Kindergarteners at Risk for Severe Mathematics Difficulties: Investigating Tipping Points of Core Mathematics Instruction

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Abstract

A concerning number of students enter kindergarten facing an intractable variation of mathematics difficulties (MD). This study investigated the impact of an explicit, core kindergarten mathematics program on the mathematical outcomes of kindergartners who demonstrated severe MD at kindergarten entry and examined whether these students improved from a category of high MD risk (i.e., <10th percentile) to a lower risk of MD (i.e., at or above the 10th, 20th, and 40th percentiles) between the fall and spring of kindergarten. Differential response to the program based on the classroom-level proportion of students with severe MD was also explored. A total of 795 kindergarteners with severe MD from 122 classrooms were included in the analyses. Results suggested students with severe MD in treatment classrooms improved from fall to spring at a greater rate than their control peers. Treatment students also demonstrated higher rates of improvement from below the 10th percentile to at or above the 10th, 20th, and 40th percentiles across the school year. No evidence of differential efficacy of the program by the classroom-level proportion of students with severe MD was found. Implications for using explicit mathematics programs to thwart the onset of severe MD among academically vulnerable students are discussed.

Kindergarteners with Severe Mathematics Difficulties and their Response to Core Mathematics Instruction

Recently, the National Science and Technology Council (2018) tasked the field with increasing diversity, equity, and inclusion in science, technology, engineering, and mathematics (STEM). At the forefront of the council's charge is promoting strong foundations in all areas of STEM so that students can achieve overall academic success throughout public school, obtain meaningful postsecondary experiences, and contribute to the nation's STEM workforce. In mathematics, this attention toward STEM literacy is timely as compelling research suggests that a considerable number of students enter kindergarten lacking mathematical readiness and that such skill deficits are consequential for later mathematics learning (Bodvoski & Farkas, 2007; Duncan et al., 2007; Morgan, Farkas, & Wu, 2009). For example, examining longitudinal data from nearly 8,000 students in the Early Childhood Longitudinal Study – Kindergarten (ECLS-K), Morgan et al. (2009) found that students entering and exiting kindergarten with performances below the 10th percentile on a nationally normed mathematics assessment had a 70% chance of experiencing persistent difficulties in mathematics throughout the later grades. These data clearly suggest that unless educational research and practice focuses on the instructional needs of these struggling learners during the kindergarten year, many will experience low growth trajectories in mathematics and face an eventual a pathway of mathematical failure.

The current study focuses specifically on the mathematics achievement of students who begin their kindergarten year performing below the 10th percentile on a standardized mathematics assessment. Aligned with existing research (i.e., Fletcher, Lyon, Fuchs, & Barnes, 2019; Morgan et al., 2009), we considered those students with beginning-of-kindergarten mathematical performances at the bottom 10% as demonstrating "severe" mathematics

difficulties (MD). Students who exhibit such profound learning problems at the start of kindergarten are at high risk for experiencing a more persistent and intractable type of MD. From a perspective of a multi-tiered system of support (MTSS; Fuchs & Vaughn, 2012), students who display severe MD in kindergarten are academically vulnerable. Consequently, they have a strong propensity for needing Tier 3 intervention supports and becoming eligible for special education services (Compton et al., 2012).

Among the considerable number of kindergarten students who exhibit severe MD, many come from economically and educationally disadvantaged households, and thus do not obtain support at home or in preschool to develop adequate readiness for kindergarten mathematics (Barnes et al., 2016). And unlike many of their peers, including even those who perform in the low average range at the start of kindergarten (e.g., at or below the 25th percentile), students with severe MD enter kindergarten with deep knowledge gaps around number and numeration. Consequently, this lack of “number sense” (Berch, 2005) impacts their ability to learn and understand the kindergarten mathematics curriculum. Despite compelling evidence that severe MD in kindergarten leads to long-term difficulties in mathematics (Morgan et al., 2009), few, if any, methodologically rigorous studies have investigated whether and to what extent these at-risk learners respond to the core mathematics instruction delivered in kindergarten classrooms.

Research on Core Mathematics Instruction in Kindergarten Classrooms

Core (Tier 1) mathematics instruction is integral at all grade levels. In kindergarten, however, core mathematics instruction is of critical importance because it serves as a valuable window of mathematical learning for all students (Doabler et al., 2015; Jordan et al., 2009; Morgan et al., 2009). For many students, the core instruction delivered during the kindergarten year represents their initial exposure to the foundational concepts and skills of early

mathematics, such as the principle of cardinality, magnitude comparisons among numerical quantities, and basic number combinations (Geary et al., 2018). Moreover, because the time and resources put towards beginning reading in kindergarten classrooms often result in limited time for mathematics instruction (La Paro et al., 2009), core instruction may represent the totality of mathematics instruction for kindergarten with severe MD. Therefore, the core mathematics instruction delivered in kindergarten classrooms has important implications for all kindergarten students (Doabler et al., 2015). Not only is core mathematics instruction charged with allowing typical achieving kindergarten students to learn and progress successfully, it is also responsible for establishing a positive trajectory of mathematical learning among those kindergarteners who face severe MD beginning at the start of the kindergarten year.

