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Instructional Research Group

A Systematic Review and Meta-Analysis of Rational Number Interventions for Students Experiencing Difficulty with Mathematics

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The data set compiled for this project (Schumacher et al., 2021) is available on the Harvard Dataverse Network.

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## 1. Executive Summary

Our goal was to provide valuable information, grounded in evidence, on which aspects of rational numbers interventions were associated with positive student outcomes. We examined which instructional components (e.g., visual representations, use of number lines, teaching and using of mathematical language) and study features (e.g., group size, interventionist training) may have contributed to the effectiveness of intervention. In choosing to focus on instructional components and study features, the findings have the capacity to inform state and district leaders, as well as teachers providing intervention, on which instructional components are necessary for a rational numbers intervention to be effective.

We searched for studies that focused on teaching rational numbers concepts to students experiencing mathematics difficulties in Grades 3 through 9. Approximately 1,500 studies were identified and screened, leaving 77 reports to be screened more in-depth for relevance. Those still eligible were then screened for rigor by applying the What Works Clearinghouse (WWC) evidence standards. A total of 26 studies with 4,237 unique participants from 30 independent samples were included in the meta-analysis.

Findings indicated that rational numbers interventions are effective. The estimate of the mean effect size was statistically significant ($g = 0.65$; $p < .001$). To assess which specific instructional components were effective, a multivariate meta-regression model with robust variance estimation (RVE) was used. The majority of the instructional components were non-significant at $p < 0.05$, with the exception of the teaching and use of mathematical language ($p < .047$), indicating that studies including this component were associated with significantly larger positive effects on mathematics outcomes. Using this rigorous approach, the significant finding of teaching and using of mathematical language as a moderator ($g = 0.57$) can be interpreted with confidence.

When exploring study features, we found that interventions had stronger impacts when they were (a) implemented for students in Grades 3–6, (b) delivered to small groups of 2–6 students, (c) provided by research project personnel, (d) longer than nine hours—and even more effective when 20 hours or longer, and (e) implemented by interventionists who received ongoing training.

The results from this meta-analysis suggest that intervention programs devoting time to the teaching and use of mathematical language can substantially enhance outcomes. Mathematical language is a type of abstract academic language (terms such as *equivalent*, *reciprocal*, *circumference*) that helps students learn mathematics concepts more precisely. When students understand and use mathematical language, it is believed that the students will more deeply understand the mathematics they are learning.

## 2. Introduction

According to many mathematicians, mathematics educators, and cognitive psychologists, understanding rational numbers–especially fractions–holds the key to future success in algebra and higher-level mathematics (National Mathematics Advisory Panel [NMAP], 2008; Schneider et al., 2017; Siegler & Lortie-Forgues, 2014). Unfortunately, the topic of rational numbers gets increasingly complex through upper elementary and early middle school (e.g., Common Core State Standards [CCSS]; National Governor's Association Center for Best Practices & Council of Chief State School Officers, 2015), the gap between lower- and higher-performing students tends to widen. Providing rational numbers intervention to students experiencing mathematics difficulties is therefore critical for long-term outcomes.

Fortunately, over the last decade there has been a growing body of intervention research focused on rational numbers topics designed for elementary and middle school students (e.g., Barbieri et al., 2019; Dyson et al., 2020; Fuchs et al., 2014; Jitendra et al., 2017). A quantitative synthesis of this research base would help educators better understand which aspects of interventions focused on the critical, yet challenging topic of rational numbers, are effective (Therrien et al., 2020). Our goal, therefore, is to investigate which *instructional components* (e.g., visual representations, timed fluency-building activities) and *study features* (e.g., group size, interventionist training) may impact the effectiveness of intervention. In choosing to focus on instructional components and study features, the present meta-analysis has the capacity to inform state and district leaders, as well as teachers providing intervention, on which instructional components are necessary for a rational number intervention to be effective.

## 2.1 Previous Related Reviews

We identified five related narrative reviews and meta-analyses conducted previously on the topic of fractions alone—just one aspect of rational numbers intervention. Three are narrative reviews (Misquitta, 2011; Roesslein & Codding, 2019; Shin & Bryant, 2015) and therefore use qualitative approaches to summarize the evidence from single-case and group design studies. Two applied quantitative methodologies (Ennis & Losinski, 2019; Hwang et al., 2019). The present study extends these previous reviews by (a) including only studies appraised using the rigorous WWC standards, (b) using the most current meta-analytic techniques, and (c) focusing on instructional components and study features associated with impacts.

## 2.2 Research Questions

Through this meta-analysis, answers to the following research questions will be generated to better understand which instructional practices and study features might lead to stronger, weaker, or even non-existent impacts.

1. How effective are rational number interventions for students in Grades 3 through 9 who

are experiencing mathematics difficulties?

2. Do specific *instructional components* in intervention (explicit instruction, representations, strategic prompting tools, cumulative review, teaching and use of mathematical language, timed fluency-building activities) moderate the impacts on mathematical knowledge?
3. Does inclusion of *number lines* as a central visual representation of the magnitude of rational numbers in intervention moderate the impacts on mathematical knowledge?
4. Do certain *study features* (grade level, group size, interventionist, technology usage, duration, initial training, ongoing training, instructional setting, type of measure, outcome domain) moderate the impacts of rational number interventions on mathematics outcomes?

## 3. Method

### 3.1 Search Strategy

We used a comprehensive search strategy to identify all studies published over the last three decades focused on rational number interventions for struggling students in Grades 3 through 9. An initial search was completed in September 2017. The literature search was then extended through August 2019. A targeted, database-specific keyword search of the following databases was conducted using ProQuest: ERIC, PsycINFO, Dissertation Abstracts, and the Social Sciences Citation Index. We then conducted extensive supplementary searches of the awards databases of both the National Science Foundation and the Institute of Educational Sciences along with the bibliographies of relevant screened studies and prior narrative reviews and meta-analyses—including those used by the WWC to evaluate the effectiveness of relevant rational number intervention programs. Finally, key researchers in the field were solicited for recommendations of studies likely to meet eligibility criteria.

To locate all studies that potentially met the eligibility criteria, we searched titles and abstracts of studies using the following set of keywords and their variants and the Boolean operators "*," "AND," and "OR": *fraction, rational number, proportion, ratio, decimal, intervention, response to intervention, tutor, multi-tiered system of support, third grade, fourth grade, fifth grade, sixth grade, seventh grade, eighth grade, ninth grade, elementary, upper elementary, middle school, junior high, studies, experimental, random, experiment*.

### 3.2 Inclusion and Exclusion Criteria

All studies were screened for eligibility based on the title, keywords, and abstracts. Studies had to be published in English during or after 1987 as a dissertation, ERIC document, or article published in a peer-reviewed journal. The studies were then screened for eligibility using the PICOS framework (i.e., population, intervention, comparison, outcomes, and study design) to support identification of critical aspects of eligible studies.

#### *Population*

Eligible studies had to focus on students with difficulties in mathematics in Grades 3 through 9. This included students identified with a learning disability in mathematics by their school or district and students who are experiencing mathematics difficulties if they met one of the following criteria: performance below the $35^{th}$ percentile on a valid screening measure of general mathematics knowledge or rational number topic(s), or attendance at a very low-performing school. To be included in the meta-analysis, the sample had to involve a minimum of 60% of students who met this criterion. Studies involving students in other grades were not included unless the authors disaggregated the results of the students in eligible grades or students in eligible grades comprised more than 50% of the mixed-age sample.

#### *Interventions*

For the purpose of this study, the term *intervention* was defined as one-on-one, small-group (2−6 students), or large-group (6 or more students) programs aimed at helping students considered at risk for future problems in learning rational numbers or mathematics. We included studies that were implemented in three instructional settings: (a) the general classroom, (b) supplemental intervention settings (e.g., pull-out for additional instruction to support core classroom mathematics instruction), or (c) special education settings including resource rooms. Eligible interventions were not merely tutoring or helping students with homework or class assignments. Rather, eligible interventions were designed to address key concepts from previous grades or to support the learning of challenging material with instructional approaches not typically found in core instruction.

