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A graph isomorphism with didactic connections to QED

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Abstract: This manuscript is meant to support secondary school teachers in their constant effort to find novel ways to engage students. Adolescents seem particularly stimulated by time-travelling scenarios, like the famous "wormhole billiard ball paradox" proposed by J. Polchinski in 1990, which are usually solved through closed time-like curves (CTCs). The concept of *causal loop* has been popularized by a vast sci-fi literature, so that it sounds familiar to high school pupils. We present an adaptation of the Polchinski's puzzle to the possible scatterings (Møller or Bhabha) of an electron entering a time-travel tunnel so that it can collide with its earlier self at low energy. In order to avoid a discouraging mathematical formulation, our analysis is based merely on graph isomorphism and can be viewed, in educational terms, as an introduction to quantum electrodynamics at undergraduate level. In fact, all the electron interactions along the CTCs result in Feynman exchange diagrams (*s*-channel, *t*-channel, *u*-channel).

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Keywords: school teaching, closed time-like curves, graph isomorphism, Feynman diagrams, quantum electrodynamics.

1. Introduction

Mainly oriented to high school teachers of physics (Bonacci, 2020), this is a formula-free investigation about the possible scatterings of an electron entering a time-travel tunnel so that it can collide with its earlier self at low energy (Bonacci, 2021).

We use accessible tools, like the graph isomorphism (GI for short) and the Feynman diagrams, to analyze the related quantum electrodynamics (QED) in the framework of the famous wormhole billiard ball paradox (Friedman et al., 1990), i.e., along closed time-like curves (acronym CTCs).

We explore two alternative cases about the exiting particle:

1) If it is still an *electron*, then the collision (Møller scattering) deflects the trajectory of the incoming particle just towards the tunnel entrance within a stable time loop.

2) If it is a *positron*, i.e., the antiparticle of an electron (Feynman, 1949), the interaction with the incoming electron (Bhabha scattering) is a process of pair production which is reversed inside the tunnel as annihilation.

For a full comprehension, we suggest the preliminary study of the Mandelstam variables (s, t, u).

Although the application to elementary particles of the Polchinski's paradox is not new (Bishop et al., 2020) and its solution via Novikov's self-consistent causal loops is even trivial (Echeverria et al., 1991), our work seems original in form and in some contents, such as open questions ranging from the role of a preferential arrow of time to the validity of the law of inertia in chronology violations.

2. A three-fold framework for our paper

2.1 A simplified model of wormhole

We do not scrutinize neither the physical nature of the traversable wormhole, according to General Relativity equations, nor the feasibility of a time machine at quantum level (Moldoveanu, 2007).

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For our purposes, it is just a static toroidal tunnel (tubular shape) for time travelling, with two converging mouths called *entrance* and *exit* (Fig. 1).

The entering and exiting particles are supposed slow enough (compared to *c*) to avert high-energy quantum processes and to neglect relativistic effects.

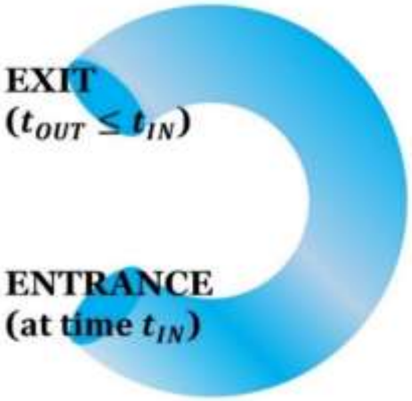

Figure 1. A time-travel static toroidal tunnel.

2.2 The Theory of Positrons

According to (Feynman, 1949), we regard the positrons as electrons moving backwards in time (Fig. 2).