Against that backdrop, researchers have begun to investigate different instructional approaches for improving the efficacy of core mathematics instruction in kindergarten classrooms. One instructional approach that has shown strong promise in these instructional settings for supporting mathematics proficiency among the full range of learners, including kindergarten students at risk for MD, is explicit mathematics instruction. For example, Sood and Jitendra (2013) investigated the impact of an explicitly-designed, Tier 1 kindergarten mathematics program focused on promoting students' knowledge of early number sense concepts. Results suggested significant differences in student mathematics achievement favoring the number sense intervention program relative to the comparison condition. Reported effect sizes (Hedges' *g*) ranged from 0.55 to 1.44.

In a more recent randomized controlled trial, Clarke et al. (2015) investigated the efficacy of the Early Learning in Mathematics (ELM) program. ELM is a yearlong, core (Tier 1) kindergarten mathematics program that is delivered in whole-class settings and aimed at

supporting students' development of mathematical proficiency with concepts of whole numbers, measurement, and geometry identified in the Common Core State Standards for Mathematics (CCSS-M, 2010). The program's lessons are invariably grounded in empirically-validated principles of explicit mathematics instruction (Coyne et al., 2011; Gersten et al., 2009). In this way, the program systematically facilitates instructional interactions around critical mathematics content. Such interactions are comprised of teachers offering overt demonstrations of new mathematical content, facilitating guided and independent practice opportunities for students to develop mathematical proficiency, and providing timely academic feedback to confirm student responses and address potential misconceptions.

Participating in the ELM efficacy trial were approximately 2,600 kindergarten students from 129 kindergarten classrooms (Clarke et al., 2015). In the study, Clarke et al. randomly assigned the 129 kindergarten classrooms, blocking on schools, to either treatment or control conditions. Classrooms in the treatment condition implemented the ELM program, while control classrooms continued to use standard district practices (business-as-usual). Results indicated that ELM classrooms did not significantly differ from control classrooms (g = 0.11). Findings did, however, reveal evidence of a moderation effect, suggesting that students who began the kindergarten year below the 25th percentile on the Test of Early Mathematics Ability – Third Edition (TEMA-3; Ginsburg & Baroody, 2003) demonstrated the strongest gains across the school year. Clarke et al. also reported that treatment students who tested above the 25th percentile on the pretest distribution of the TEMA-3 remained "on track" for developing mathematics proficiency relative to their non-risk control peers.

While findings from Clarke et al. (2015) suggested a pattern of differential response among students considered at "some" risk for MD (i.e., <25th percentile), it did not fully

establish the impact of the ELM program on the mathematics achievement of kindergartners with severe MD (i.e., <10th percentile). For example, it did not examine whether ELM enabled accelerated learning gains for these students such that their mathematical performance improved beyond a threshold of severe MD by the end of kindergarten. Moreover, Clarke et al. (2015) did not investigate the composition of kindergarten classrooms to determine whether and to what extent the proportion of kindergarten students with severe MD in a kindergarten classroom might moderate the effects of the ELM program. Recognizing that schools have few resources available to provide instructional support in kindergarten mathematics beyond Tier 1 educational settings, it seems paramount to examine and unpack the potential benefits of explicit, core mathematics instruction for kindergartens who enter school facing severe MD.

Explicit Instruction and Tipping Points of Efficacy

Despite the potential benefits of explicitly-designed core mathematics instruction (Gersten et al., 2009), it is likely that this instructional approach may reach a point at which its effects on student mathematics achievement begin to slow or completely level off. In kindergarten classrooms, this tipping point may be a function of the proportion of students who perform well below average in a given classroom. Thus, a large proportion of students with severe MD in a given classroom may stifle a teacher's ability to deliver effective core mathematics instruction, such as facilitating the necessary number of mathematics verbalizations that students with severe MD, require to build proficiency with early mathematical concepts. Interestingly, research has found this notion of "diminishing returns" (Doabler et al., 2019; Ritter & Schooler, 2001) to be true even when teachers are equipped with empirically-validated, explicitly-designed core academic programs.