To be considered for inclusion, the intervention needed to comprise specific areas of rational number knowledge (e.g., fractions, decimals, proportions, and rates). If topics unrelated to rational numbers were also part of an intervention, rational number topics needed to be at least 50% of the intervention content. Additionally, studies needed to provide details about the knowledge and skills that were targeted, the instructional approach, the unit of instructional delivery (e.g., one-on-one or small group), and mode of delivery (e.g., teacher-led instruction, computer-mediated intervention) to meet inclusion criteria.

***Comparisons***

Comparison conditions included business-as-usual control or an alternative treatment. We coded for the comparison condition as either business-as-usual (i.e., instruction provided by the school and not by the research team) or a researcher-implemented alternative treatment in the analysis and used comparison type as a covariate. We took this approach because of previous findings that effects can be differential based on the type of comparison condition (Lemons et al., 2014).

***Outcome Measures***

Eligible studies were required to include at least one relevant student outcome that demonstrated sufficient reliability (i.e., internal consistency minimum of 0.50; temporal stability/test-retest reliability minimum of 0.40; or inter-rater reliability minimum of 0.50; U.S. ED & WWC, 2013) and content validity. Relevant outcome measures included nationally normed achievement tests; other standardized tests such as state assessments, published progress monitoring, and formative assessments; as well as research-based or locally developed tests or instruments.

To be considered eligible, outcomes had to fall within the following domains: knowledge of rational numbers topics, rational numbers magnitude, fractions computation (i.e., measures that present computation involving all four operations), fractions addition and subtraction, word problems (i.e., measures that present arithmetic word problems), problem solving (i.e., measures that include complex problem solving situations), or general mathematics achievement which included rational number topics and other areas of mathematics (e.g., measurement, whole-numbers concepts).

***Study Designs***

Only randomized controlled trials or quasi-experimental group design studies were included. This includes cluster-randomized control trials.

**3.3 Screening and Coding Procedures**

Under the supervision of the principal investigators, a team of Ph.D.-level researchers conducted eligibility screening and coding of the studies. An Excel-based codebook was used to capture all variables of interest. Screening first occurred at the title and abstract level. At this stage, if there was ambiguity about a study's potential eligibility, the screeners leaned toward inclusivity. Eligible studies were then retrieved for full-text screening, during which the two screeners independently reviewed each article for eligibility. Potential discrepancies were resolved to consensus via discussion with one of the principal investigators.

For all studies screened eligible, a three-phase coding process was used to extract data. In Phase 1, two WWC-certified reviewers independently examined each study design for strength and quality using the *WWC Procedures and Standards Handbook* (Version 3.0; U.S. ED & WWC, 2013). Only studies meeting WWC standards (with or without reservations) were eligible for Phases 2 and 3. During Phase 2, data were extracted for study features (i.e., grade level, group size, interventionist, technology usage, duration, initial training, ongoing training, instructional setting, type of measure, outcome domain; see Tables 1 & 2). Outcome characteristics, which are part of the study features, are presented with effect sizes in (Table 3). Study features were used either for descriptive purposes, explored as potential moderators in the meta-analysis as part of research question three, or included in analyses as control variables. Researchers discussed and resolved any discrepancies of the data coded in Phase 2. During Phase 3, we coded for the presence or absence of the major instructional components based on authors' descriptions of the interventions. Some instructional components of interest we could not reliably code for (e.g., feedback, guided inquiry) and others (e.g., student explanations, comparing worked examples) we did not have enough studies to include. Potential coding discrepancies were again resolved by consensus with one of the principal investigators. All study features and instructional components were coded by two researchers, and reliability exceeded 95%. All coding discrepancies were then reconciled to 100% agreement. See Table 4 for operational definitions of instructional components and study features.

## 3.4 Risk of Bias

The majority of potential sources of bias within studies included in this review were addressed by applying the WWC standards to appraise study quality. Bias sources that would determine that a study does not meet WWC standards include sources of bias such as those arising from the randomization process, incomplete outcome data, selection of the reported results, or lack of pretest equivalence. We also cross-checked the WWC standards against the Cochrane Collaboration tool for assessing the risk of bias in randomized trials (Higgins & Green, 2011). Areas of bias included in the Cochrane Collaboration risk-of-bias tool that are not addressed through the WWC review process include the assessment of bias arising from deviations from intended interventions. Because of this, the Cochrane Collaboration tool was used as an additional means to assess risk of bias in this area for the studies in our sample.

## 3.5 Data Analysis

### *Effect Size Metric*

The outcomes of interest in the meta-analyses were calculated using means and pooled standard deviations for intervention and comparison groups, which includes an adjustment for small samples (Hedges, 1981). All effect sizes were reported to indicate that positive values indicate a favorable performance on outcome measures (e.g., more precise estimation on fraction number lines, stronger fractions magnitude understanding). This primarily involved reverse coding of number line estimation measures.

### *Meta-Analytic Procedures*

The majority of studies in our sample ($n$ = 23) provided multiple effect sizes of interest. To include all effect sizes from each study and account for dependencies within the dataset (e.g., studies reporting multiple effect sizes), we used the random effects robust variance estimation

(RVE) technique developed by Hedges et al. (2010). This approach allows for clustered data (i.e., effect sizes nested within studies or samples) by applying a correction to the study standard errors to account for the correlations between effect sizes from the same sample. When studies included multiple treatment arms, generally, the two treatments were aggregated for the analyses. In two of these studies, aggregating treatments did not make sense. In those cases, the most relevant treatment for this meta-analysis was chosen.

Robust variance estimation does not result in the loss of any information, does not require knowledge of the underlying correlation structure, and can accommodate multiple sources of dependencies. Thus, RVE was used to estimate the overall effect size and conduct moderator analyses. All RVE analyses were run in Stata version 16.0 (StataCorp, 2019) using the *ROBUMETA* package (Hedberg, 2014). Because RVE requires an estimate of the mean correlation ($\rho$) between all pairs of effect sizes within a cluster (i.e., study) to be specified, we first conducted sensitivity analyses using $\rho$ values of 0 to .90. Results showed that findings were robust across differing reasonable estimates of $\rho$. Thus, we estimated $\tau^2$ using a value of .80 (Tanner-Smith & Tipton, 2013).

We used the intercept from the RVE meta-regression models to estimate the mean effect sizes and examined heterogeneity by using $\tau^2$ as the between-study variance component. All analyses used random effects modeling to account for the expected heterogeneity in effect sizes. We also tested for moderators using a meta-regression approach. We estimated a series of both multivariate and univariate random-effects meta-regression models using *ROBUMETA* (Hedberg, 2014) to summarize effect sizes and examine associations between effect sizes and the different candidate moderators.

To answer research question two, a multivariate random-effects RVE meta-regression was conducted to assess whether the specific instructional components in interventions (explicit instruction, representations, strategic prompting tools, cumulative review, teaching and use of mathematical language, and timed fluency-building activities) moderate the impacts on mathematical knowledge.

Methodological and procedural characteristics of studies are often at risk of confounding their substantive features and—by distorting measures of association of meta-regressions—risk leading to incorrect conclusions (Deeks et al., 2019). To address this, we used variables we believe may be potential confounders (i.e., group size, grade level, intervention duration, and the nature of the comparison condition) as statistical controls in the moderator analyses using multivariate meta-regression models.

We explored the moderating effects of the use of number lines in instruction for our third research question. To do so, we first ran a univariate meta-regression testing number line individually, then we ran a multivariate meta-regression including the four potential confounding variables to adjust for their potential influence.

Last, we applied a series of univariate random-effects meta-regression models to explore research question four. For variables with more than two categories, one was chosen as the *reference category*, or the category to which the other categories were compared. Univariate analyses were conducted due to the large number of variables of interest (three study-level, four intervention, and two outcome characteristics). While multivariate models are ideal, many models that included multiple moderators resulted in fewer than four degrees of freedom, which likely underestimates the true Type 1 error (Tipton, 2015). To avoid this, a series of univariate RVE meta-regression models were run as exploratory analyses. However, results should be interpreted with caution because other moderators that are unaccounted for may have potential confounding effects.