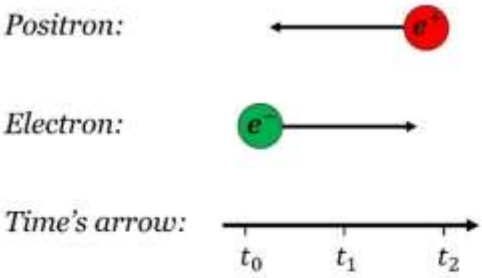

Figure 2. The Feynman-Stueckelberg interpretation.

2.3 The Principle of Reciprocity in physics

We assume that the Principle of Self-Consistency (acronym PSC) by Novikov (Echeverria et al., 1991) is a specific case of the Reciprocity Principle (RP for brevity), i.e., the physical description given by the permutation between subject (cause) and direct object (effect) in a well-formulated proposition (Bonacci, 2007a, 2007b, 2007c, 2007d, 2008a, 2008b, 2008c).

When the outgoing particle is still an *electron* (Sections 3 and 4), the collision between the two particles deflects the trajectory of the incoming electron just towards the tunnel entrance according to the PSC.

When the outgoing particle is a *positron* (Section 5), we apply directly the RP within a stable time loop (Fig. 3).

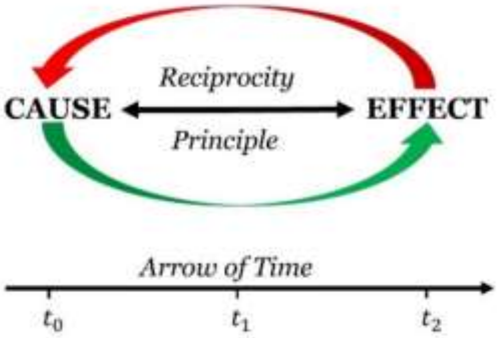

Figure 3. Reciprocity Principle and time loops.

3. The *t*-channel exchange diagram

We examine two electron-electron interactions (matter-only scenarios) whose graph isomorphism leads to the *t*-channel Feynman diagram associated with Møller scattering (Fig. 4).

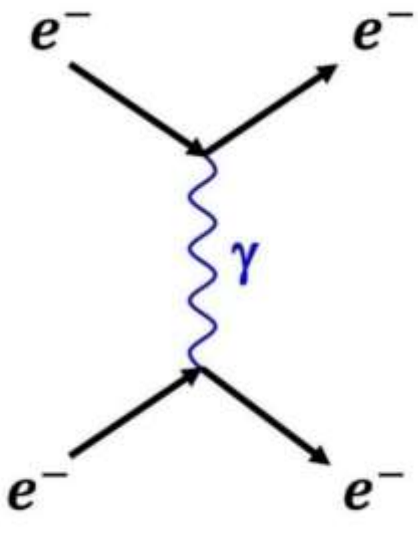

Figure 4. Møller scattering's *t*-channel Feynman graph.

According to the PSC, the collision between the two electrons deflects the trajectory of the incoming particle just towards the tunnel entrance (within a stable time loop). The trajectories of particles do not intersect.

3.1 First matter-only scenario

We consider an electron that enters the static wormhole at the instant t_2, leaves it at the past time $t_0 < t_2$ and interacts with its earlier self (Fig. $5-6$) at $t_1 > t_0$.

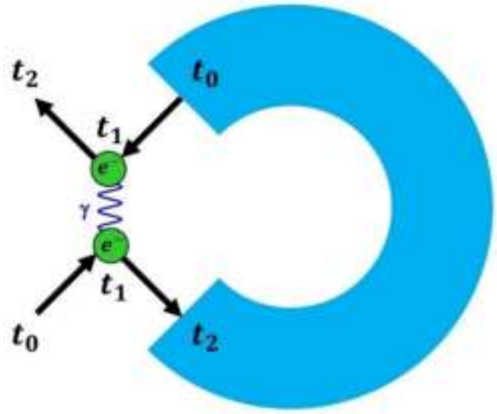

Figure 5. First electron-electron interaction.