In the area of reading, for example, Vaughn et al. (2017) investigated the impact of Promoting Adolescents Comprehension of Text (PACT) on the reading comprehension of

middle school English learners (ELs) with learning difficulties. PACT represents a bundle of explicit instructional practices designed to promote middle school students' knowledge in the content areas of science and social studies. While the primary aim of the randomized controlled trial was to test the treatment effects of PACT on the social studies outcomes of middle school ELs, Vaughn et al. (2017) also explored whether the proportion of ELs in classrooms might moderate the intervention's impact. In other words, Vaughn and colleagues were interested in establishing whether there was a specific classroom-level proportion of ELs that would predict a tipping point in the beneficial effect of PACT. Results suggested that when the percentage of ELs in a given classroom exceeded 14%, the effects of PACT began to mitigate. A more recent randomized controlled trial involving a sample of non-ELs reported similar thresholds for the PACT intervention (Vaughn et al., 2019).

The work of Vaughn and colleagues (2017; 2019) on classroom composition was particularly cogent in that it helped shed light on a blank spot in the empirical literature on mathematics intervention research. We know of no mathematics intervention studies that have investigated whether the classroom-level percentage of students with severe MD at the start of the school year influences the yearlong impact of core mathematics instruction that teachers provide in kindergarten classrooms. This is surprising given that core mathematics instruction in kindergarten plays a pivotal role in supporting students' development of mathematical proficiency (Morgan et al., 2009).

Similar to the findings of Vaughn et al. (2017; 2019), large proportions of students with severe MD in kindergarten classrooms may mitigate teachers' capacity to deliver highly effective core mathematics instruction even when they are equipped with empirically-validated, core mathematics programs. When the classroom-level proportion of students with severe MD crests

a particular threshold (e.g., > .25), teachers may be forced to slow the pace of instruction, reteach concepts, and limit the number of instructional interactions for students to engage in critical mathematics content. For example, students may have limited time to work with concrete manipulatives and engage in meaningful mathematical discourse. Thus, establishing the point at which the effects of core mathematics instruction on student mathematics achievement slows or completely levels off as a function of the classroom-level proportion of students with severe MD could help pinpoint when additional classroom-level strategies or support systems (e.g., Tier 2 or 3) are necessary to accelerate the mathematical learning of students who enter kindergarten significantly "off track" for developing mathematical proficiency. This study aimed to identify these particular thresholds.

Purpose of the Study

The current study sought to expand the work of Clarke et al. (2015) by specifically examining the impact of the ELM mathematics program on the mathematical outcomes of students who entered kindergarten with severe MD (i.e., < 10th percentile on the TEMA-3). Additionally, we were interested in the notion of whether a well-designed and delivered, core mathematics program could accelerate students' mathematical performance to a point where students who experience severe MD at the beginning of kindergarten get "on track" for developing mathematics proficiency by the end of the school year. It was hypothesized that the rate of improvement across the school year for students in ELM classrooms who experience severe MD at pretest would exceed that of students with severe MD in control classrooms. Progress on a norm-referenced mathematics outcome measure from a category of "high risk" or severe MD (< 10th percentile) at the start of kindergarten to at or above the 10th, 20th, 30th, and 40th percentiles at the end of kindergarten served as targeted indices of improvement. Finally,

we examined whether and to what extent the classroom-level proportion of kindergarten students with severe MD moderated the effects of the ELM program. As with other core instructional programs (e.g., Vaughn et al., 2017), the efficacy of ELM may begin to depreciate once the proportion of students with severe MD reaches a particular magnitude. Three research questions were addressed:

1. What is the effect of the ELM program on the mathematics achievement of students with severe MD at the start of the school year?
2. To what extent does ELM accelerate student mathematics performance above various MD thresholds (i.e., norm-referenced performance at or above the 10th, 20th, 30th, and 40th percentiles)?
3. Does the classroom-level proportion of students with severe MD moderate the effect of ELM on the outcomes of students with severe MD?

Method

Research Design and Database

This study analyzed data collected during a federally-funded efficacy trial aimed at investigating the impact of the ELM kindergarten mathematics program (Clarke et al., 2015). The ELM Efficacy Trial was conducted in Oregon and Texas, respectively, during the 2008-2009 and 2009-2010 school years. Blocking on schools, 129 kindergarten classrooms were randomly assigned to either treatment (ELM; $n = 68$) or control (district-approved kindergarten mathematics instruction; $n = 61$) conditions. In the aggregate, the original sample included 2,598 kindergarten students attending 129 classrooms in 46 schools.