### *Publication Bias*

Extensive sensitivity analyses were conducted to assess the potential presence of publication bias within our sample. First, we visually inspected the symmetry of the funnel plot with pseudo 95% confidence limits. Next, to statistically test for funnel plot asymmetry, we conducted Egger regression tests (Egger et al., 1997). Finally, we conducted trim-and-fill analyses (Duval & Tweedie, 2000) to adjust for potential publication bias by correcting for funnel plot asymmetry. All analyses were run in Stata version 16.0 (StataCorp, 2019) using the *metafunnel*, *metabias*, and *metatrim* commands.

### *Outliers*

To minimize the potential bias due to disproportionate influence of effect size outliers on the meta-analysis results, outliers were Winsorized to less extreme values (Lipsey & Wilson, 2001). We conducted an 80% Winsorization, which modified 10% of potential outliers from each tail area. Using this method, four studies were identified as outliers and were assigned effect estimates closer to other effect estimates in the set. We also re-ran the meta-analyses with these four outliers removed from the meta-analysis and found no substantive difference from the Winsorized results. As such, we conducted subsequent analyses with outliers included and Winsorized.

### *Missing Data*

To meet eligibility inclusion criteria, studies were required to provide sufficient information for estimation of a pretest and posttest effect size on at least one relevant outcome measure. As a result, there were no missing data in the outcomes of interest. There were, however, missing data for some of the coded intervention components and study features. Our sample size was not large enough to conduct any defensible imputation of missing data. Instead, we first queried authors for missing data and then, in the event of non-response from author queries, we used listwise deletion.

This meta-analysis adheres to the Preferred Reporting Items for Systematic Review and Meta-Analysis (PRISMA) statement (Moher et al., 2009). The methods were based on a protocol developed prior to data collection and analysis (Newman-Gonchar, et al., 2019).

## 4. Results

### 4.1 Search and Screening Results

We concluded our literature search in August 2019. See Figure 1 for a flow diagram detailing the eligibility coding for the 1,424 candidate reports yielded from our initial and extended search procedures. Following title and abstract screening, a majority of reports ($n$ = 1,347) were excluded, leaving 77 reports to be screened at the full-text level for final eligibility status. After full-text screening, 51 ineligible reports were excluded, primarily due to: (a) a participant group that did not include a sufficient proportion of students with or at risk for difficulties in mathematics; (b) failure to meet WWC group design standards; (c) lack of relation of intervention content to rational number concepts or operations; (d) ineligibility of research design; or (e) absence of eligible outcomes in the study. A total of 26 studies with 4,237 unique participants from 30 independent samples were deemed eligible for inclusion in the final meta-analysis. These samples provided a total of 115 effect sizes.

Overall, 97% of the student sample for included studies (25 of the 26 included studies; $n$ = 4,109) was identified as with or at risk for difficulties in mathematics either through a study-implemented screening process or formal identification of a disability. The exception (Delacruz, 2011; $n$ = 128), did not identify students through a screening process but targeted at-risk students by only including schools determined to primarily comprise students from "groups that historically do poorly in math" (p. 27).

**4.2 Study Features**

Table 1 shows descriptive statistics for some of the study features of the included studies. Sample sizes ranged from 22 to 755 students, with publication dates ranging from 1993 to 2019. Median publication year was 2014, and the majority of studies ($n$ = 16) were published during or after 2014. Of the included studies, 21 were journal articles, three were dissertations, and two were technical reports.

Duration of initial interventionist training (prior to the beginning of intervention implementation) was more than one day (8 hours) for the majority of studies ($n$ = 15). Five studies reported initial interventionist training duration to be less than one day, and the remaining studies ($n$ = 6) either reported that no training was conducted or did not report on the presence or absence of initial training. Ongoing training (i.e., interventionist training and implementation monitoring/feedback that continued throughout the duration of intervention implementation) was reported in half of the studies ($n$ = 13).

Twenty studies were appraised to meet WWC standards without reservations (i.e., RCTs with low attrition). The remaining six studies met WWC standards with reservations (i.e., QEDs or RCTs with high attrition and/or a lack of equivalence at baseline). With the exception of Turner (2012), a QED, all studies were RCTs. The sample of studies was divided approximately evenly between students in Grades 3 through 6 ($n$ = 14) and Grades 7 through 9 ($n$ = 12).

The 26 studies included in our analysis represent interventions that differ on several important characteristics. Intervention descriptions with study features related to implementation are presented in Table 2. These include the types of interventionists, the content type and level, group size, duration, instructional setting, and extent of technology use during the intervention.

Overall, the majority of studies included interventions lasting more than 20 hours ($n$ = 13); these interventions were delivered by research program staff ($n$ = 12) and included no use of technology ($n$ = 17). Intervention content focused on fractions-only instruction in 17 studies while the remaining nine studies included rations, proportions or fractions. The majority of intervention content was developed to target foundational knowledge combined with skills related to grade-level content (i.e., *integrated content*; $n$ = 13) for students in a small-group setting ($n$ = 14).

**4.3 Overall Intervention Effect**

Table 3 provides key features of the samples and outcomes along with outcome-level effect sizes. The estimate of the mean effect size across all 26 studies (115 effect sizes) included in the analysis was 0.65 and differed significantly from zero ($p < .001$, 95% CI [0.50, 0.80]). Figure 3 shows the corresponding forest plot for the 26 effect sizes. The forest plot shows the average effect size of each study (outliers Winsorized) as the dot in the center of each line (which represents the length of the 95% confidence interval). The $\tau^2$ estimate of the true variance in the population of effects is 0.01. These results address research question one and indicate that students receiving rational number intervention report significantly larger impacts in relevant aspects of mathematics performance than those in control conditions.

**4.4 Efficacy of Instructional Components**

To better understand the instructional components that relate to the efficacy of rational number interventions as posed in research question two, we examined each instructional component for which we could identify 10 or more studies—thus excluding the instructional component category of student explanations from the analysis due to an inadequate number of studies (e.g., Borenstein et al., 2009). Many of the studies included instructional components that were nonexclusive (i.e., overlapping). That may mean that the studies have included both the use of timed fluency-building activities and cumulative review. See Table 7 for the instructional components included in each study.

We used a multivariate meta-regression model with RVE to assess the possible moderating effects of instructional components. The model included all instructional components simultaneously and controlled for important study and intervention characteristics within a single model (Pigott & Polanin, 2019).

As indicated by the mean effect sizes (i.e., the intercepts from the RVE meta-regression model; see Table 5), the majority of the instructional components were non-significant at $p < 0.05$, with the exception of the teaching and use of mathematical language ($p < .047$). Ten studies with 62 effect sizes reflected interventions including the teaching and use of mathematical language, indicating that studies including this instructional component were associated with significantly larger positive effects on the outcomes.

To better understand the role of the number line in rational number instruction we explored whether its use explained any variability while also including important controls in the meta-regression model (i.e., group size, grade level, intervention duration, and the nature of the comparison condition). We found no significant relationship ($b = 0.68$, $p = 0.07$, 95% CI [−0.13, 1.48]). However, degrees of freedom were less than four, which likely underestimates the true Type 1 error (Tipton, 2015). A sensitivity analysis examining the meta-regression results without the presence of the control variables yielded significant results ($p < 0.0001$).

**4.5 Exploratory Moderator Analyses of Study Features**

Using univariate meta-regression models, we explored 10 categorial moderator variables related to study and intervention characteristics as raised in research question three. We used univariate models in lieu of recommended simultaneous, multivariate models due to a lack of power resulting from the relatively small number of studies and the moderately large set of multivariate moderators. We found some evidence of moderator effects related to intervention components (see Table 6). Here we discuss only those moderators significant at a Bonferroni corrected critical $p$ value of 0.005.

In the outcome domain moderator category, outcomes in the *rational number magnitude* domain and the *fractions addition and subtraction* domain had significantly larger effect sizes than outcomes in the *knowledge of rational numbers topics* domain (the referent group). In this outcome domain analysis, we excluded the domain of general mathematics achievement because only two studies had outcomes in this outcome domain. We found a significant relationship ($p < 0.005$) between grade level and effect size: interventions for students in elementary grades (3 through 6) had larger effects than those for students in middle school (7 through 9). Interventions delivered to small groups had significantly larger ($p < 0.005$) effects than those delivered in large-group settings.