By graph isomorphism, from Fig. 6 we obtain the *t*-channel Feynman diagram for Møller scattering (Fig. 4).

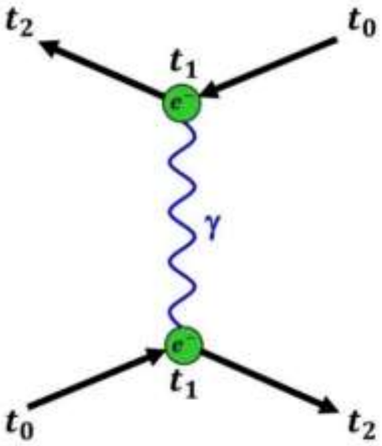

Figure 6. Linear older-younger electron collision.

3.2 Second matter-only scenario

We consider an electron that enters the static wormhole at the instant t_2, leaves it at the past time $t_0 < t_2$ and interacts with another electron (Fig. $7-8$) at $t_1 > t_0$.

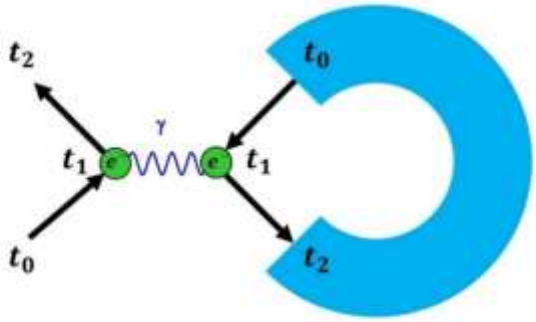

Figure 7. Second electron-electron interaction.

By graph isomorphism, from Fig. 8 we obtain the *t*-channel Feynman diagram for Møller scattering (Fig. 4).

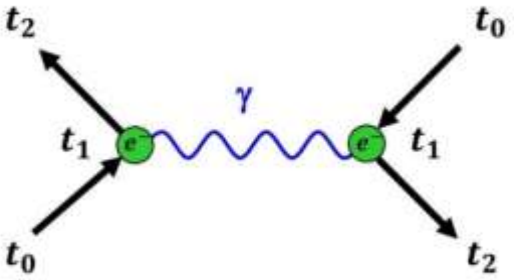

Figure 8. Linear collision between different electrons.

4. The *u*-channel exchange diagram

We examine two electron-electron crossed interactions (matter-only scenarios) whose graph isomorphism leads to the *u*-channel Feynman diagram associated with Møller scattering (Fig. 9).

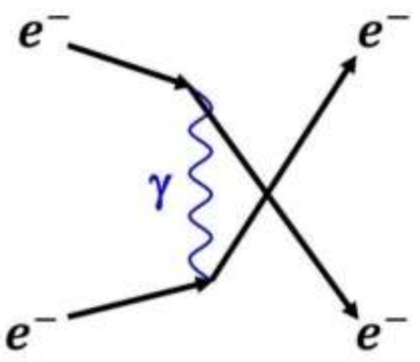

Figure 9. Møller scattering's *u*-channel Feynman graph.

According to the PSC, the collision between the two electrons deflects the trajectory of the incoming particle just towards the tunnel entrance (within a stable time loop). The particles’ trajectories intersect in both cases, though not in the same instant.

4.1 Third matter-only scenario

We consider an electron that enters the static wormhole at the instant t_2, leaves it at the past time $t_0 < t_2$ and interacts with its past self (Fig. $10 - 11$) at $t_1 > t_0$.

With reference to the time of the incoming electron, the trajectories cross each other *before* the particles’ collision.

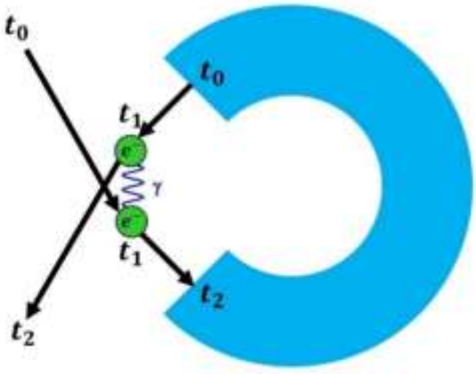

Figure 10. Third electron-electron interaction.