Because the primary focus of the current study was students with severe MD, we included only those treatment and control classrooms that enrolled students who tested below the

10th percentile at pretest on the TEMA-3. From the original sample of 129 kindergarten classrooms, 7 classrooms (2 treatment, 5 control) were dropped because they did not include students with severe MD at the start of kindergarten. In total, the analytical sample for the current study included 122 kindergarten classrooms with 2,454 students, of which 32% (n = 795) were considered as demonstrating severe MD upon entering their kindergarten year. Data analyzed in the current study included student mathematics outcome data collected from the 795 students with severe MD, documenting their gains in mathematics achievement from the beginning to the end of kindergarten.

Teacher and Student Sample

The 122 classrooms (66 treatment, 56 control) were from 45 schools located in 3 school districts in Oregon and 4 school districts in Dallas, Texas. Teachers in treatment classrooms delivered the ELM program, whereas control classrooms continued to offer standard district kindergarten mathematics instruction (business-as-usual). Of the 122 classrooms, 96 were located in public schools, 9 were in charter public schools and 17 were in private schools. All charter and private school classrooms were located in Texas. Public school classrooms were located in schools eligible for Title 1 funding. Table 1 provides descriptive information about the classrooms by condition. The majority of classrooms (87%) provided a full-day kindergarten program versus a half-day program. All half-day classrooms were located in Oregon. Mathematics instruction in all classrooms was delivered in English. Average class size for treatment and control classrooms was M = 21.3 (SD = 3.6) and M = 20.1 (SD = 3.7), respectively. The 122 participating classrooms were taught by 123 teachers. One control classroom in Oregon had two teachers, each working a half-day schedule. All teachers participated for the duration of the ELM Efficacy Trial.

Nested within the 122 classrooms were 795 kindergarten students with severe MD at the start of kindergarten. Students were determined as demonstrating severe MD based upon a pretest performance below the 10th percentile on the TEMA-3. Of the 795 students, 447 and 348 were in treatment and control classrooms, respectively. The number of students with severe MD in treatment classrooms ranged from 1 to 18, while the range in control classrooms was 1 to 15. As shown in Table 1, students with severe MD in both conditions were comparable in terms of gender, race, and percentage identified for special education. Treatment and control classrooms also had equivalent proportions of students with severe MD at pretest.

ELM Kindergarten Mathematics Program

ELM is a core kindergarten mathematics program that consists of four quarterly teacher manuals, each containing 30 daily lessons. Mathematics content is systematically introduced, reviewed, and extended through ELM's explicit instructional design framework. For example, each manual offers scripted guidelines to support teachers in demonstrating key mathematics content, delivering timely academic feedback and facilitating frequent students practice opportunities, including structured verbal interactions between teachers and students, and among students, around key mathematics content. Such practice opportunities are systematically designed to help students build mathematical proficiency, and develop mathematical language and vocabulary. To build conceptual understanding, lessons incorporate frequent opportunities for students to work with visual representations of mathematical ideas, such as 3-D shapes, counting blocks, and numbers lines.

Mathematics domains targeted in ELM include: (a) counting and cardinality, (b) operations and algebraic thinking, (c) number and operations in base ten, (d) measurement and data, and (e) geometry. Mathematics vocabulary is explicitly taught in each lesson. Daily lessons

last approximately 45 minutes in duration and include (a) whole-class and small-group activities focused on new mathematical content, (b) judicious review of previously learned material, and (c) worksheet activities that provide students extended practice with previously taught concepts and skills. Problem solving activities are introduced every five lessons to help students practice newly acquired problem solving skills and engage in complex mathematical problems, such as collecting categorical data and representing the data on a graph. The 66 treatment teachers implemented the ELM program five days per week in general education classrooms.

Professional development. Treatment teachers received four professional development workshops related to the program implementation. Each workshop lasted six hours and corresponded with the ELM quarterly teacher manuals. For example, the first workshop was conducted prior to the start of the school year and focused on lessons 1-30. Each workshop centered on evidence-based principles of mathematics instruction and the instructional design and delivery features of the ELM program. Workshops also offered ELM teachers opportunities to practice with sample lessons and receive feedback from the ELM curriculum team.

Treatment Fidelity. The ELM Efficacy Trial involved three fidelity checks in each treatment classroom. Teachers' adherence to the program was documented by project staff using a rating scale ranging from 0 (did not implement), 0.5 (partial implementation) to 1.0 (full implementation). On average, ELM teachers implemented the curriculum with moderate levels of implementation fidelity: fall (M = .86, SD =.13), winter (M = .87, SD =.15), and spring (M = .87, SD =.14). The potential for treatment diffusion was also monitored in control classrooms. Observations documented no evidence of contamination between ELM and control classrooms.