Interventions delivered by research project personnel were significantly more effective ($p < 0.005$) than those delivered by school personnel. We also found that interventions longer than nine hours (i.e., interventions 10–19 hours and interventions 20 hours or longer) were more effective than shorter interventions (0–9 hours). However, only the specific comparison with the category of interventions lasting 20 hours or longer was statistically significant ($p < 0.005$). Finally, interventions for which the interventionists participated in ongoing training were significantly more effective than those without ($p < 0.005$).

We then conducted three supplemental moderator analyses using multivariate meta-regression models which included all moderator variables in single models to explore the three outcome domains with the largest number of effect sizes: knowledge of rational numbers topics, rational numbers magnitude, and fraction addition and subtraction. Results from all three separate multivariate meta-regressions, along with the univariate model, indicated that the presence of ongoing training in an intervention remains a particularly strong moderator associated with improved effectiveness of intervention.

**4.6 Publication Bias**

To assess research quality, we conducted extensive sensitivity analyses to assess the potential presence of publication bias within our sample. The funnel plot with pseudo 95% confidence limits for the study-level effect sizes is shown in Figure 2. Visual inspection of the funnel plot suggests the potential absence of studies with large effects and small sample sizes. However, results from Egger regression tests (Egger et al., 1997) were non-significant and provided no evidence of small study bias ($b = 0.37$, $p = 0.059$). Because the Egger's test can be underpowered in small samples, these results are not confirmatory of the absence of publication bias but do suggest that its presence is unlikely. Trim-and fill analyses (Duval & Tweedie, 2000) were also conducted to adjust for potential publication bias. Estimated mean effects were attenuated, but results, again, provided no strong evidence of publication bias, such that the average effect sizes for intervention were substantively unchanged after the trim-and-fill procedure ($g = 0.613$, $p < 0.001$). Taken together, we can conclude that publication bias is likely only a minor concern for this set of studies.

### 4.7 Risk of Bias

Studies that did not adequately address potential sources of bias failed to meet the WWC standards and were excluded from the meta-analysis ($n$ = 17). We also used the Cochrane Collaboration tool for assessing risk of bias to assess potential deviations from the intended interventions. The tool assesses additional sources of variance not covered by applying WWC review standards, such as allocation concealment or blinding of assessment administration. The sample of studies included in this review would be considered of *some concern* due to potential deviations from intended interventions. Studies in our sample did not report bias protections for these sources of bias due to the nature of school-based research, within which it is most often not possible to ensure participant or researcher blinding.

## 5. Summary

Findings from this study provide valuable information that may help educators understand effective intervention components for students experiencing mathematics difficulties and the conditions under which intervention is optimal. Specifically, the positive impact for teaching and using of mathematical language may guide schools and districts in choosing interventions that include this practice. Also, the impact for interventionists receiving high-quality, ongoing training may guide districts when making decisions on how to provide training and professional development to teachers who deliver intervention. Providing evidence-based intervention on rational numbers topics addresses the ever-widening achievement gap in mathematics between low- and high-performing students as they prepare for high-stakes courses like Algebra 1. Findings from this project should be considered as schools make important decisions about the provision of mathematics services to students experiencing difficulties in mathematics.

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Table 1

*Study Features*

| Author (year) | Design | Grade Level | Quality Rating | Initial Training | Ongoing Training | Sample Size (Total *N*) | Publication Type |
|---|---|---|---|---|---|---|---|
| Adams et al. (2014) | RCT | 7th–9th | MWOR | None | None | 107 | Journal |
| Barbieri et al. (2019) | RCT | 3rd–6th | MWR | > 1 day | None | 51 | Journal |
| Bottge et al. (1993) | RCT | 7th–9th | MWOR | None | None | 29 | Journal |
| Bottge et al. (2007) | C-RCT | 7th–9th | MWOR | > 1 day | None | 90 | Journal |
| Bottge et al. (2010) | C-RCT | 7th–9th | MWOR | < 1 day | None | 54 | Journal |
| Bottge et al. (2014) | C-RCT | 7th–9th | MWOR | > 1 day | None | 317 | Journal |
| Bottge et al. (2015) | C-RCT | 7th–9th | MWOR | > 1 day | None | 123 | Journal |
| Butler et al. (2003) | C-RCT | 7th–9th | MWOR | < 1 day | Yes | 50 | Journal |
| Delacruz (2011) | RCT | 3rd–6th | MWR | None | None | 128 | Report |
| Dyson et al. (2018) | RCT | 3rd–6th | MWOR | < 1 day | Yes | 52 | Journal |
| Fuchs et al. (2013) | RCT | 3rd–6th | MWOR | > 1 day | Yes | 164 | Journal |
| Fuchs et al. (2014) | RCT | 3rd–6th | MWOR | > 1 day | Yes | 259 | Journal |
| Fuchs et al. (2016a) | RCT | 3rd–6th | MWOR | > 1 day | Yes | 139 | Journal |
| Fuchs et al. (2016b) | RCT | 3rd–6th | MWOR | > 1 day | Yes | 142 | Journal |
| Fuchs et al. (2019) | RCT | 3rd–6th | MWOR | > 1 day | Yes | 97 | Journal |
| Hughes (2011) | RCT | 7th–9th | MWR | < 1 day | Yes | 35 | Dissertation |
| Jayanthi et al. (2018) | RCT | 3rd–6th | MWOR | > 1 day | Yes | 185 | Report |
| Jitendra et al. (2016) | C-RCT | 3rd–6th | MWOR | > 1 day | None | 260 | Journal |
| Jitendra et al. (2017) | C-RCT | 3rd–6th | MWR | > 1 day | None | 755 | Journal |
| Malone et al. (2019) | RCT | 3rd–6th | MWOR | > 1 day | Yes | 152 | Journal |
| McLaren et al. (2015) | RCT | 7th–9th | MWOR | None | None | 200 | Journal |
| Turner (2012) | QED | 7th–9th | MWR | NR | None | 88 | Dissertation |
| Wang et al. (2019) | RCT | 3rd–6th | MWOR | > 1 day | Yes | 58 | Journal |
| Watt et al. (2016) | RCT | 7th–9th | MWR | < 1 day | None | 32 | Journal |
| Westenskow (2012) | RCT | 3rd–6th | MWOR | NR | Yes | 43 | Dissertation |
| Xin et al. (2005) | RCT | 7th–9th | MWOR | < 1 day | Yes | 22 | Journal |

*Note.* C-RCT = cluster RCT; MWR = meets WWC standards with reservations; MWOR = meets WWC standards without reservations; 1 day = 8 hours; NR = not reported.

Table 2

*Intervention Descriptions and Study Features*

| Author (year) | Int. | Content | Intervention Description | Grouping | Duration | Instructional Setting | Content Level | Tech. Usage |
|---|---|---|---|---|---|---|---|---|
| Adams et al. (2014) | None (Virtual) | D | Web-based tutoring; students critiqued incorrect solutions to decimal problems *Comparison*: Students solved problems with feedback from web-based tutoring | Ind. | ≤ 9 hours | Supplemental Intervention | Integrated | Yes (Full) |
| Barbieri et al. (2019) | Research Project Personnel | F | Fraction intervention centered on the number line *Comparison*: BAU | SG | ≥ 20 hours | Supplemental Intervention | Foundational | None |
| Bottge et al. (1993) | Research Project Personnel and School-Based Personnel | F | Students learned problem-solving skills to solve contextualized problems *Comparison*: Students learned problem-solving skills to solve arithmetic word problems | SG | ≤ 9 hours | SpEd | At Grade | Yes (Partial) |
| Bottge et al. (2007) | School-Based Personnel | D, R/R | Enhanced Anchored Instruction (EAI); intervention focused on applied problem solving *Comparison*: BAU | LG | ≥ 20 hours | SpEd | At Grade | Yes (Partial) |
| Bottge et al. (2010) | School-Based Personnel plus virtual | F | Formal fractions instruction plus (EAI) *Comparison*: Informal fractions instruction plus (EAI) | LG | ≥ 20 hours | SpEd | At Grade | Yes (Partial) |
| Bottge et al. (2014) | School-Based Personnel | F, D, Per, Pro, R/R | Enhanced Anchored Instruction (EAI); intervention focused on applied problem solving *Comparison*: BAU | LG | ≥ 20 hours | SpEd | Integrated | Yes (Partial) |