By graph isomorphism, from Fig. 11 we obtain the *u*-channel Feynman diagram for Møller scattering (Fig. 9).

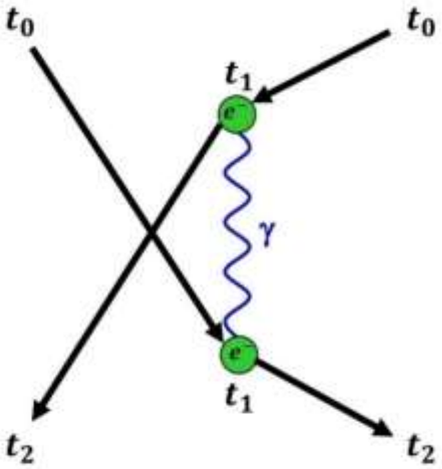

Figure 11. Pre-crossed older-younger electron collision.

4.2 Fourth matter-only scenario

We consider an electron that enters the static wormhole at the instant t_2, leaves it at the past time $t_0 < t_2$ and interacts with its past self (Fig. $12 - 13$) at $t_1 > t_0$.

With reference to the time of the incoming electron, the trajectories cross each other *after* the particles’ collision.

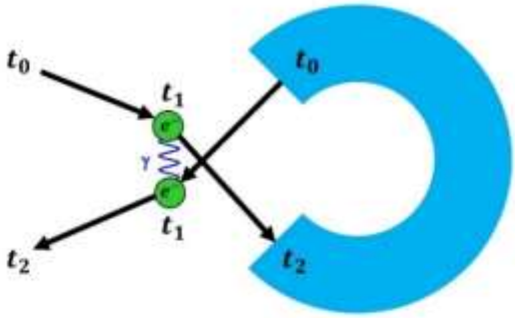

Figure 12. Fourth electron-electron interaction.

By graph isomorphism, from Fig. 13 we obtain the *u*-channel Feynman diagram for Møller scattering (Fig. 9).

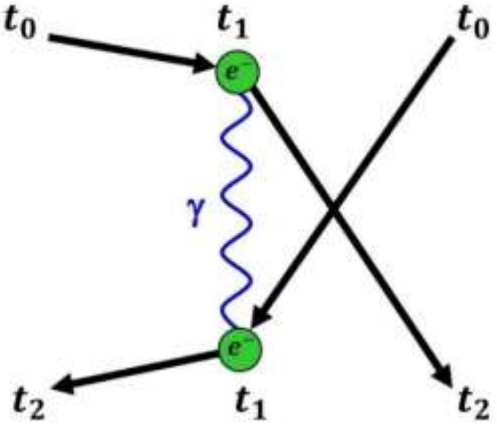

Figure 13. Post-crossed older-younger electron collision.

5. The *s*-channel exchange diagram

We examine an electron-positron interaction (matter-antimatter scenario) whose graph isomorphism leads to the *s*-channel Feynman diagram associated with Bhabha scattering (Fig. 14).

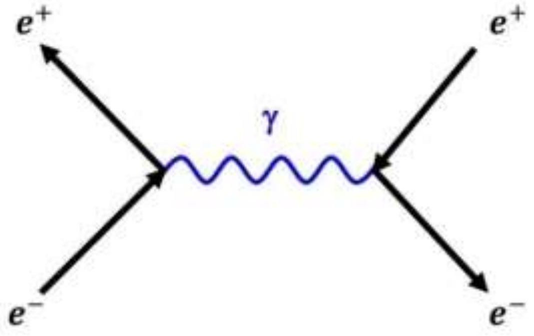

Figure 14. Bhabha scattering's *s*-channel Feynman graph.