Control Classrooms

Mathematics instruction in the 56 control classrooms consisted of business-as-usual mathematics practices. Teachers in control classrooms used a variety of instructional materials, including teacher-developed activities and a number of commercially available programs. Surveys indicated that mathematics materials used in control classrooms varied within participating districts and schools. The most widely used programs were *Everyday Mathematics*, *Houghton Mifflin*, *Scott Foresman*, *Texas Mathematics*, and *Bridges in Mathematics*. The instructional focus in control classrooms also varied. Some teachers emphasized whole number concepts, while others focused primarily on patterning and particular aspects of geometry and measurement. A variety of different mediums were employed to deliver instruction in the control classrooms, including whole-class and center-based activities.

Student Mathematics Outcome Measure

Our analyses focused on students' pretest and posttest performances on the TEMA-3 (Ginsburg & Baroody, 2003). TEMA-3 pretest occurred just prior to the start of kindergarten for participating students, whereas posttest took place at the end of the school year. Trained staff administered the TEMA-3, with data collection meeting acceptable reliability criteria (i.e., implementation fidelity of .95 or higher). The TEMA-3 is a norm-referenced, general outcome measure purported to measure students' procedural and conceptual knowledge of early number sense. Internal consistency reliabilities of the measure exceed .92 and alternate-form and test-retest reliabilities exceed .80. Concurrent validity coefficients with four widely used tests of mathematics ranged from .55 to .91. In the current study, the intraclass correlation coefficient (ICC) for standard scale scores on the pretest TEMA-3 was .26, and the average reliability across all 122 classrooms was .85. The TEMA-3 provides age norms to calculate standard scores and percentile ranks..

Statistical Analysis

For Research Question 1, we assessed intervention effects on the TEMA-3 and EN-CBM raw scores of students with severe MD using a mixed-model (multilevel) time × condition analysis (Murray, 1998) to account for the intraclass correlation associated with students nested within classrooms, the level of random assignment. The analysis tested differences between conditions on change in outcomes from the fall to spring of kindergarten, with gains for individual students clustered within classrooms. The statistical model included time, condition, and the Time × Condition interaction, with time coded 0 in the fall and 1 in the spring of kindergarten and condition coded 0 for control and 1 for ELM. Analyses were based on 122 classrooms that included at least one student with severe MD at the start of the school year.

For Research Question 2, we examined rates by study condition at which norm-referenced student TEMA scores increased from below the 10th percentile to at or above the 10th, 20th, 30th, and 40th percentiles between the fall and spring of kindergarten. Specifically, we created four binary variables indicating whether student performance increased beyond each threshold. These variables form non-mutually exclusive categories, as a student who scored at or above the 40th percentile also scored at or above the 20th by definition. We tested condition differences in these binary indicators of achievement gains using contingency table analyses and χ^2 tests.

Finally, for Research Question 3, we explored differential response to the ELM program as a function of the classroom-level proportion of students with severe MD. We expanded the statistical model described for Research Question 1 to include this potential moderator and its interaction with condition, time, and the Time × Condition term; resulting in a three-way interaction, all corresponding two-way interactions, and individual (conditional) effects. The

three-way interaction of the moderator, time, and condition provided an estimate of whether condition effects varied by the proportion of students with severe MD.

Model estimation. We fit multilevel statistical models to our data using SAS PROC MIXED version 14.2 (SAS Institute, 2016) and restricted maximum likelihood estimation. Maximum likelihood estimation with all available data produces potentially unbiased results even in the face of substantial missing data, provided the missing data were missing at random (Schafer & Graham, 2002), although nonrandom missingness "is often not sufficient to affect the internal validity of an experimental study to any practical extent" (Graham, 2009, p. 568). In the present study, we did not believe that missing data represented a meaningful departure from the missing at random assumption, meaning that missing data did not likely depend on unobserved determinants of the outcomes of interest (Little & Rubin, 2002). The majority of missing data involved students who were absent on the day of assessment (e.g., due to illness) or transferred to a new school (e.g., due to their families moving).

The models assume independent and normally distributed observations. We addressed the first, more important assumption (van Belle, 2008) by explicitly modeling the multilevel nature of the data. Multilevel regression methods are also quite robust to violations of normality (e.g., Hannan & Murray, 1996).

Effect sizes. To interpret condition differences we computed effect sizes, the Hedges' *g* for continuous measures and Cox's *d* for dichotomous measures, using What Works Clearinghouse procedures (WWC, 2020).

Results

Herein we summarize demographic information and present results for the comparison between study conditions (ELM versus control) among the subgroup of students with severe MD

at the start of the school year. We then present results for condition differences moderated by the classroom-level proportion of students with severe MD.