| Author (year) | Int. | Content | Intervention Description | Grouping | Duration | Instructional Setting | Content Level | Tech. Usage |
|---|---|---|---|---|---|---|---|---|
| Bottge et al. (2015) | School-Based Personnel | F, D, Per, Pro, R/R | Enhanced Anchored Instruction (EAI); intervention focused on applied problem solving *Comparison*: BAU | LG | ≥ 20 hours | General Classroom | Integrated | Yes (Partial) |
| Butler et al. (2003) | School-Based Personnel | F | Concrete-representational-abstract instructional sequence (CRA) *Comparison*: Representational-abstract instruction only | LG | ≤ 9 hours | SpEd | Foundational | None |
| Delacruz (2011) | None (virtual) | F | Computer-based pre-algebra math game with tutorials and feedback *Comparison*: Computer-based math game with different learning objectives than treatment group | Ind. | ≤ 9 hours | Supplemental Intervention | Foundational | Yes (Full) |
| Dyson et al. (2018) | Research Project Personnel | F | Fractions-sense intervention centered on the number line *Comparison*: BAU | SG | 10−19 hours | Supplemental Intervention | Foundational | None |
| Fuchs et al. (2013) | Research Project Personnel | F | Fraction Challenge; intervention focused on the measurement interpretation of fractions *Comparison*: BAU | SG | 10−19 hours | Supplemental Intervention | Integrated | None |
| Fuchs et al. (2014) | Research Project Personnel | F | Fraction Face-Off!; intervention focused on the measurement interpretation of fractions with two variants: fluency or conceptual activities *Comparison*: BAU | SG | 10−19 hours | Supplemental Intervention | Integrated | None |

| Author (year) | Int. | Content | Intervention Description | Grouping | Duration | Instructional Setting | Content Level | Tech. Usage |
|---|---|---|---|---|---|---|---|---|
| Fuchs et al. (2016a) | Research Project Personnel | F | Fraction Face-Off!; intervention focused on the measurement interpretation of fractions with two variants: word problem solving or providing explanations<br>*Comparison*: BAU | SG | ≥ 20 hours | Supplemental Intervention | Integrated | None |
| Fuchs et al. (2016b) | Research Project Personnel | F | Fraction Face-Off!; intervention focused on the measurement interpretation of fractions with two variants: additive or multiplicative word problems<br>*Comparison*: BAU | SG | ≥ 20 hours | Supplemental Intervention | Integrated | None |
| Fuchs et al. (2019) | Research Project Personnel | F | Super Solvers; intervention focused on fractions magnitude with two variants: fractions calculations and error analysis<br>*Comparison*: BAU | SG | ≥ 20 hours | Supplemental Intervention | Integrated | None |
| Hughes (2011) | School-Based Personnel | F | Concrete-representational-abstract instructional sequence (CRA)<br>*Comparison*: BAU | WC | 10−19 hours | General Classroom | Foundational | None |
| Jayanthi et al. (2018) | Research Project Personnel | F | Adapted TransMath®; intervention focused on foundational fractions concepts and procedures with a focus on student explanations<br>*Comparison*: BAU | SG | ≥ 20 hours | Supplemental Intervention | Integrated | None |
| Jitendra et al. (2016) | School-Based Personnel | Per, Pro, R/R | Schema-Based Instruction (SBI); students learned proportional problem-solving<br>*Comparison*: BAU | LG | ≥ 20 hours | General Classroom | At Grade | None |
| Jitendra et al. (2017) | School-Based Personnel | Per, Pro, R/R | Schema-Based Instruction (SBI); students learned proportional problem-solving<br>*Comparison*: BAU | LG | 10−19 hours | General Classroom | At Grade | None |

| Author (year) | Int. | Content | Intervention Description | Grouping | Duration | Instructional Setting | Content Level | Tech. Usage |
|---|---|---|---|---|---|---|---|---|
| Malone et al. (2019) | Research Project Personnel | F, D | Fraction Face-Off!; intervention focused on fraction and decimal magnitudes *Comparison*: BAU | SG | ≥ 20 hours | Supplemental Intervention | Integrated | None |
| McLaren et al. (2015) | None (virtual) | D | Web-based; students found, explained, and fixed errors in decimal problems *Comparison*: Web-based; students solved the same decimal problems and explained their solutions | Ind. | ≤ 9 hours | Supplemental Intervention | Integrated | Yes (Full) |
| Turner (2012) | School-Based Personnel | F | Constructivist-based fraction intervention *Comparison*: BAU | LG | ≥ 20 hours | General Classroom | Foundational | None |
| Wang et al. (2019) | Research Project Personnel | F | Super Solvers; intervention focused on fractions magnitude and word problems with two variants: with or without self-regulation *Comparison*: BAU | SG | ≥ 20 hours | Supplemental Intervention | Integrated | None |
| Watt et al. (2016) | Research Project Personnel | F | Preteaching fraction computations using the concrete-representational-abstract (CRA) instructional sequence *Comparison*: Students received supplemental reading instruction | SG | ≤ 9 hours | Supplemental Intervention | Integrated | None |
| Westenskow (2012) | Research Project Personnel | F | Students learned equivalent fractions concepts using physical manipulatives *Comparison*: Students learned the same concepts using virtual manipulatives | SG | ≤ 9 hours | Supplemental Intervention | Foundational | Yes (Partial) |

| Author (year) | Int. | Content | Intervention Description | Grouping | Duration | Instructional Setting | Content Level | Tech. Usage |
|---|---|---|---|---|---|---|---|---|
| Xin et al. (2005) | Research Project Personnel and School-Based Personnel | F, Pro, R/R | Schema-based instruction (SBI); students learned mathematical word-problem solving *Comparison*: Student learned mathematical word-problem solving using general strategy instruction (GSI) | SG | 10−19 hours | SpEd | At Grade | None |

*Note.* All studies with individual grouping are also computer-based (virtual) interventions. CRA (concrete-representational-abstract) is equivalent to CSA (concrete-semiconcrete-abstract). F = fractions; D = decimals; Per = percentages; Pro = proportions; R/R = rates and ratios; Ind. = individual; Int. = interventionist; SG = small group; LG = large group; BAU = business-as-usual.

Table 3

*Outcome Characteristics*

| Author(s) (year) | TX n | CON n | Outcome Domain | Measure Type | Outcome-Level Effect Sizes (SE) |
|---|---|---|---|---|---|
| Adams et al. (2014) | 44 | 63 | KN | IND | 0.58 (0.20), 0.30 (0.20) |
| Barbieri et al. (2019) | 28 | 23 | FR | RD | 0.29 (0.28), 0.28 (0.28) |
| | 28 | 23 | KN | IND | 1.28 (0.31), 0.78 (0.29) |
| Bottge et al. (1993) | 15 | 14 | PS | RD | 1.07 (0.40) |
| | 15 | 14 | WP | RD | 0.52 (0.38), 0.07 (0.37) |
| Bottge et al. (2007) | 48 | 42 | PS | RD | 1.06 (0.22) |
| Bottge et al. (2010) | 29 | 25 | FR | RD | 0.93 (0.29) |
| | 29 | 25 | PS | IND | −0.03 (0.27) |
| | 29 | 25 | CO | IND | −0.07 (0.27) |
| | 29 | 25 | PS | RD | 0.01 (0.27) |
| Bottge et al. (2014) | 154 | 171 | PS | IND | 0.21 (0.11) |
| | 146 | 171 | FR | IND | 0.55 (0.11) |
| | 153 | 171 | PS | RD | 0.29 (0.11) |
| Bottge et al. (2015) | 56 | 67 | FR | RD | 0.72 (0.19) |
| | 56 | 70 | PS | IND | 0.36 (0.18) |
| | 55 | 68 | FR | IND | 0.31 (0.18) |
| | 62 | 70 | PS | RD | 0.47 (0.18) |
| Butler et al. (2003) | 26 | 24 | MAG | IND | 0.04 (0.28) |
| | 26 | 24 | KN | IND | −1.99 (0.28), 0.92 (0.30) |
| | 26 | 24 | WP | RD | 0.07 (0.28) |
| | 26 | 24 | KN | RD | 0.46 (0.28) |
| Delacruz (2011) | 112 | 16 | KN | RD | 0.17 (0.27) |
| Dyson et al. (2018) | 25 | 27 | FR | RD | 0.48 (0.28), 0.25 (0.28) |
| | 25 | 27 | KN | IND | 0.99 (0.29), 0.63 (0.28) |
| | 25 | 27 | MAG | IND | 0.92 (0.29), 1.03 (0.30) |
| Fuchs et al. (2013) | 129 | 130 | MAG | RD | 1.87 (0.15) |
| | 129 | 130 | FR | RD | 2.49 (0.17) |
| | 129 | 130 | MAG | IND | 1.03 (0.13) |
| | 129 | 130 | KN | IND | 0.92 (0.13) |
| Fuchs et al. (2014) | 79 | 80 | CO | RD | 1.41 (0.21) |
| | 79 | 80 | MAG | IND | 1.22 (0.39) |
| | 79 | 80 | KN | IND | 0.64 (0.31) |
| | 84 | 80 | FR | RD | 1.44 (0.17) |
| | 84 | 80 | MAG | IND | 0.98 (0.17) |
| | 84 | 80 | KN | IND | 0.63 (0.16) |