According to the RP, the interaction with the incoming electron is a process of pair production, reversed inside the tunnel as annihilation, which directs the trajectories of the particles each towards a tunnel mouth (Fig. $15 - 17$).

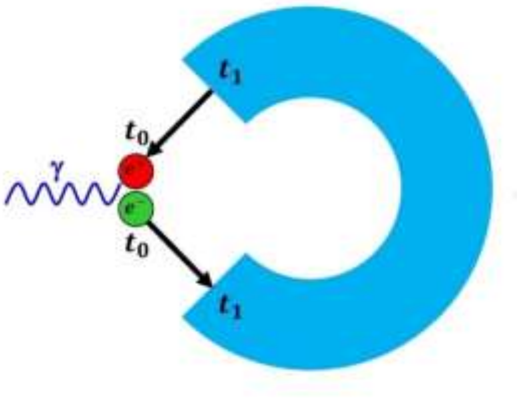

Figure 15. Electron-positron pair production.

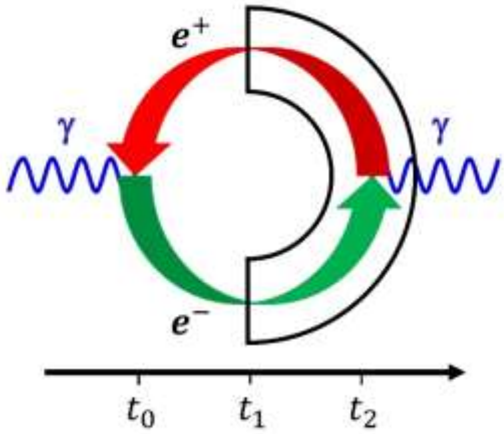

Figure 16. Annihilation process inside the tunnel.

The electron enters the static wormhole at the instant t_1, leaves it (as a positron) at the same time t_1 and interacts with its earlier self in the past time $t_0 < t_1$.

By graph isomorphism, from Fig. 17 we obtain the *s*-channel Feynman diagram for Bhabha scattering (Fig. 14).

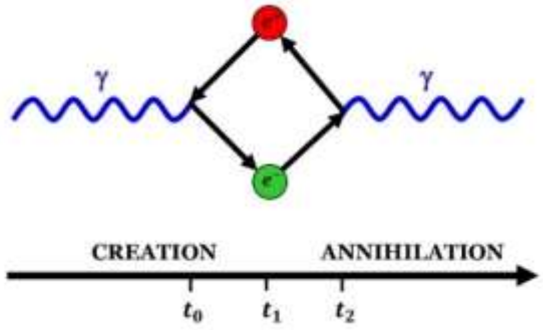

Figure 17. The creation-annihilation processes.

5. Implicit hypotheses and open questions

5.1 Antimatter as unique solution

One of our solutions of the Polchinski's conundrum, is based on the electron travelling backwards in time as positron (Fig. 18).

What would happen if the *only* way to travel back in time was to be (or to become) antimatter? We should rule out all the situations where the particle leaving the tunnel is still an electron.

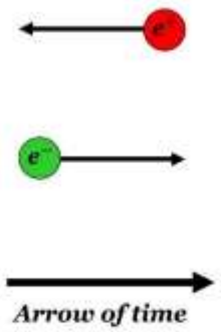

Figure 18. Antimatter as time-reversed matter.

5.2 A missing collision?

We have implicitly referred to the Law of Inertia by assuming that the exiting particle leaves the wormhole at a certain time as if the tunnel was at rest, keeping its spatial coordinates (Fig. 19).

What would happen if the *inertia principle* was not valid in chronology-violating regions? The outgoing particle would appear in a different locality, with respect to the tunnel entrance mouth, avoiding any interaction with its past self.