Descriptive Results and Baseline Equivalence

Demographic characteristics are reported in Table 1. ELM and control classrooms did not meaningfully differ on proportions of students who were male, Hispanic, or eligible for special education services ($d < 0.10$). Table 2 reports the descriptive statistics for TEMA-3 and EN-CBM scores by assessment time and intervention condition.

Attrition

The overall rate of missing TEMA-3 data at posttest was 12.8%, and the difference in rates of missingness between conditions was 1.0%. "The proportions of the treatment and control groups that provide information are not particularly important, at least for internal validity" (Foster & Bickman, 1996, p. 698), so we tested the potential for *differential attrition effects* to identify potential threats to internal validity. To do so, we conducted a mixed-model analysis of variance designed to test whether attrition differentially affected condition differences for outcome variables. Specifically, the analyses tested the association between pretest outcome and (a) study condition (ELM versus control), (b) attrition status, and (c) the interaction between the two (Graham & Donaldson, 1993). We found no interaction between attrition and condition that predicted baseline outcome large enough to suggest that attrition threatened internal validity (p = .5215).

ELM Efficacy for Students with Severe MD

We tested the hypothesis that students with severe MD in ELM classrooms would experience greater gains in TEMA-3 and EN-CBM scores during kindergarten than students with severe MD in control classrooms. The g and p values reported in the left columns of Table 3

represent the tests of ELM efficacy. Students with severe MD in ELM classrooms made greater gains from fall to spring than students in the control condition on the TEMA-3 (g = 0.28, 95% CI [0.07, 0.49]) and EN-CBM (g = 0.18, 95% CI [-0.01, 0.38]).

Contingency table analyses of norm-referenced TEMA-3 scores revealed that higher rates of students in ELM classrooms scored at or above the 20th (54% versus 41%; χ^2 [1, 795] = 11.88, p = .0006, d = 0.32, odds ratio [*OR*] = 1.64), 30th (44% versus 32%; χ^2 [1, 795] = 11.95, p = .0005, d = 0.31, *OR* = 1.67), and 40th (29% versus 20%; χ^2 [1, 795] = 9.95, p = .0016, d = 0.30, *OR* = 1.71) percentiles at the end of the school year than their control peers. Rates at which student TEMA-3 scores increased to at or above the 10th percentile did not differ between ELM and control conditions (68% versus 64%, respectively; χ^2 [1, 795] = 1.31, p = .2519, d = 0.11, odds ratio [*OR*] = 1.19).

Moderation by Class-Level Proportion of Students with Severe MD

The right columns of Table 3 presents tests of differential response to ELM as a function of the classroom-level proportion of students with severe MD at the start of the school year. Tests of moderation require additional fixed effects of the moderator and its interaction with condition, time, and the Time × Condition term. This three-way interaction provided no statistical evidence of differential efficacy of the ELM program by the classroom-level proportion of students with severe MD (p = .3657 and .4831 for TEMA-3 and EN-CBM outcomes, respectively).

Discussion

Results Summary and Directions for Future Research

Today’s kindergarten classrooms are becoming increasingly diverse with students who face severe difficulties in mathematics at the start of their kindergarten year. While many of

these knowledge gaps may be attributable to students' lack of informal learning opportunities in mathematics prior to school entry, the implications for core mathematics instruction are visible. The purpose of this study was to extend the work of Clarke et al. (2015) by examining the impact of a core, explicitly-designed mathematics program on the mathematical outcomes of students who entered kindergarten with severe MD. Three research questions were addressed.

Efficacy of the ELM mathematics program. First, we tested the efficacy of the ELM core mathematics program on the mathematics achievement of students who performed below the 10th percentile on the TEMA-3 at the start of kindergarten. We hypothesized that these students would reap positive benefit from ELM based on its explicit design. Our findings confirmed this hypothesis. Results suggested ELM had a substantively positive effect for students with severe MD, producing an effect size (Hedges' *g*) of 0.28 on the TEMA-3 and 0.18 on EN-CBM. Overall, this finding complements the growing line of research that suggests explicit, core mathematics instruction is beneficial for students who struggle to develop mathematical proficiency in the early grades (Agodini & Harris, 2010; Sood & Jitendra, 2013). Thus, delivering explicit mathematics programs at the preventative instructional tier in an MTSS model may result in thwarting the onset of severe MD among the most academically vulnerable students.

Because the current study involved only one core mathematics program, additional research is needed to investigate the impact of other core programs on the mathematics outcomes of students with severe MD. Over the past 10 years, significant development efforts have been made in the area of mathematics. For example, to date, the WWC (n.d.) has examined over 20 elementary mathematics programs (i.e., kindergarten to fifth grade). While only a few have demonstrated positive or potentially positive effects on student mathematics achievement, future

research is still warranted that examines whether these core programs and those that are still in the development pipeline improve the mathematics achievement of students with severe MD.

