples and self-explanation prompts with traditional algebra problems. In order to receive maximum benefit, students should be exposed to this approach consistently throughout the entire school year, not just in a single instance. Furthermore, such interventions should be interleaved in algebra textbooks, rather than simply displaying a few correct worked-examples at the beginning of a section. Finally, both correct and incorrect worked-examples should be used in the classroom to promote maximum benefit.

As previously noted, success in Algebra I is a known gatekeeper to later mathematics and science success. However, many students enter algebra with persistent misconceptions that obstruct their achievement in algebra. The findings from this study suggest that using worked-examples combined with self-explanation prompts as classroom practice materials can improve student's conceptual knowledge, consequently decreasing their misconceptions. The findings from this study are particularly exciting as they come from a study that took place in actual classrooms and not research laboratories. Due to the setting of the current study, our findings illustrate that even when precision, like that provided in a laboratory, cannot be guaranteed the positive effect of using worked-examples paired with self-explanation prompts is still seen.

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