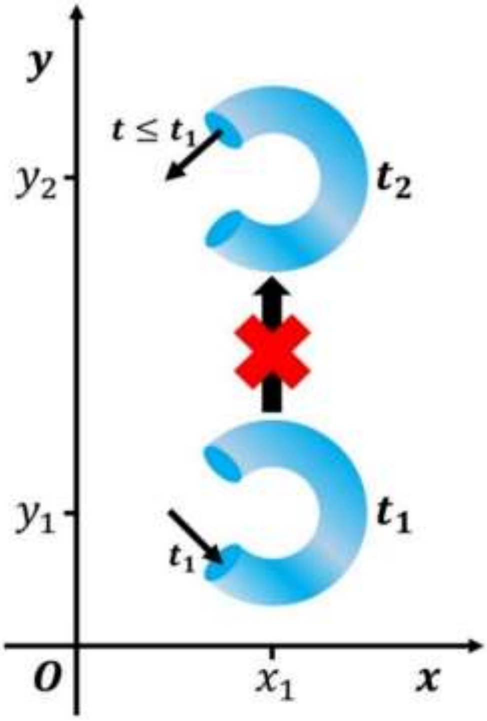

Figure 19. Non-inertial chronology violations.

5.3 A new paradigm in physics?

The ordinary causal-chain representation of the events is incomplete for stable time loops whose description (Fig. 20) is supplied by pairs of reciprocal propositions (cause-effect permutation).

What would happen if *everything* had also a retro-causal explanation? We should see any physical phenomenon as a *whole*, beyond our common logic of observers who experience the conventional arrow of time.

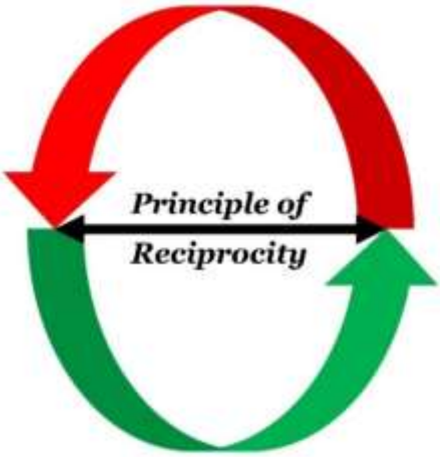

Figure 20. Causality without a preferential arrow of time.

6. Conclusions

We firmly believe that high school students can be successfully introduced to basic concepts and methods of QED through the captivating idea, widely celebrated in cinematography, of *causal loop*.

Since time travels are genuinely appealing to pupils whilst pre-university quantum physics raises several didactics issues (Bonacci, 2020), we have developed a purely topological study on the low-energy scatterings of an electron with its earlier self in the space between the two converging mouths (*entrance* and *exit*) of a time-travel static toroidal tunnel (Bonacci, 2021).

We have encountered five scenarios (four matter-only and one particle-antiparticle) all resulting in well-known Feynman diagrams (*space*, *time*, and *u*-channel) via a simple graph isomorphism.

We have assumed that the past determines the future as well as the future determines the past according to Novikov's PSC (Echeverria et al., 1991) and to our RP (Bonacci, 2007a, 2007b, 2007c, 2007d, 2008a, 2008b, 2008c).

We have eventually explored the limits of:

1) time-travelling for ordinary matter (Feynman, 1949);

2) inertia in chronology-violating regions;

3) logical arrows of time in defining the causality.

Our GI analysis enriches the debate about the possibility to extend a classical paradox (Friedman et al.,

1990) to elementary particles (Bishop et al., 2020; Moldoveanu, 2007).

Remarks

Approaching quantum physics from popular fiction is a tested didactic strategy for secondary schools (Bonacci, 2020); it aims at improving the learners' motivation and activating their emotional intelligence.

The reader must, however, be aware that *time-travelling* is a highly controversial topic, especially when dealing with the quantum realm (Moldoveanu, 2007).

Acknowledgments

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