Rates of improvement beyond thresholds of severe MD. Our second research question examined whether ELM accelerated student mathematics performance above various MD thresholds (i.e., performance at or above the 10th, 20th, 30th, and 40th percentiles). It was hypothesized that the rate of improvement across the school year for students with severe MD in ELM classrooms would exceed that of students with severe MD in control classrooms. Findings indicated the odds of improving from below the 10th percentile at the start of kindergarten to at or above the 20th, 30th, and 40th percentiles at the end of kindergarten was nearly two times higher for students in ELM classrooms than their control peers.

While preliminary, findings from our second research question suggest systematically-designed and explicitly-delivered, core mathematics instruction can accelerate the mathematical performance of students with severe MD beyond a threshold of high risk by the end of the school year. Thus, well-designed and delivered core mathematics instruction not only has the capacity to meet the needs of students with some of most significant mathematical needs, but can also position them on a positive learning trajectory in mathematics. This is critical as being "on track" for mathematical success at the end of kindergarten significantly increases the likelihood of experiencing positive mathematics outcomes in the later grades (Claessens & Engel, 2013).

We contend that the implications of this finding are significant for schools and particularly noteworthy for future research. The capability of a core mathematics program to move students from a category of high risk to on track for developing mathematics proficiency substantially alleviates the pressure of mathematics intervention services to act as a backstop to mathematics failure. Consequently, this can have major implications for schools. Therefore,

future studies are needed to understand how to deploy the resources saved by effective core mathematics instruction in ways that are targeted and cost effective for at-risk learners.

Moderation by class-level proportion of students with severe MD. For our third research question, we investigated whether the classroom-level proportion of students with severe MD moderated the effects of core mathematics instruction delivered in kindergarten classrooms. A growing line of empirical research suggests that explicitly-designed, core mathematics instruction in kindergarten classrooms offers a plausible mechanism to promote positive student outcomes for students, particularly students who enter school at risk for long-term MD (Clarke et al., 2011; Sood & Jitendra, 2013). Yet, as in the area of reading (e.g., Vaughn et al., 2017), the effectiveness of empirically-validated, core mathematics program is likely susceptible to the notion of diminishing returns. That is, as the classroom-level proportion of kindergarten students with severe MD increases in a given classroom, it may suppress a teacher's capacity to deliver effective core mathematics instruction.

While we hypothesized that ELM would demonstrate a high threshold for the classroom-level percentage of students with severe MD given the program's explicit and systematic instructional design, it was also expected that the program would exhibit an observable tipping point. That is, we figured that once the classroom proportion of students with severe MD exceeded a certain magnitude, the effect of ELM would begin to diminish. Interestingly, our findings indicated otherwise. A non-significant moderation effect for the proportion of students with severe MD was reported, suggesting comparable performances in ELM classrooms regardless of classroom composition by students' initial skill level in mathematics.

It may be that ELM's explicit instructional design was able to support teachers in successfully managing classrooms that contained students with severe MD. When core

mathematics programs are engineered to embrace a systematic and explicit instructional framework, they have particular design features that differentiate them from other programs, such as ones that utilize more student-centered instructional approaches. One key design feature of explicit mathematics programs is the incorporation of scaffolds or temporary supports to promote a high success rate with new and complex mathematics content. For example, scaffolds in the ELM program include carefully sequenced instructional examples and judicious review of previously learned content. Another design feature that sets explicit mathematics programs apart from other programs is their capacity to support teachers in (a) offering overt demonstrations and explanations of new mathematical content, (b) providing specific academic feedback to confirm student responses and address potential misconceptions, and (c) facilitating important practice opportunities for students to demonstrate their mathematical thinking and reasoning, such as mathematics verbalizations. In the ELM program, such practice opportunities not allow struggling learners to build mathematical proficiency but also to collaborate and learn from their typically-achieving peers. Combined, these design features may allow kindergarten teachers to meet the instructional needs of students who are at high risk for failure in mathematics.

While the current study focused specifically on students experiencing severe MD, future research should consider investigating whether other student-related factors at the classroom level serve as potential moderators of explicit mathematics programs. For example, the number of students identified with behavior disorders in a given classroom may influence a teacher's ability to effectively implement an evidence-based core mathematics program. In these situations, teachers may spend more time redirecting or attending to non-academic, off-task behaviors than delivering the program with fidelity. Future research that includes larger student

samples with diverse categories of disabilities may be needed to gain a deep understanding for how the student composition of mathematics classrooms influences core instruction.