| Author(s) (year) | TX n | CON n | Outcome Domain | Measure Type | Outcome-Level Effect Sizes (SE) |
|---|---|---|---|---|---|
| Fuchs et al. (2016a) EXP Contrast | 73 | 70 | FR | RD | 1.91 (0.23) |
| | 73 | 70 | KN | IND | 0.59 (0.32) |
| | 73 | 70 | MAG | RD | 1.34 (0.29), 1.16 (0.17) |
| | 73 | 70 | MAG | IND | 0.62 (0.32) |
| | 73 | 70 | WP | RD | -0.11 (0.20) |
| Fuchs et al. (2016a) WP Contrast | 69 | 70 | FR | RD | 1.96 (0.21) |
| | 69 | 70 | KN | IND | 0.74 (0.17) |
| | 69 | 70 | MAG | RD | 0.57 (0.18), 0.88 (0.18) |
| | 69 | 70 | MAG | IND | 0.47 (0.17) |
| | 69 | 70 | WP | RD | 1.18 (0.17) |
| Fuchs et al. (2016b) AWP Contrast | 71 | 70 | FR | RD | 1.68 (0.17) |
| | 71 | 70 | KN | IND | 0.32 (0.17) |
| | 71 | 70 | MAG | IND | 0.62 (0.17) |
| | 71 | 70 | WP | RD | 1.35 (0.17), 0.08 (0.17) |
| Fuchs et al. (2016b) MWP Contrast | 72 | 70 | FR | RD | 1.19 (0.18) |
| | 72 | 70 | KN | IND | 0.41 (0.17) |
| | 72 | 70 | MAG | IND | 0.93 (0.18) |
| | 72 | 70 | WP | RD | 0.99 (0.18), 0.89 (0.17) |
| Fuchs et al. (2019) SS Contrast | 46 | 52 | FR | RD | 1.86 (0.26) |
| | 46 | 52 | KN | IND | −0.04 (0.27) |
| | 46 | 52 | MAG | IND | 1.56 (0.37) |
| | 46 | 52 | MAG | RD | 1.26 (0.23) |
| Fuchs et al. (2019) SSE Contrast | 45 | 52 | FR | RD | 1.99 (0.24) |
| | 45 | 52 | KN | IND | 0.10 (0.20) |
| | 45 | 52 | MAG | IND | 1.42 (0.23) |
| | 45 | 52 | MAG | RD | 1.33 (0.22) |
| Hughes (2011) | 20 | 15 | CO | RD | 0.16 (0.34), 1.02 (0.36) |
| Jayanthi et al. (2018) | 86 | 99 | MAG | IND | 1.08 (0.16), 0.79 (0.15) |
| | 86 | 99 | KN | IND | 0.66 (0.15) |
| | 87 | 99 | KN | IND | 0.78 (0.15) |
| | 86 | 99 | CO | IND | 1.06 (0.16) |
| Jitendra et al. (2016) | 149 | 111 | WP | RD | 0.40 (0.13), 0.42 (0.13), 0.08 (0.13) |
| Jitendra et al. (2017) | 381 | 374 | WP | RD | 0.26 (0.07) |
| | 372 | 357 | WP | RD | 0.21 (0.07) |

| Author(s) (year) | TX n | CON n | Outcome Domain | Measure Type | Outcome-Level Effect Sizes (SE) |
|---|---|---|---|---|---|
| Malone et al. (2019) | 76 | 76 | FR | RD | 1.54 (0.19) |
| | 76 | 76 | KN | IND | 0.21 (0.16) |
| | 76 | 76 | MAG | IND | 0.47 (0.17), 0.81 (0.17) |
| | 76 | 76 | MAG | RD | 0.39 (0.16), 1.58 (0.18) |
| | 76 | 76 | WP | RD | 0.49 (0.17) |
| McLaren et al. (2015) | 98 | 102 | KN | IND | 0.27 (0.14), 0.20 (0.14) |
| Turner (2012) | 45 | 43 | GEN | IND | 0.88 (0.22) |
| | 45 | 43 | KN | IND | 0.82 (0.22) |
| Wang et al. (2019) SS Contrast | 26 | 29 | CO | RD | 0.57 (0.31) |
| | 26 | 29 | FR | RD | 1.05 (0.27) |
| | 26 | 29 | KN | IND | 1.10 (0.32) |
| | 26 | 29 | MAG | IND | 0.90 (0.30) |
| | 26 | 29 | MAG | RD | 1.01 (0.30) |
| | 26 | 29 | WP | RD | 0.96 (0.28) |
| Wang et al. (2019) SS-SR Contrast | 29 | 29 | CO | RD | 0.84 (0.29) |
| | 29 | 29 | FR | RD | 0.97 (0.28) |
| | 29 | 29 | KN | IND | 0.70 (0.28) |
| | 29 | 29 | MAG | IND | 0.85 (0.28) |
| | 29 | 29 | MAG | RD | 1.25 (0.28) |
| | 29 | 29 | WP | RD | 0.91 (0.28) |
| Watt et al. (2016) | 17 | 15 | GEN | IND | 0.11 (0.35) |
| Westenskow (2012) | 15 | 14 | KN | RD | −0.02 (0.38) |
| | 15 | 14 | MAG | RD | 0.20 (0.37), 0.20 (0.37) |
| Xin et al. (2005) | 11 | 10 | WP | IND | 1.33 (0.46) |
| | 11 | 11 | WP | RD | 1.87 (0.49) |
| | 9 | 10 | WP | RD | 2.86 (0.62) |

*Note.* CO = fractions computation; FR = fractions addition and subtraction; GEN = general mathematics achievement; KN = knowledge of rational numbers topics; MAG = rational numbers magnitude; PS = problem solving; WP = word problems; IND = independent; RD = researcher developed.

Table 4

*Operational Definitions for Instructional Components and Study Features*

| Variable | Definition |
|---|---|
| Comparison | TvsC=Treatment vs Business-as-usual control; TvsT=Treatment vs. Alternative active treatment; T+TvsC=Aggregated treatments vs Business-as-usual control |
| Cumulative Review | Yes=Intervention provided evidence of cumulative review; No=Authors did not report including cumulative review |
| Design | RCT=Randomized controlled trial; QED=Quasi-experimental design; Cluster RCT=RCT in which groups of students were randomized; Cluster QED=QED in which groups of students were assigned to conditions |
| Duration | >9= Intervention was 9 hours in overall duration; 10−19=Intervention was between 10 and 19 hours; >20=Intervention lasted longer than 20 hours |
| Explicit Instruction | Yes=Intervention included clear explanatory language, used scripts, included direct teaching and modeling of concepts and procedures;<br>No=Intervention was constructivist in nature, computer-mediated intervention, discovery/explore description, student-driven rather than teacher driven |
| Timed Fluency Activities | Yes=Study included timed activities designed to build students' fluency;<br>No=Authors did not report including timed fluency-building activities |
| Grade Level | Elementary=Grades 3−6; Middle School=Grades 7−9 |
| Group Size | Small Group=Instruction was provided in small groups of 2−6 students; Large Group=Instruction was provided to a large group or whole class (greater than 6 students); No study provided one-on-one instruction delivered by an interventionist. The only instruction considered to be individualized was virtual (computer-based) |
| Initial Training | No=Interventionists were provided no initial training; <1 day=Interventionists participated in pre-intervention training lasting less than or equal to one day (8 hours); >1 day= Interventionists participated in pre-intervention training lasting greater than one day (8 hours), N/R=Presence or absence of initial training was not reported |
| Interventionist | RSCH=Research Project Personnel; SCH=School-Based Personnel |
| Measure Type | Researcher Developed=Developed by the research team to test the intervention; Independent=Standardized or norm-referenced measures developed independently of research project personnel. Contains specific standard instructions used when the measure is administered (usually not developed by the current researchers), or referenced data from a normative sample |
| Number Line | Yes=Intervention incorporated the use of the number line to support learning;<br>No=Authors did not report the use of a number line during intervention |
| Ongoing Training | Yes=Interventionists participated in periodic training that continued throughout the intervention; No=Interventionists participated in only initial training or received no training |