Limitations

When interpreting the findings from this study, a number of limitations should be considered. First, the study included only 795 participating students. Tests of moderation may require a larger sample for including classroom-level moderating variables, such as the proportion of students with severe MD in a kindergarten classroom. Relatedly, the misidentification of students with "some risk" for MD at the time of pretest (i.e., false positives) may have increased the size of our analytic sample. However, a consistent educational finding is that a considerable number of U.S. students enter kindergarten lacking a level of number sense knowledge required for early success in mathematics (Duncan et al., 2007; Morgan et al., 2009). As such, we feel confident that the observed pretest results were an accurate representation of students' mathematical performance at the start of the kindergarten year.

Additionally, while our operationalization of severe MD (i.e., < 10th percentile on a nationally-recognized mathematics outcome measure) was similar to cutoffs used in existing research (e.g., Morgan et al., 2009), a more conservative cutoff (i.e., performance ≤ 5th percentile) likely would have yielded different results. To provide teachers with actionable recommendations for improving the effectiveness of their core mathematics instruction, we, as a field, need to agree upon what constitutes categories of MD, particularly severe MD. Fields such as the medical field have established cutoffs for conditions as varied as hypertension, obesity, diabetes, and thyroid disease (Nettina, 2019). Agreed upon cutoffs of MD would increase consistency in the way researchers and educators conceptualize, operationalize, and provide needed supports in early mathematics. Moreover, they would help improve methodologies for

investigating rates of improvement in mathematics, such as whether kindergarten students with severe MD transition to a less at-risk category of MD across the school year. Relatedly, our study focused exclusively on immediate outcomes and thus did not include a longitudinal follow-up assessment (Watts, Bailey, & Li, 2019). Fadeout effects are a common finding of intervention research in the field of education (Bailey, Duncan, Odgers, & Yu, 2017). As such future studies are required to determine whether explicit, core mathematics programs produce lasting impacts on the long-term learning trajectories of students with severe MD.

Implications for Practice

Research on global climate change has sparked a spate of attention to critical thresholds or tipping points that signal irreversible climate change effects on our natural environment (Lenton et al., 2008; Russill & Nyssa, 2009). Relative to climate science, investigations of tipping points in education are still in its infancy. However, recent research has begun to unveil classroom-level variables that contribute to the point at which validated core programs begin to produce diminishing returns (Vaughn et al., 2017; 2019). While the current study did not reveal a tipping point for ELM based on the class-level proportion of students with severe MD, we encourage researchers to continue this line of work as it has important implications for how schools can maximize the effectiveness of evidence-based, core mathematics programs.

For example, consider Program A, a core kindergarten mathematics program that has a solid evidentiary base yet demonstrates a tipping point once the classroom proportion of students with significant MD exceeds 20%. Theoretically, under a MTSS model (Fuchs & Vaughn, 2012), a school would use a Tier 2 intervention to supplement Program A. However, extra resources are often scarce and consequently many schools lack the support systems needed to boost the effectiveness of core mathematics instruction. Therefore, schools need cost effective

ways to thwart the point at which the impact of validated core mathematics programs, such as Program A, begin to diminish. Below, we briefly make two practical recommendations.

One approach is to monitor fidelity of implementation of core mathematics programs as effective program delivery is a plausible way to mitigate the diminishing returns of evidence-based programs. To fit local contexts, teachers will likely adapt validated core mathematics programs. While fidelity and program adaptability often co-occur (Durlak & DuPre, 2008), such adaptations or modifications may affect program effectiveness. As such, some measure that documents the preservation of the essential components of core mathematics programs, such as Program A, is likely necessary. A second recommendation is for schools examine the procedures they use to assemble students in classrooms. When administrators and teachers are deciding which students should be placed in particular classrooms, as much information as possible about students' prior and current performance in mathematics should be gathered. Such information may prove invaluable for maximizing the effectiveness of validated core mathematics programs.

Conclusion

The research suggesting that a considerable number of U.S. students facing severe MD in kindergarten is clear and compelling. In light of this evidence, the average kindergarten classroom likely has little support for students' mathematics development beyond the core curriculum. Given that core mathematics instruction delivered in kindergarten classrooms may represent the totality of a student's exposure to foundational mathematics content, it is therefore critical that it positively impact the development of mathematical proficiency of all students, including those who face severe MD. We believe that core mathematics programs that are purposefully designed to meet the instructional needs of the full range of learners, such as those programs that incorporate explicit and systematic design principles, serve as a valuable first line

of defense in impacting the mathematics achievement of students with severe MD.

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