| Variable | Definition |
| --- | --- |
| Outcome Domain | Knowledge of rational number topics, Rational numbers magnitude, Fractions computation (including all four operations), Fractions addition and subtraction, Word problems, Problem solving, General mathematics achievement |
| Strategic Prompting Tools | Yes=Intervention included cue card or notes, a step-by-step strategy, a monitoring strategy, strategic prompt cards, strategic cards; No=Authors did not report using prompting tools during intervention |
| Representations | Yes=Intervention included use of concrete or semi-concrete representations of mathematical concepts such as fraction tiles, number lines, or shaded models; No=Authors did not report using concrete or semi-concrete representations to teach concepts |
| Technology | Yes=Intervention included the full or partial use of technology (i.e., computers); No=Intervention did not include the use of technology |
| Instructional Setting | SPED=Instruction was delivered in a special education setting including resource rooms; General Classroom=Instruction was delivered in core/general mathematics classes; Supplemental Intervention=Instruction was supplemental and provided one-on-one or in small groups |
| Teaching and use of mathematical language | Yes=Intervention included instruction on mathematical language such as learning mathematical terminology or novel vocabulary in word problems; No=Authors did not report teaching mathematical terminology |

Table 5

*Relationships Between Instructional Components and Effect Sizes Using Multivariate Meta-Regression Models*

| Variable | *k*(*n*) | *b* [95% CI] | SE | *df* | *p* | $\tau^2$ |
|---|---|---|---|---|---|---|
| Instructional Components | | | | | | 0.00 |
| Explicit | 19(101) | −0.34 [−0.87, 0.18] | 0.18 | 3.76[a] | 0.142[a] | |
| Representations | 23(108) | −0.38 [−1.73, 0.96] | 0.52 | 4.76 | 0.491 | |
| Strategic Prompting Tools | 14(87) | 0.18 [−0.27, 0.63] | 0.19 | 6.88 | 0.371 | |
| Cumulative Review | 14(79) | −0.22 [−0.59, 0.14] | 0.15 | 6.90 | 0.183 | |
| Teaching and Use of Mathematical Language | 10(62) | 0.57 [0.01, 1.14] | 0.23 | 6.53 | 0.047* | |
| Timed Fluency-Building Activities | 10(67) | 0.17 [−0.40, 0.74] | 0.14 | 2.13[a] | 0.337[a] | |
| Intervention and Design Characteristics (Controls) | | | | | | |
| Group Size (small group, large group) | | −0.34 [−1.04, 0.35] | 0.28 | 6.08 | 0.275 | |
| Grade Level (3rd–6th, 7th–9th) | | −0.20 [−1.17, 0.77] | 0.39 | 5.69 | 0.631 | |
| Duration (≤ 9 hrs, 10−19 hrs, ≥ 20 hrs) | | 0.09 [−0.16, 0.34] | 0.11 | 8.58 | 0.445 | |
| Comparison Condition (BAU, active alt. treatment) | | 0.02 [−0.65, 0.68] | 0.22 | 3.32[a] | 0.944[a] | |
| Number Line Representation | | | | | | 0.00 |
| Number line (with controls) | 10(74) | 0.68 [–0.13, 1.48] | 0.19 | 1.98[a] | 0.069 | |
| Number line (without controls) | | 0.52 [0.34, 0.69] | 0.08 | 22.89 | 0.000* | |

*Note.* The assumed average intercorrelation across all variables, rho ($\rho$), is set at .80. A multivariate meta-regression model (simultaneous model) was estimated using robust variance estimation (RVE) to handle statistically dependent effect sizes. *k* = number of studies; *n* = number of effect sizes; CI = confidence interval.

[a] Degrees of freedom is less than four, thus the *p*-value is untrustworthy due to small sample size (Tipton, 2015).

* $p < .05$.

Table 6

*Exploratory Analyses: Relationships Between Study Features and Effect Sizes*

| Moderator | *k*(*n*) | *b* [95% CI] | SE | df | *p* | $\tau^2$ |
|---|---|---|---|---|---|---|
| Type of Measure[a] | | | | | | 0.02 |
| Researcher Developed | 21(62) | 0.10 [−0.16, 0.35] | 0.12 | 24.27 | 0.44 | |
| Independent | 19(53) | 0 (Ref) | | | | |
| Outcome Domain[a] | | | | | | 0.00 |
| Knowledge of Rational Numbers | 16(28) | 0 (Ref) | | | | |
| Rational Numbers Magnitude | 11(31) | 0.47 [0.22, 0.72] | 0.12 | 16.85 | 0.00*** | |
| Fractions Computation | 5(7) | 0.24 [−0.20, 0.69] | 0.17 | 4.32 | 0.21 | |
| Fractions Addition and Subtraction | 12(19) | 0.52 [0.14, 0.90] | 0.18 | 17.03 | 0.01* | |
| Word Problems | 9(20) | −0.02 [−0.39, 0.35] | 0.16 | 9.18 | 0.89 | |
| Problem Solving | 5(8) | 0.06 [−0.57, 0.69] | 0.25 | 5.12 | 0.81 | |
| Grade Level | | | | | | 0.00 |
| Elementary (3rd–6th) | 12(78) | 0.43 [0.22, 0.63] | 0.10 | 22.56 | 0.00*** | |
| Middle School (7th–9th) | 14(37) | 0 (Ref) | | | | |
| Group Size | | | | | | 0.00 |
| Large Group (> 6) | 9(26) | 0 (Ref) | | | | |
| Small Group (2−6 students) | 14(84) | 0.47 [0.28, 0.66] | 0.09 | 21.38 | 0.00*** | |
| Interventionist | | | | | | 0.00 |
| Research Project Personnel | 12(78) | 0 (Ref) | | | | |
| School-Based Personnel | 9(26) | −0.43 [−0.67, −0.20] | 0.11 | 14.85 | 0.00*** | |
| Technology Usage | | | | | | 0.00 |
| None | 17(92) | 0 (Ref) | | | | |
| Partial or full | 9(23) | −0.33 [−0.61, −0.05] | 0.13 | 12.63 | 0.03* | |
| Duration | | | | | | 0.00 |
| ≤ 9 hours | 7(17) | 0 (Ref) | | | | |
| 10−19 hours | 6(23) | 0.41 [−0.28, 1.11] | 0.30 | 8.05 | 0.21 | |
| ≥ 20 hours | 13(75) | 0.45 [0.24, 0.66] | 0.09 | 8.61 | 0.00*** | |
| Initial Training | | | | | | 0.02 |
| None | 4(8) | 0 (Ref) | | | | |
| < 1 day | 5(17) | 0.23 [−0.25, 0.70] | 0.20 | 6.83 | 0.30 | |
| > 1 day | 15(85) | 0.37 [0.02, 0.72] | 0.13 | 4.19 | 0.04* | |
| Ongoing Training | | | | | | 0.00 |
| No | 13(32) | 0 (Ref) | | | | |
| Yes | 13(83) | 0.48 [0.29, 0.67] | 0.09 | 22.27 | 0.00*** | |
| Instructional Setting | | | | | | 0.00 |
| Special Education | 6(19) | 0 (Ref) | | | | |
| General Classroom | 5(13) | −0.18 [−0.65, 0.30] | 0.20 | 6.73 | 0.41 | |
| Supplemental Intervention | 15(83) | 0.19 [−0.26, 0.64] | 0.18 | 6.14 | 0.35 | |

[a] Meta-regression models were estimated at the outcome level, not the study level. The variables labeled "(Ref)" are the reference categories to which the other categories are being compared. *k* = number of studies; *n* = number of effect sizes; CI = confidence interval.
* $p < .05$. ** $p < .01$. *** $p < .005$ (the Bonferroni corrected critical *p* value).

Table 7

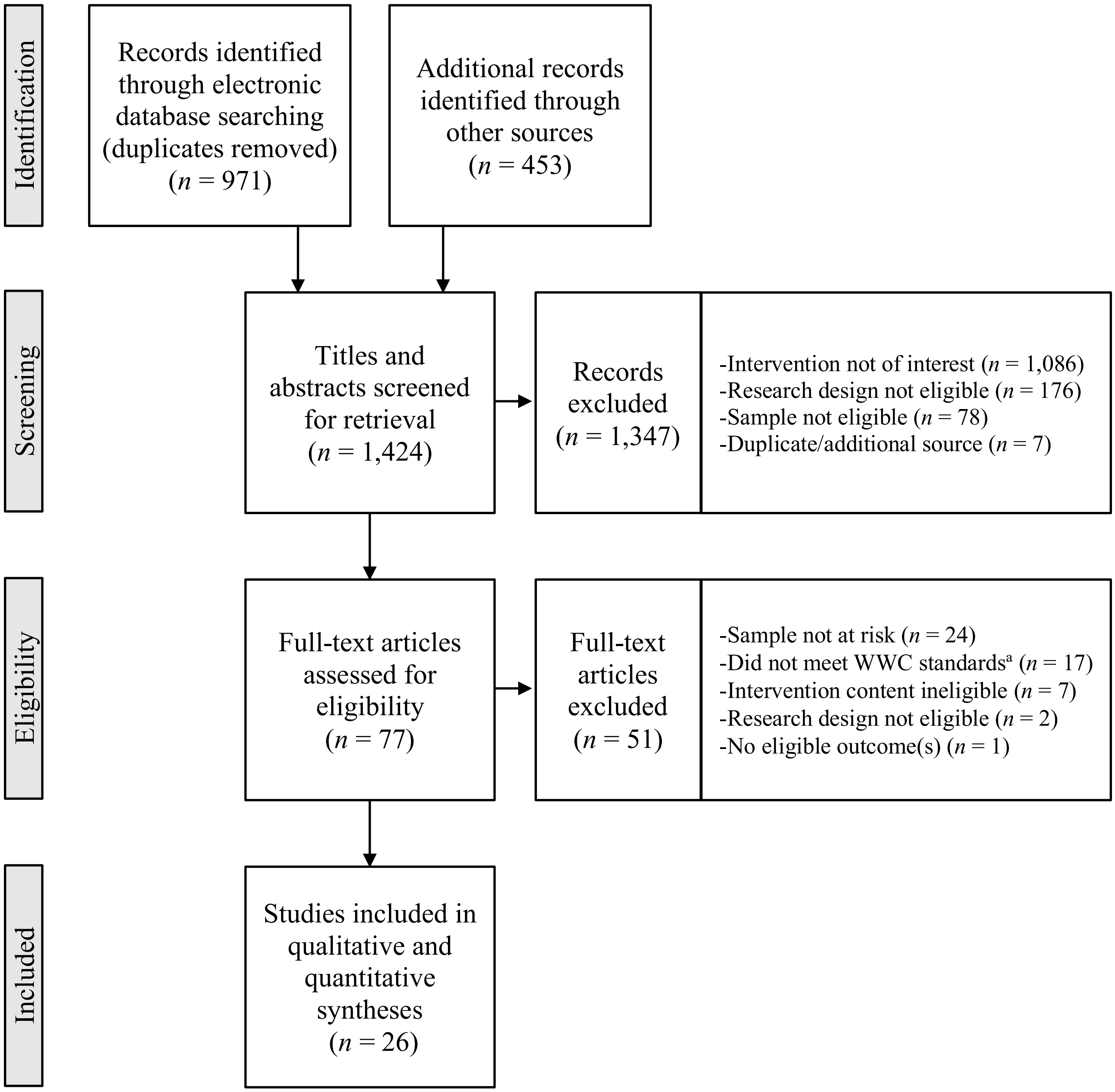
*Instructional Components Included in Each Study*

| Author | Instructional Components | | | | | |
|---|---|---|---|---|---|---|
| | Explicit Instruction | Representations | Strategic Prompting Tools | Cumulative Review | Teaching and Use of Mathematical Language | Timed Fluency-Building Activities |
| Adams et al. (2014) | | | | | | |
| Barbieri et al. (2019) | X | X[a] | X | | | X |
| Bottge et al. (1993) | | | | X | | |
| Bottge et al. (2007) | | X | | | | |
| Bottge et al. (2010) | X | X | | X | | |
| Bottge et al. (2014) | X | X | | | | |
| Bottge et al. (2015) | X | X | | | | |
| Butler et al. (2003) | X | X | X | | | |
| Delacruz (2011) | | X | | | | X |
| Dyson et al. (2018) | X | X[a] | X | | | X |
| Fuchs et al. (2013) | X | X[a] | X | X | X | X |
| Fuchs et al. (2014) | X | X[a] | X | X | X | X[b] |
| Fuchs et al. (2016a) | X | X[a] | X | X | X[b] | X |
| Fuchs et al. (2016b) | X | X[a] | X | X | X | X |
| Fuchs et al. (2019) | X | X[a] | X | X | X | X |
| Hughes (2011) | X | X | | X | | |
| Jayanthi et al. (2018) | X | X[a] | X | X | X | |
| Jitendra et al. (2016) | X | X | X | X | | |
| Jitendra et al. (2017) | X | X | X | X | | |
| Malone et al. (2019) | X | X[a] | X | X | X | X |
| McLaren et al. (2015) | | | | | | |
| Turner (2012) | | X | | | | |
| Wang et al. (2019) | X | X[a] | X | X | X | X |
| Watt et al. (2016) | X | X | | X | X | |
| Westenskow (2012) | | X | | | | |
| Xin et al. (2005) | X | X | X | | X | |

*Note*. [a] indicates that the study's use of representations included number lines. [b] indicates that some, but not all, contrasts of this study included the instructional component.

**Figure 1**

*Study identification flow diagram following PRISMA guidelines*




[a] The study is a randomized controlled trial with high attrition or a quasi-experimental design study with analysis groups that are not shown to be equivalent.

**Figure 2**

*Funnel Plot*

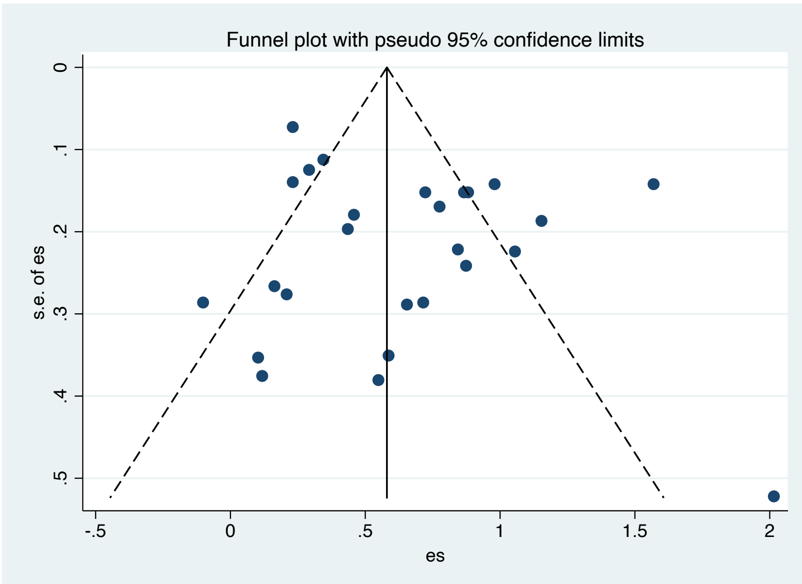


**Figure 3**

*Forest Plot*